Dr Oliver Mathematics
GCSE Mathematics
2008 June Paper 3H: Non-Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Here are the ingredients needed to make 8 pancakes.

   Pancakes
   Ingredients to make 8 pancakes
   300 ml milk
   1 egg
   120 g flour
   5 g butter

   Jacob makes 24 pancakes.

   (a) Work out how much milk he needs. (2)

   Solution

   $$\frac{24}{8} \times 300 = 3 \times 300 = \underline{\underline{900 \text{ ml}}}.$$

   Cathie makes 12 pancakes.

   (b) Work out how much flour she needs. (2)

   Solution

   $$\frac{12}{8} \times 120 = \frac{3}{2} \times 120 = \underline{\underline{180 \text{ g}}}.$$

2. Kaysha has a part-time job. (3)
   She is paid £5.40 for each hour she works.
   Last week Kaysha worked for 24 hours.
   Work out Kaysha's total pay for last week.

Solution

$\times$	20	4
5	100	20
0.4	8	1.6

Hence,

$$£5.40 \times 24 = \underline{\underline{£129.60}}.$$

3. Here are the ages, in years, of 15 teachers. (3)

35	52	42	27	36
23	31	41	50	34
44	28	45	45	35

Draw an ordered stem and leaf diagram to show this information.
You must include a key.

Solution

5	0	2	3		
4	1	2	4	5	5
3	1	4	5	6	
2	3	7	8		

Key: 2|3 means 23 years of age.

4. Using the information that

$$4.8 \times 34 = 163.2,$$

write down the value of

(a) 48×34, (1)

Solution

$$48 \times 34 = \underline{\underline{1\,632}}.$$

(b) 4.8×3.4, (1)

Solution

$$4.8 \times 3.4 = \underline{16.32}.$$

(c) $163.2 \div 48$. (1)

Solution

$$163.2 \div 48 = \underline{3.4}.$$

5. A cuboid is shown on a 3-dimensional grid.

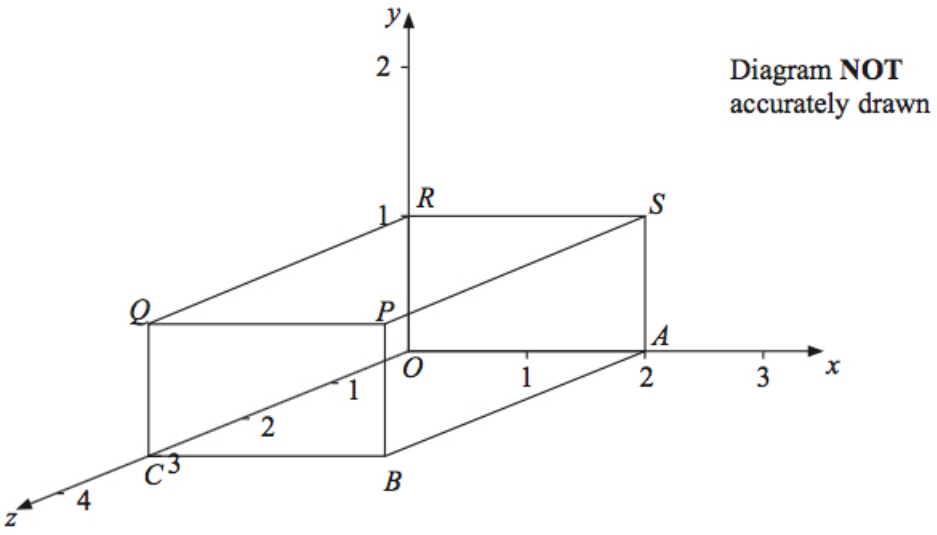

Diagram **NOT** accurately drawn

(a) Write down the letter of the point with coordinates $(2, 1, 0)$. (1)

Solution

$\underline{S}$.

(b) Write down the coordinates of the point P. (1)

Solution

$\underline{(2, 1, 3)}$.

6. This rule is used to work out the total cost, in pounds, of hiring a carpet cleaner.

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Multiply the number of days' hire by 4.
Add 6 to your answer.

Peter hires a carpet cleaner.
The total cost is £18.

(a) Work out for how many days he hires the carpet cleaner. (2)

Solution

Let x be the number of days. Then

$$4x + 6 = 18 \Rightarrow 4x = 12$$
$$\Rightarrow \underline{\underline{x = 3}}.$$

(b) Write down an expression, in terms of n, for the total cost, in pounds, of hiring a carpet cleaner for n days. (2)

Solution

The total cost is $\underline{\underline{4n + 6}}$.

7. Work out the total surface area of the triangular prism. (4)

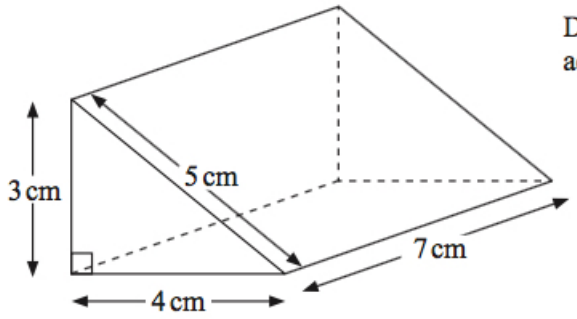


Diagram **NOT** accurately drawn

Give the units with your answer.

