Dr Oliver Mathematics
Mathematics
Trigonometry Part 1
Past Examination Questions

This booklet consists of 24 questions across a variety of examination topics.
The total number of marks available is 203.

1. (a) Show that the equation (2)

$$5\cos^2 x = 3(1 + \sin x)$$

can be written as

$$5\sin^2 x + 3\sin x - 2.$$

(b) Hence solve, for $0^\circ \leqslant x < 360^\circ$, the equation (5)

$$5\cos^2 x = 3(1 + \sin x),$$

giving your answer to 1 decimal place where appropriate.

2. Solve, for $0^\circ \leqslant x \leqslant 180^\circ$, the equation

(a) $\sin(x + 10)^\circ = \frac{\sqrt{3}}{2}$, (4)

(b) $\cos 2x = -0.9$, giving your answer to 1 decimal place. (4)

3. (a) Find all the values of θ, to 1 decimal place, in the interval $0^\circ \leqslant \theta < 360^\circ$ for which (4)

$$5\sin(\theta + 30)^\circ = 3.$$

(b) Find all the values of θ, to 1 decimal place, in the interval $0^\circ \leqslant x < 360^\circ$ for which (5)

$$\tan^2 \theta = 4.$$

4. (a) Given that $\sin\theta = 5\cos\theta$, find the value of $\tan\theta$. (1)

(b) Hence, or otherwise, find the values of θ in the interval $0^\circ \leqslant x < 360^\circ$ for which (3)

$$\sin\theta = 5\cos\theta,$$

giving your answer to 1 decimal place where appropriate.

5. Find all the solutions, in the interval $0 \leqslant x < 2\pi$, of the equation (6)

$$2\cos^2 x + 1 = 5\sin x,$$

giving each solution in terms of π.

6. (a) Sketch, for $0 \leqslant x \leqslant 2\pi$, the graph of $y = \sin(x + \frac{\pi}{6})$. (2)

(b) Write down the exact coordinates of the points where the graph meets the coordinates axes. (3)

(c) Solve, for $0 \leqslant x \leqslant 2\pi$, the equation (5)

$$\sin(x + \tfrac{\pi}{6}) = 0.65,$$

giving your answer in radians to 2 decimal places.

7. (a) Show that the equation (2)

$$3\sin^2\theta - 2\cos^2\theta = 1$$

can be written as

$$5\sin^2\theta = 1.$$

(b) Hence solve, for $0^\circ \leqslant x < 360^\circ$, the equation (7)

$$3\sin^2\theta - 2\cos^2\theta = 1,$$

giving your answer to 1 decimal place.

8. Solve, for $0^\circ \leqslant x < 360^\circ$,

(a) $\sin(x - 20)^\circ = \dfrac{1}{\sqrt{2}}$, (4)

(b) $\cos 3x^\circ = -\dfrac{1}{2}$. (6)

9. (a) Show that the equation (2)

$$4\sin^2 x + 9\cos x - 6 = 0$$

can be written as

$$4\cos^2 x - 9\cos x + 2 = 0.$$

(b) Hence solve, for $0^\circ \leqslant x < 720^\circ$, (6)

$$4\sin^2 x - 9\cos x - 6 = 0$$

giving your answer to 1 decimal place.

10. (a) Solve, for $-180^\circ \leqslant \theta < 180^\circ$, (4)

$$(1 + \tan\theta)(5\sin\theta - 2) = 0.$$

(b) Solve, for $0^\circ \leqslant x < 360^\circ$, (6)

$$4\sin x = 3\tan x.$$

11. (a) Show that the equation (2)

$$5\sin x = 1 + 2\cos^2 x$$

can be written as

$$2\sin^2 x + 5\sin x - 3 = 0.$$

(b) Solve, for $0^\circ \leqslant x < 360^\circ$, (4)

$$2\sin^2 x + 5\sin x - 3 = 0.$$

12. (a) Given that $5\sin\theta = 2\cos\theta$, find the value of $\tan\theta$. (1)

(b) Solve, for $0^\circ \leqslant x < 360^\circ$, (5)

$$5\sin 2x = 2\cos 2x,$$

giving your answer to 1 decimal place.

13. (a) Show that the equation (2)

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

can be written in the form

$$4\sin^2 x + 7\sin x + 3 = 0.$$

(b) Hence solve, for $0^\circ \leqslant x < 360^\circ$, (7)

$$3\sin^2 x + 7\sin x = \cos^2 x - 4,$$

giving your answer to 1 decimal place where appropriate.

