

Dr Oliver Mathematics
Mathematics: Higher
2018 Paper 2: Calculator
1 hour 30 minutes

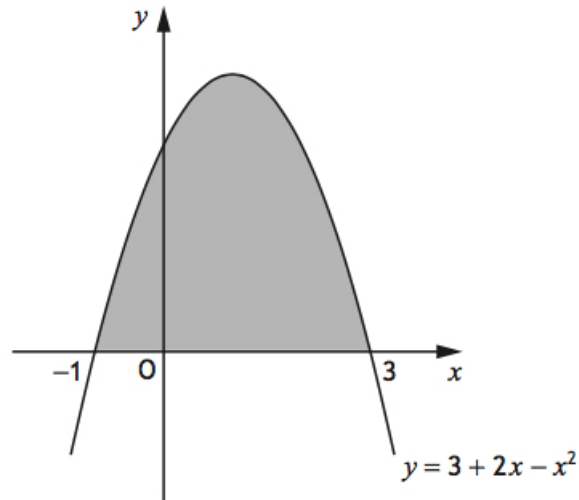
The total number of marks available is 70.

You must write down all the stages in your working.

1. The diagram shows the curve with equation

(4)

$$y = 3 + 2x - x^2.$$



Calculate the shaded area.

2. Vectors \mathbf{u} and \mathbf{v} are defined by

$$\mathbf{u} = \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} -7 \\ 8 \\ 5 \end{pmatrix}.$$

(a) Find $\mathbf{u} \cdot \mathbf{v}$.

(1)

(b) Calculate the acute angle between \mathbf{u} and \mathbf{v} .

(4)

3. A function, f , is defined on the set of real numbers by

(3)

$$f(x) = x^3 - 7x + 6.$$

Determine whether f is increasing or decreasing when $x = 2$.

4. Express

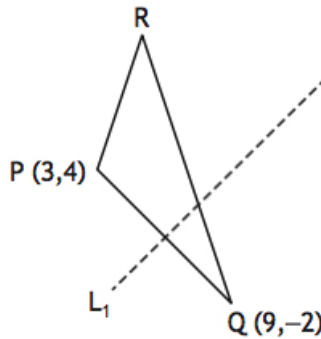
$$-3x^2 - 6x + 7$$

(3)

in the form

$$a(x + b)^2 + c.$$

5. PQR is a triangle with $P(3,4)$ and $Q(9,-2)$.

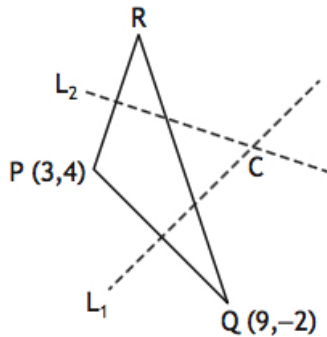


(a) Find the equation of L_1 , the perpendicular bisector of PQ .

(3)

The equation of L_2 , the perpendicular bisector of PR is

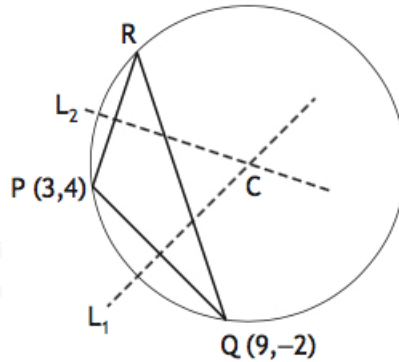
$$3y + x = 25.$$



(b) Calculate the coordinates of C , the point of intersection of L_1 and L_2 .

(2)

C is the centre of the circle which passes through the vertices of triangle PQR .



(c) Determine the equation of this circle. (2)

6. Functions, f and g , are given by

$$f(x) = 3 + \cos x \text{ and } g(x) = 2x, x \in \mathbb{R}.$$

(a) Find expressions for

(i) $f(g(x))$ and (2)

(ii) $g(f(x))$. (1)

(b) Determine the value(s) of x for which (6)

$$f(g(x)) = g(f(x))$$

where $0 \leq x < 2\pi$.

7. (a) (i) Show that $(x - 2)$ is a factor of (2)

$$2x^3 - 3x^2 - 3x + 2.$$

(ii) Hence, factorise (2)

$$2x^3 - 3x^2 - 3x + 2$$

fully.

The fifth term, u_5 , of a sequence is

$$u_5 = 2a - 3.$$

The terms of the sequence satisfy the recurrence relation

$$u_{n+1} = au_n - 1.$$

(b) Show that (1)

$$u_7 = 2a^3 - 3a^2 - a - 1.$$

For this sequence, it is known that

- $u_7 = u_5$ and
- a limit exists.

(c) (i) Determine the value of a . (3)

(ii) Calculate the limit. (1)

8. (a) Express (4)

$$2 \cos x^\circ - \sin x^\circ$$

in the form

$$k \cos(x - a)^\circ, \quad k > 0, \quad 0 < a < 360.$$

(b) Hence, or otherwise, find (1)

(i) the minimum value of

$$6 \cos x^\circ - 3 \sin x^\circ,$$

(ii) the value of x for which it occurs where $0 \leq x < 360$. (2)

9. A sector with a particular fixed area has radius x cm. (6)

The perimeter, P cm, of the sector is given by

$$P = 2x + \frac{128}{x}.$$

Find the minimum value of P .

10. The equation (4)

$$x^2 + (m - 3)x + m = 0$$

has two real and distinct roots.

Determine the range of values for m .

11. A supermarket has been investigating how long customers have to wait at the checkout.

During any half hour period, the percentage, $P\%$, of customers who wait for less than t minutes, can be modelled by

$$P = 100(1 - e^{kt}),$$

where k is a constant.

(a) If 50% of customers wait for less than 3 minutes, determine the value of k . (4)

(b) Calculate the percentage of customers who wait for 5 minutes or longer. (2)

12. Circle C_1 has equation

$$(x - 13)^2 + (y + 4)^2 = 100.$$

Circle C_2 has equation

$$x^2 + y^2 + 14x - 22y + c = 0.$$

(a) (i) Write down the coordinates of the centre of C_1 . (1)

The centre of C_1 lies on the circumference of C_2 .

(ii) Show that $c = -455$. (1)

The line joining the centres of the circles intersects C_1 at P .

(b) (i) Determine the ratio in which P divides the line joining the centres of the circles. (2)

(ii) Hence, or otherwise, determine the coordinates of P . (2)

P is the centre of a third circle, C_3 .

C_2 touches C_3 internally.

(c) Determine the equation of C_3 . (1)