Solution

$$\begin{aligned}\text{Total surface area} &= (2 \times \tfrac{1}{2} \times 3 \times 4) + (3 \times 7) + (4 \times 7) + (5 \times 7) \\ &= 12 + 21 + 28 + 35 \\ &= 33 + 63 \\ &= \underline{\underline{96 \text{ cm}^2}}.\end{aligned}$$

8. Work out an estimate for (3)

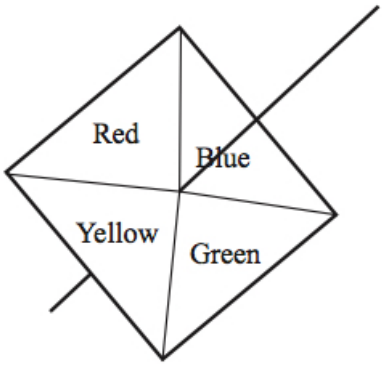
$$\frac{302 \times 9.96}{0.51}.$$

Solution

Use 1 significant figure:

$$\begin{aligned}\frac{302 \times 9.96}{0.51} &\approx \frac{300 \times 10}{0.5} \\ &= \frac{3\,000}{0.5} \\ &= \underline{\underline{6\,000}}.\end{aligned}$$

9. Here is a 4-sided spinner. (2)



The sides of the spinner are labelled Red, Blue, Green, and Yellow.
The spinner is biased.
The table shows the probability that the spinner will land on each of the colours Red, Yellow, and Green.

Colour	Red	Blue	Green	Yellow
Probability	0.2		0.3	0.1

Work out the probability the spinner will land on Blue.

Solution

Let x be the probability it lands on Blue. Then

$$0.2 + x + 0.3 + 0.1 = 1 \Rightarrow x + 0.6 = 1$$
$$\Rightarrow \underline{\underline{x = 0.4}}.$$

10. (a) Simplify $4p \times 5q$. (1)

Solution

$$4p \times 5q = \underline{\underline{20pq}}.$$

(b) Simplify $d \times d \times d \times d$. (1)

Solution

$$d \times d \times d \times d = \underline{\underline{d^4}}.$$

(c) Expand $4(3a - 7)$. (2)

Solution

$$4(3a - 7) = \underline{\underline{12a - 28}}.$$

(d) Expand and simplify $2(2n + 3) + 3(n + 1)$. (2)

Solution

$$2(2n + 3) + 3(n + 1) = 4n + 6 + 3n + 3$$
$$= \underline{\underline{7n + 9}}.$$

(e) Simplify $t \times t^2$. (1)

Solution

$$t \times t^2 = \underline{\underline{t^3}}.$$

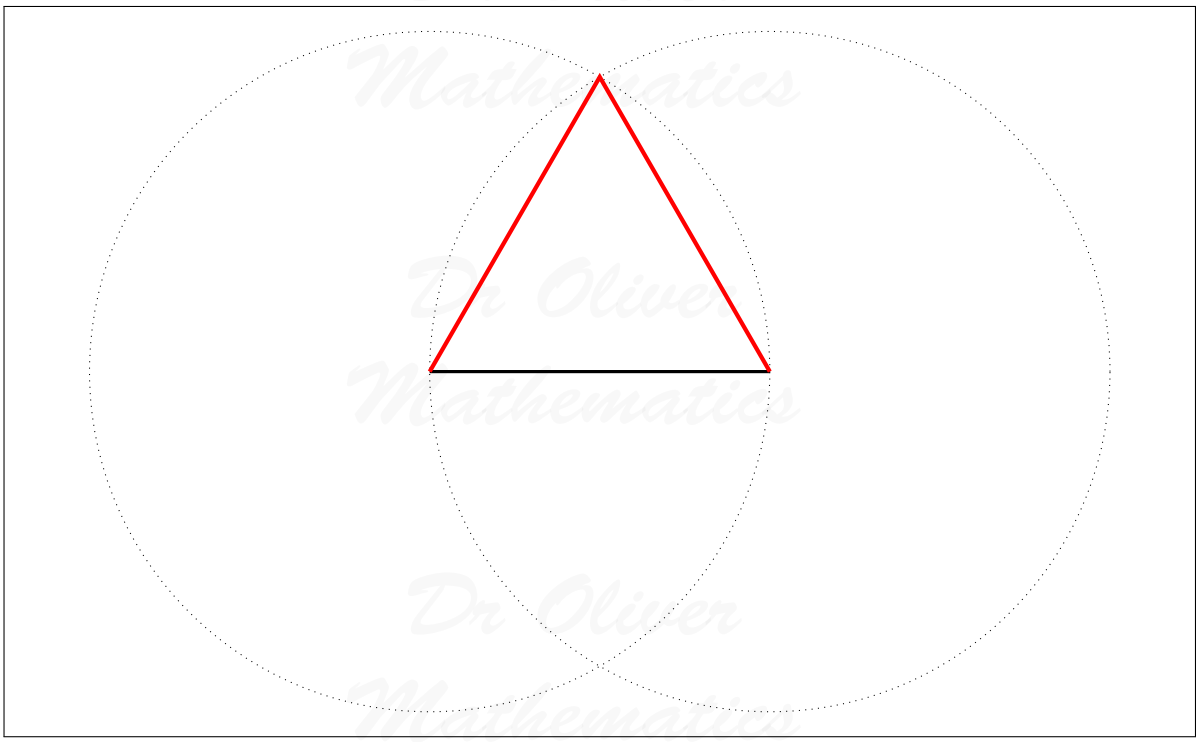
(f) Simplify $m^5 \div m^3$. (1)

Solution

$$m^5 \div m^3 = \underline{\underline{m^2}}.$$

11. In the space below, use ruler and compasses to **construct** an equilateral triangle with sides of length 6 centimetres. (2)
You must show all your construction lines.
One side of the triangle has already been drawn for you.

Solution



12. $-2 \leqslant x < 3$. (2)

x is an integer.

Write down all the possible values of x.

Solution

$\underline{\underline{-2, -1, 0, 1, 2}}$.

13. (a) Write down the reciprocal of 4. (1)

Solution

The reciprocal is $\underline{\underline{\frac{1}{4}}}$.