14. (a) Solve for $0^\circ \leqslant x < 360^\circ$, giving your answer to 1 decimal place, (4)

$$3\sin(x + 45)^\circ = 2.$$

(b) Find, for $0 \leqslant x < 2\pi$, all the solutions of (6)

$$2\sin^2 x + 2 = 7\cos x,$$

giving your answer in radians.

15. (a) Find all the solutions of the equation $\sin(3x - 15)^\circ = \frac{1}{2}$, for which $0^\circ \leqslant x \leqslant 180^\circ$. (6)

Figure 1 shows part of the curve with equation

$$y = \sin(ax - b),$$

where $a > 0$, $0 < b < \pi$.

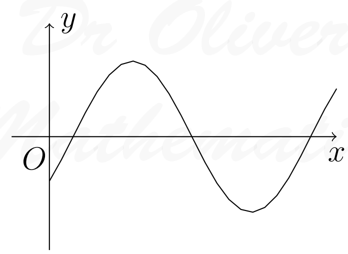

Figure 1: $y = \sin(ax - b)$

The curve cuts the x-axis at the points P, Q, and R.

(b) Given that the coordinates of $P(\frac{\pi}{10}, 0)$, $Q(\frac{3\pi}{5}, 0)$, and $R(\frac{11\pi}{10}, 0)$ respectively, find the values of a and b. (4)

16. (a) Show that the equation (2)

$$\tan 2x = 5 \sin 2x$$

can be written in the form

$$(1 - 5\cos 2x)\sin 2x = 0.$$

(b) Hence solve, for $0^\circ \leqslant x \leqslant 180^\circ$, (5)

$$\tan 2x = 5 \sin 2x,$$

giving your answer to 1 decimal place where appropriate.

17. Solve, for $0^\circ \leqslant x < 180^\circ$, (7)

$$\cos(3x - 10)^\circ = -0.4,$$

giving your answer to 1 decimal place.

18. (a) Solve, for $-180^\circ \leqslant x < 180^\circ$, (3)

$$\tan(x - 40)^\circ = 1.5,$$

giving your answer to 1 decimal place.

(b) Show that the equation (3)

$$\sin\theta\tan\theta = 3\cos\theta + 2$$

can be written in the form

$$4\cos^2\theta + 2\cos\theta - 1.$$

(c) Hence solve, for $0^\circ \leqslant \theta < 360^\circ$, (5)

$$\sin\theta\tan\theta = 3\cos\theta + 2,$$

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19. (a) Solve, for $0^\circ \leqslant \theta < 180^\circ$, (5)

$$\sin(2\theta - 30)^\circ + 1 = 0.4,$$

giving your answer to 1 decimal place.

(b) Find all the values of x, in the interval for $0^\circ \leqslant x < 360^\circ$, for which (7)

$$9\cos^2 x - 11\cos x + 3\sin^2 x = 0,$$

giving your answer to 1 decimal place.

20. (a) Solve, for $0^\circ \leqslant \theta < 360^\circ$, (4)

$$9\sin(\theta + 60)^\circ = 4,$$

giving your answer to 1 decimal place.

(b) Solve, for $-\pi \leqslant x < \pi$, the equation (6)

$$2\tan x - 3\sin x = 0,$$

giving your answer to 2 decimal place where appropriate.

21. (a) Solve, for $0^\circ \leqslant \theta \leqslant 180^\circ$, the equation (3)

$$\frac{\sin 2\theta}{4\sin 2\theta - 1} = 1,$$

giving your answer to 1 decimal place.

(b) Solve, for $0 \leqslant x < 2\pi$, the equation (5)

$$5\sin^2 x - 2\cos x - 5 = 0,$$

giving your answer to 2 decimal places.

22. (a) Solve, for $0 \leqslant \theta < \pi$, the equation (3)

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0,$$

giving your answer in terms of π.

Given that

$$4\sin^2 x + \cos x = 4 - k,\ 0 \leqslant k \leqslant 3,$$

(b) find $\cos x$ in terms of k. (3)

(c) When $k = 3$, find the values of x in the range $0° \leqslant x < 360°$. (3)

23. (a) Solve, for $-\pi < \theta \leqslant \pi$, (3)

$$1 - 2\cos(\theta - \tfrac{\pi}{5}) = 0,$$

giving your answers in terms of π.

(b) Solve, for $0° \leqslant x < 360°$, (6)

$$4\cos^2 x + 7\sin x - 2 = 0,$$

giving your answer to 1 decimal place.

24. (a) Show that the equation (3)

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2.$$

(b) Hence solve, for $0° \leqslant x < 360°$, (5)

$$\cos^2 x = 8\sin^2 x - 6\sin x,$$

giving your answers to 2 decimal place