Dr Oliver Mathematics
Mathematics: Higher
2013 Paper 2: Calculator
1 hour 10 minutes

The total number of marks available is 60.
You must write down all the stages in your working.

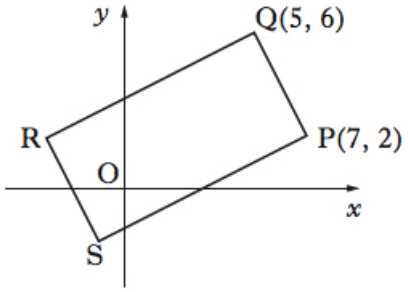
1. The first three terms of a sequence are 4, 7, and 16. (4)

   The sequence is generated by the recurrence relation

   $$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

   Find the values of m and c.

2. The diagram shows rectangle $PQRS$ with $P(7, 2)$ and $Q(5, 6)$.

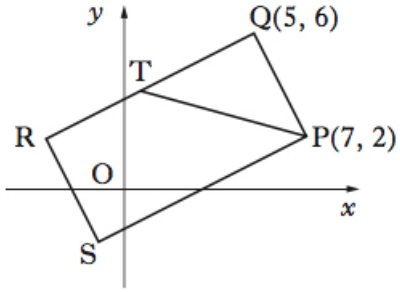


   (a) Find the equation of QR. (3)

   The line from P with the equation

   $$x + 3y = 13$$

   intersects QR at T.



(b) Find the coordinates of T. (3)

(c) Given that T is the midpoint of QR, find the coordinates of R and S. (3)

3. (a) Given that $(x-1)$ is a factor of (4)

$$x^3 + 3x^2 + x - 5,$$

factorise this cubic fully.

(b) Show that the curve with equation (5)

$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

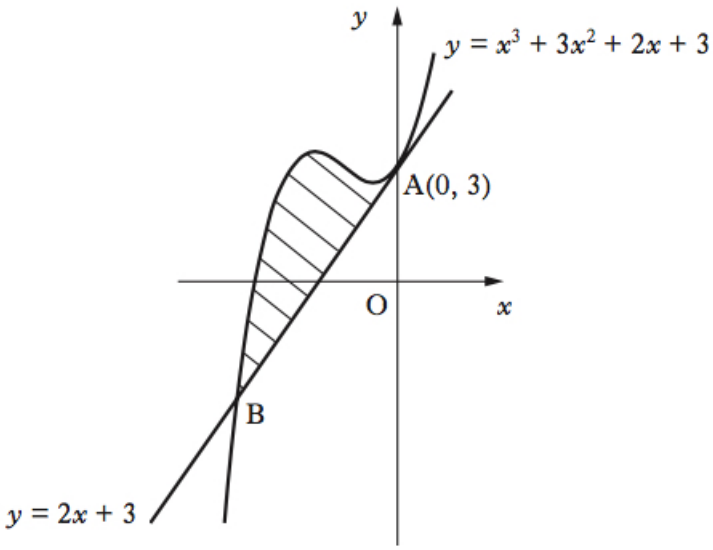
has only one stationary point.

Find the x-coordinate and determine the nature of this point.

4. The line with equation $y = 2x + 3$ is a tangent to the curve with equation (6)

$$y = x^3 + 3x^2 + 2x + 3$$

at $A(0, 3)$, as shown in the diagram.



The line meets the curve again at B.

Show that B is the point $(-3, -3)$ and find the area enclosed by the line and the curve.

5. Solve the equation (4)

$$\log_5(3 - 2x) + \log_5(2 + x) = 1,$$

where x is a real number.

6. Given that (5)

$$\int_0^a 5\sin 3x \, \mathrm{d}x = \tfrac{10}{3},\ 0 \leqslant a < \pi,$$

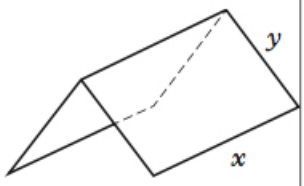
calculate the value of a.

7. A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is $24\,\mathrm{m}^2$.

(a) Show that the total length, L metres, of the rods used in a shelter is given by (3)

$$L = 3x + \frac{48}{x}.$$

These rods cost £8.25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(b) (i) Find the value of x for which L is a minimum. (7)

(ii) Calculate the minimum cost of a frame.

8. Solve algebraically the equation (6)

$$\sin 2x = 2\cos^2 x \text{ for } 0 \leqslant x < 2\pi.$$

9. The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation

$$P_t = P_0 \mathrm{e}^{-kt},$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t; and
- t is the time, in days, after the application of the pesticide.

Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

(a) If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures. (4)

(b) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*? (3)