(b) Work out the value of (3)

$$2\tfrac{4}{5} - 1\tfrac{3}{4}.$$

Give your answer as a fraction in its simplest form.

Solution

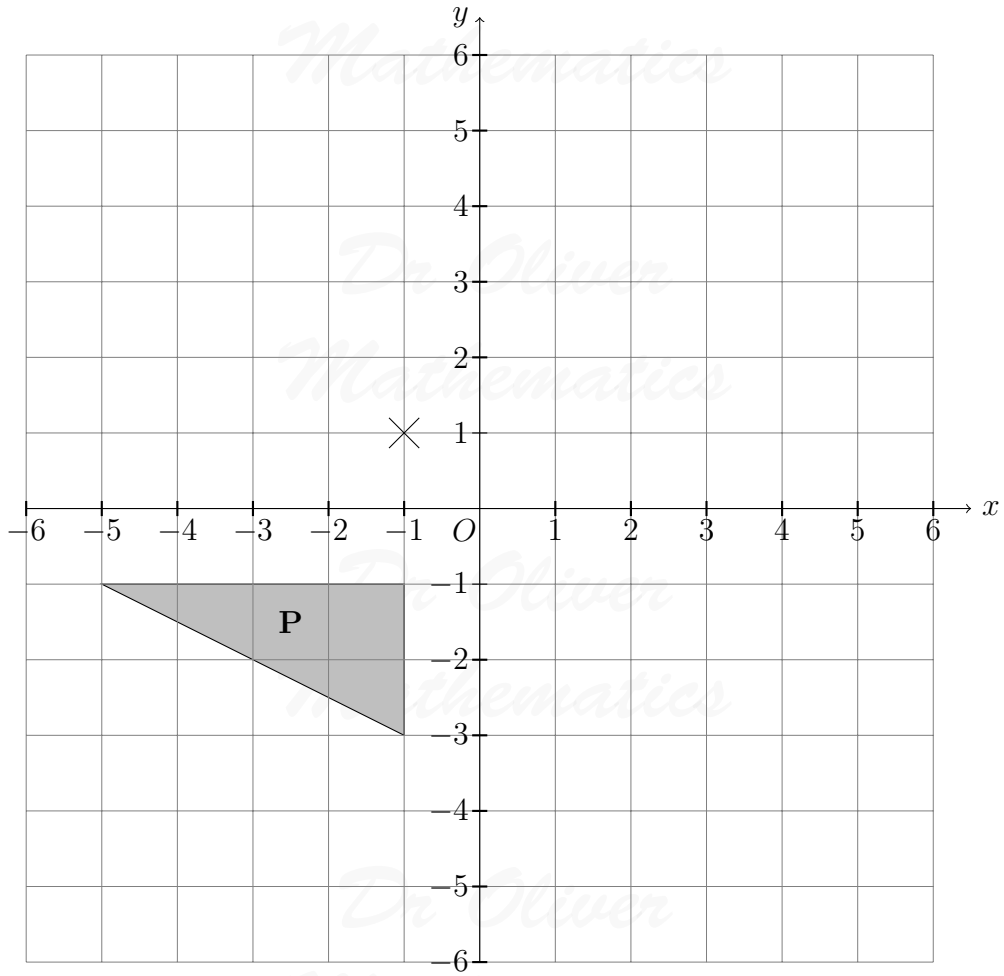
$$2\tfrac{4}{5} - 1\tfrac{3}{4} = 1\tfrac{16}{20} - \tfrac{15}{20}$$
$$= 1\tfrac{1}{20}.$$

(c) Sundas says that $4\frac{1}{3}$ is equal to 4.3. (1)
Sundas is **wrong**.
Explain why.

Solution

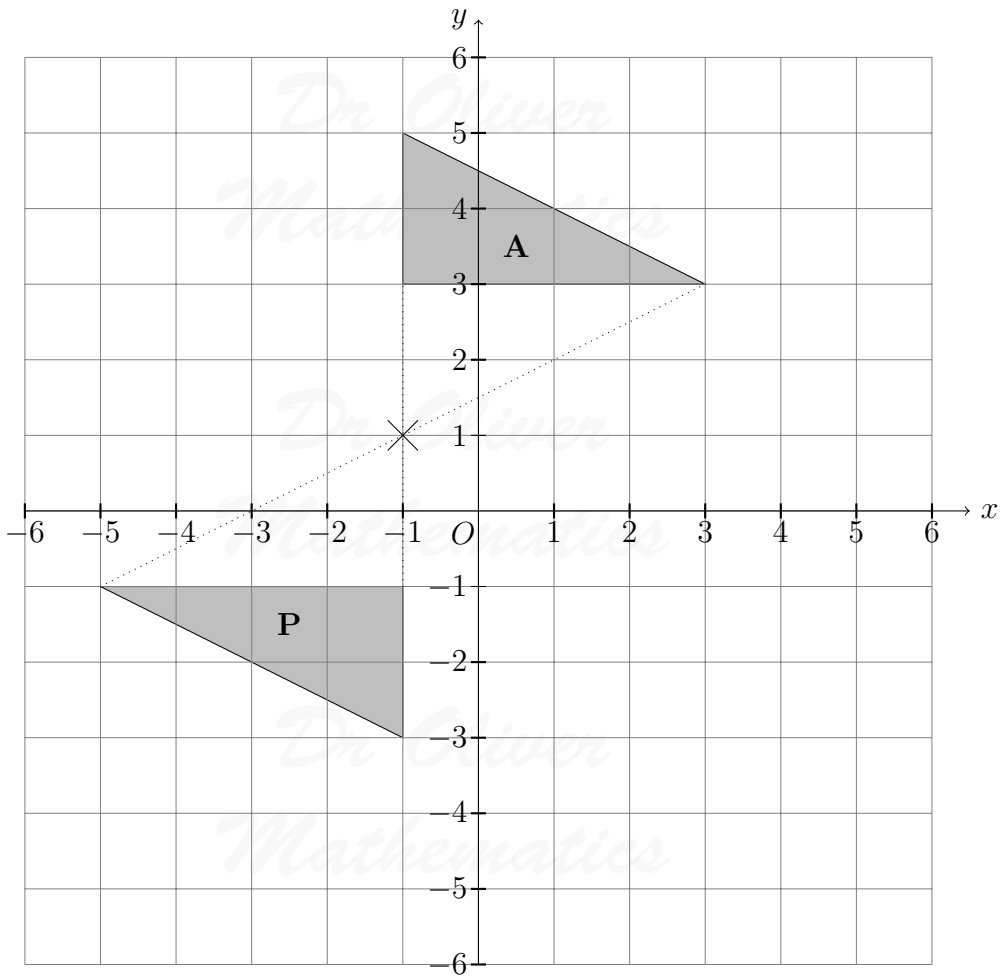
Because $4\frac{1}{3} = 4.333\,333\ldots$ and it is bigger than 4.3.

14. Here is a picture.



(a) Rotate triangle **P** 180° about the point $(-1, 1)$. (2)
Label the new triangle **A**.

Solution



(b) Translate triangle **P** by the vector $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$. (1)
Label the new triangle **B**.

Solution

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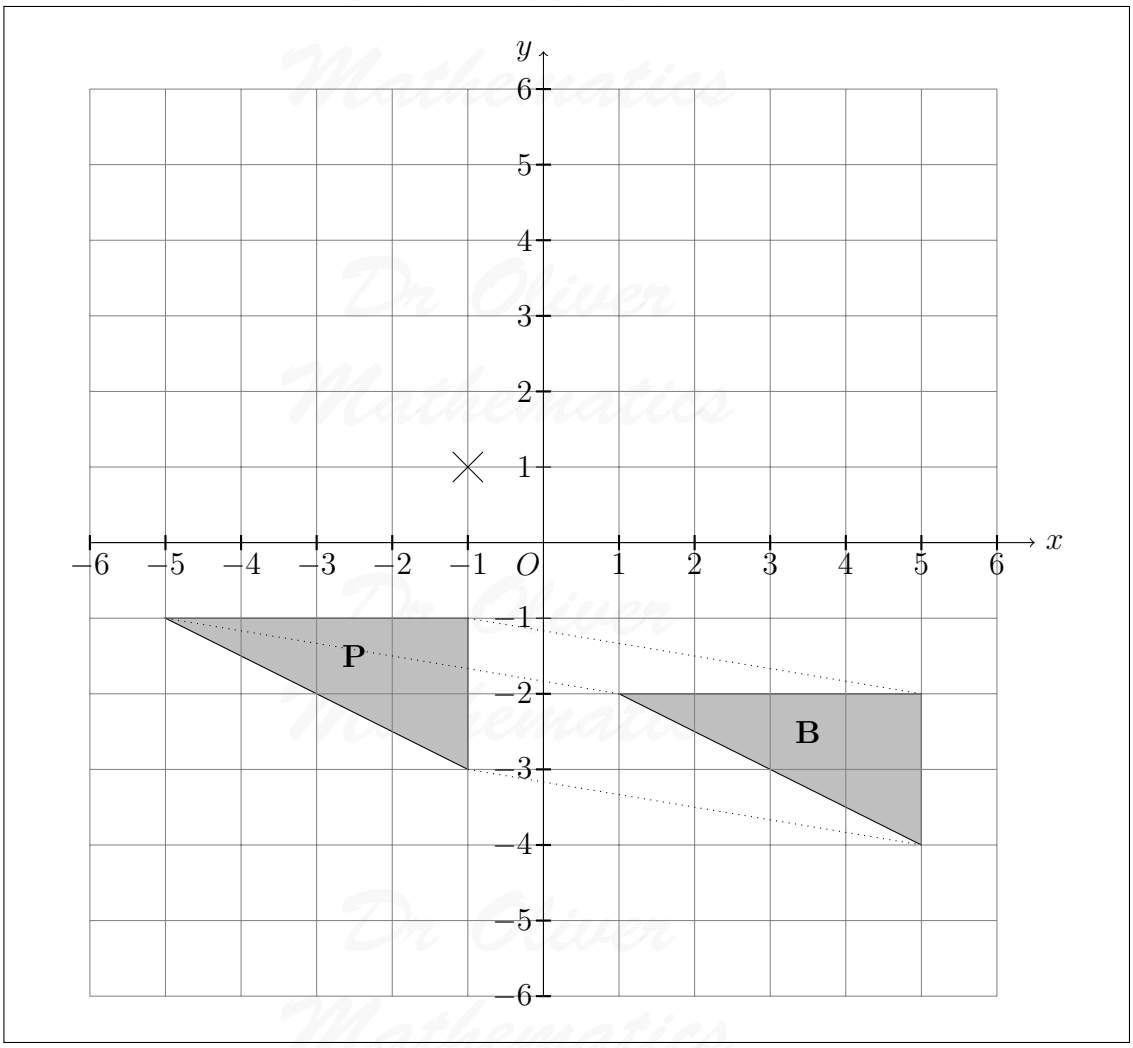

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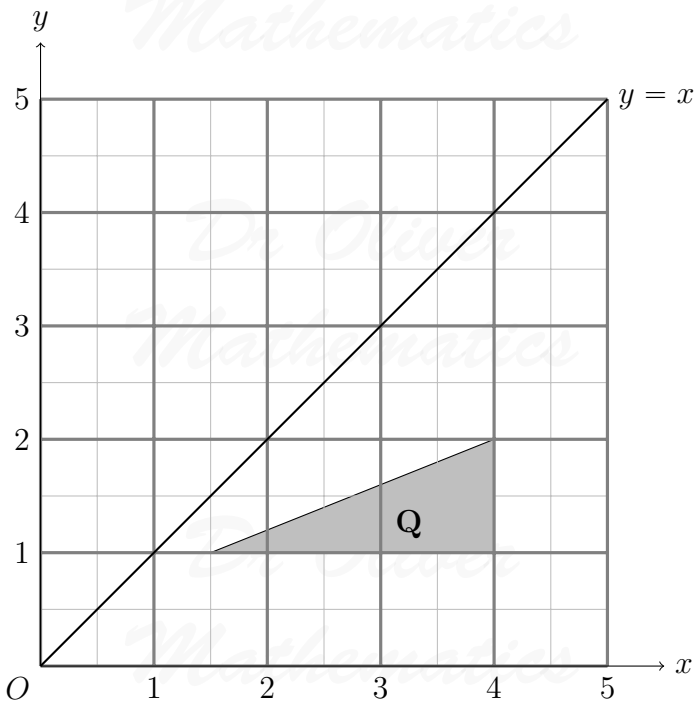
Here is another picture.



(c) Reflect triangle **Q** in the line $y = x$. (2)
Label the new triangle **C**.

Solution

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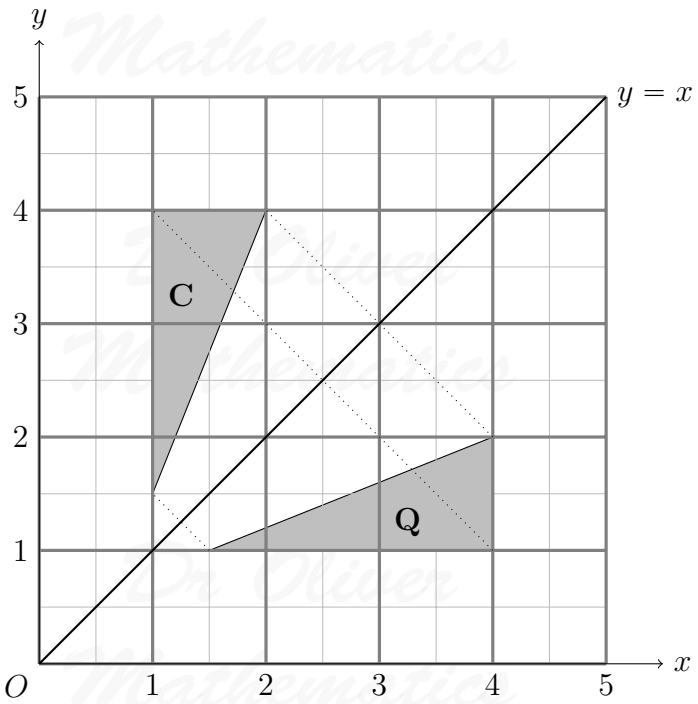


15. (a) Expand $x(3x - 5y)$. (2)

Solution

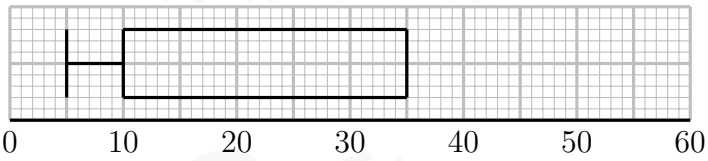
$$x(3x - 5y) = \underline{\underline{3x^2 - 5xy}}.$$

(b) Factorise $x^2 - 36$. (1)

Solution

$$x^2 - 36 = \underline{\underline{(x + 6)(x - 6)}}.$$

16. The incomplete box plot and table show some information about some marks.



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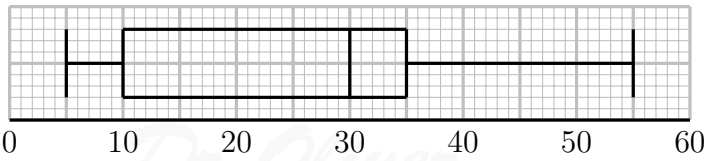
	Mark
Lowest mark	5
Lower quartile	
Median	30
Upper quartile	35
Highest mark	55

(a) Use the information in the table to complete the box plot. (2)

Solution



(b) Use the information in the box plot to complete the table. (1)

Solution

	Mark
Lowest mark	5
Lower quartile	$\underline{\underline{10}}$
Median	30
Upper quartile	35
Highest mark	55

17. (a) Write 6.4×10^4 as an ordinary number. (1)

Solution

$$6.4 \times 10^4 = \underline{\underline{64\,000}}.$$

(b) Write $0.003\,9$ in standard form. (1)

Solution

$$0.003\,9 = \underline{\underline{3.9 \times 10^{-3}}}.$$

(c) Write 0.25×10^7 in standard form. (1)

Solution

$$0.25 \times 10^7 = \underline{\underline{2.5 \times 10^6}}.$$

18. In the diagram, A, B, C, and D are points on the circumference of a circle, centre O.

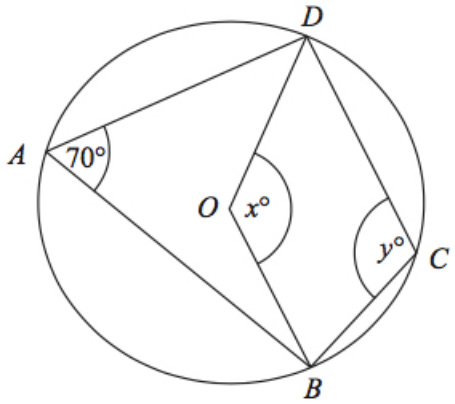


Diagram **NOT** accurately drawn

Angle $BAD = 70°$.
Angle $BOD = x°$.
Angle $BCD = y°$.

(a) (i) Work out the value of x. (2)

Solution

$\underline{\underline{140°}}$.

(ii) Give a reason for your answer.

Solution

$\underline{\underline{\text{The angle at the circumference is twice the angle at the centre}}}$.

(b) (i) Work out the value of y. (2)

Solution

$\underline{\underline{110°}}$.

(ii) Give a reason for your answer.

Solution
The angle at the circumference is twice the angle at the centre.

19. Solve the simultaneous equations (3)

$$2x + 3y = 0$$
$$x - 3y = 9.$$

Solution
Add:

$$3x = 9 \Rightarrow x = 3$$
$$\Rightarrow y = -2.$$

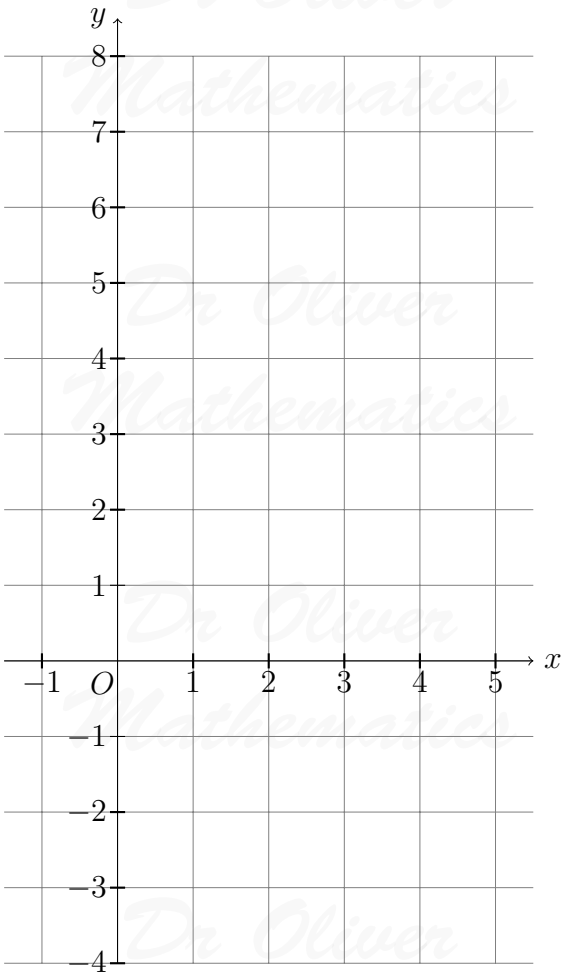
20. (a) Complete the table of values for $y = x^2 - 4x + 2$. (2)

x	-1	0	1	2	3	4	5
y		2	-1		-1		7

Solution

x	-1	0	1	2	3	4	5
y	7	2	-1	-2	-1	2	7

(b) On the grid, draw the graph of $y = x^2 - 4x + 2$. (2)



Solution

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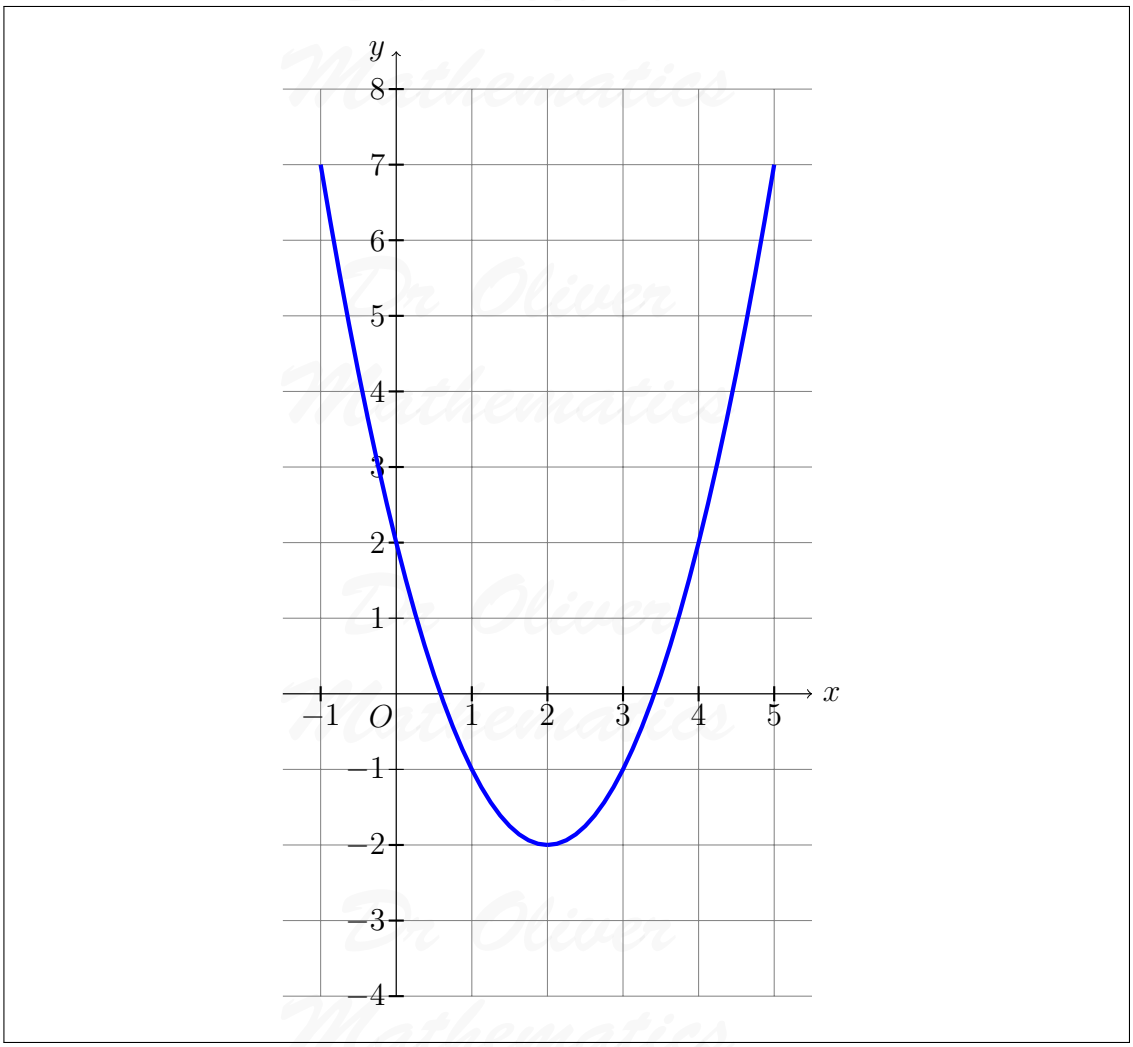

21. Tom and Sam each take a driving test.
The probability that Tom will pass the driving test is 0.8.
The probability that Sam will pass the driving test is 0.6.

(a) Complete the probability tree diagram. (2)

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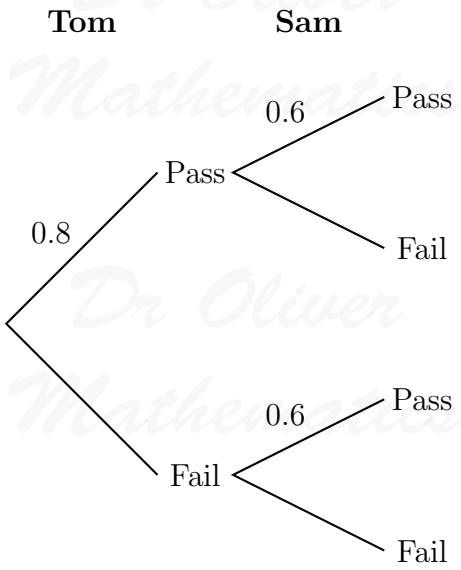
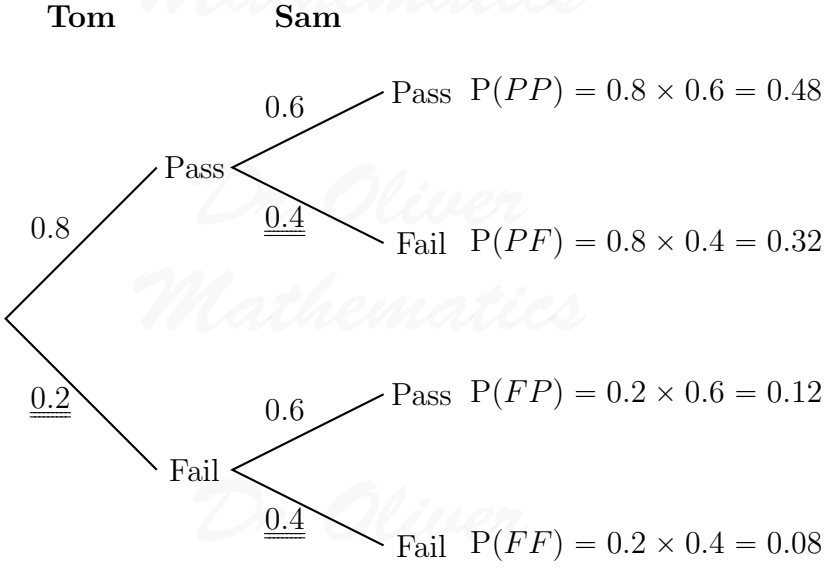


Solution



(b) Work out the probability that both Tom and Sam will pass the driving test. (2)

Solution

$$\mathrm{P}(PP) = 0.8 \times 0.6 = \underline{\underline{0.48}}.$$

(c) Work out the probability that only one of them will pass the driving test. (3)

Solution

$$\begin{aligned}\text{P(only one of them)} &= \text{P}(PF) + \text{P}(FP)\\ &= 0.32 + 0.12\\ &= \underline{\underline{0.44}}.\end{aligned}$$

22. Make b the subject of the formula (4)

$$a = \frac{2-7b}{b-5}.$$

Solution

$$\begin{aligned}a = \frac{2-7b}{b-5} &\Rightarrow a(b-5) = 2-7b\\ &\Rightarrow ab - 5a = 2 - 7b\\ &\Rightarrow ab + 7b = 2 + 5a\\ &\Rightarrow b(a+7) = 2 + 5a\\ &\Rightarrow \underline{\underline{b = \frac{2+5a}{a+7}}}.\end{aligned}$$

23. (a) Rationalise the denominator of $\frac{1}{\sqrt{3}}$. (1)

Solution

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \underline{\underline{\frac{\sqrt{3}}{3}}}.$$

(b) Expand (2)

$$(2+\sqrt{3})(1+\sqrt{3}).$$

Give your answer in the form $a + b\sqrt{3}$, where a and b are integers.

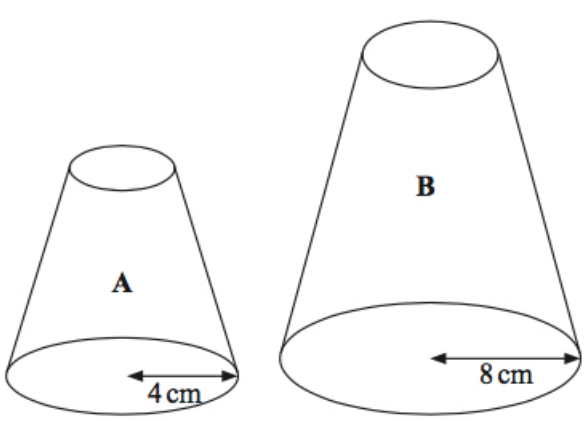
Solution

$\times$	2	$+\sqrt{3}$
1	2	$+\sqrt{3}$
$+\sqrt{3}$	$+2\sqrt{3}$	$+3$

Hence

$$(2+\sqrt{3})(1+\sqrt{3}) = \underline{\underline{5+3\sqrt{3}}}.$$

24. Two solid shapes, **A** and **B**, are mathematically similar.



Diagrams **NOT** accurately drawn

The base of shape **A** is a circle with radius 4 cm.
The base of shape **B** is a circle with radius 8 cm.
The surface area of shape **A** is 80 cm^2.

(a) Work out the surface area of shape **B**. (2)

Solution

$$\text{LSF} = 2 \Rightarrow \text{ASF} = 2^2 = 4.$$

Now,

$$\text{surface area of } \mathbf{B} = 80 \times 4 = \underline{\underline{320 \text{ cm}^2}}.$$

The volume of shape **B** is 600 cm^3.

(b) Work out the volume of shape **A**. (2)

Solution

$$\text{LSF} = 2 \Rightarrow \text{VSF} = 2^3 = 8.$$

Now,

$$\text{volume of } \mathbf{A} = \frac{600}{8} = \underline{\underline{75 \text{ cm}^3}}.$$

25. $OABC$ is a parallelogram.

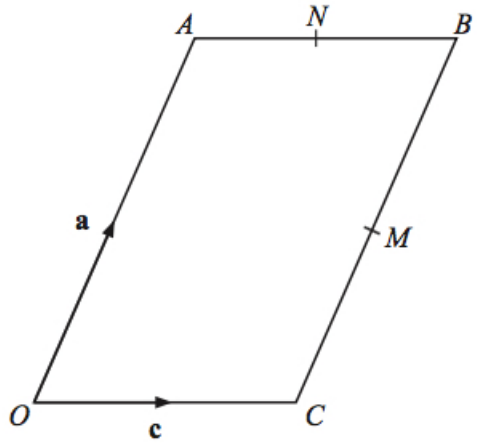


Diagram **NOT** accurately drawn

M is the midpoint of CB.
N is the midpoint of AB.
$\overrightarrow{OA} = \mathbf{a}$.
$\overrightarrow{OC} = \mathbf{c}$.

(a) Find, in terms of $\mathbf{a}$ and/or $\mathbf{c}$, the vectors (2)

(i) $\overrightarrow{MB}$,

Solution

$$\overrightarrow{MB} = \tfrac{1}{2}\overrightarrow{OA} = \underline{\underline{\tfrac{1}{2}\mathbf{a}}}.$$

(ii) $\overrightarrow{MN}$.

Solution

$$\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BN}$$
$$= \underline{\underline{\tfrac{1}{2}\mathbf{a} - \tfrac{1}{2}\mathbf{c}}}.$$

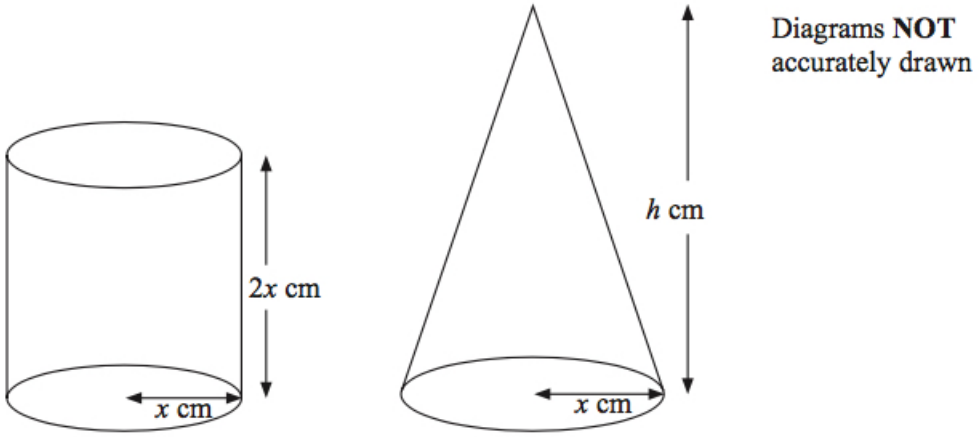
(b) Show that CA is parallel to MN. (2)

Solution

$$\begin{aligned}\overrightarrow{CA} &= \overrightarrow{CO} + \overrightarrow{ON} \\ &= \mathbf{a} - \mathbf{c} \\ &= 2\overrightarrow{MN};\end{aligned}$$

thus, CA is parallel to MN.

26. A cylinder has base radius x cm and height $2x$ cm. (3)
A cone has base radius x cm and height h cm.



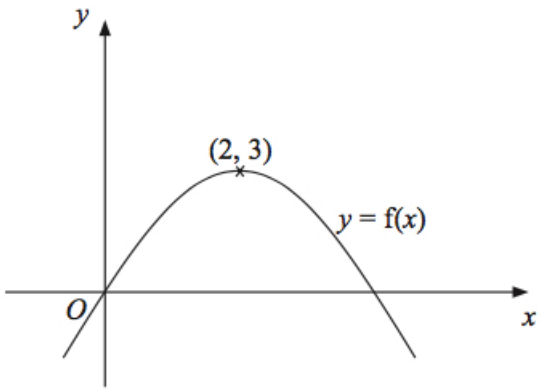
The volume of the cylinder and the volume of the cone are equal.
Find h in terms of x.
Give your answer in its simplest form.

Solution

As the volumes are equal,

$$\pi \times x^2 \times 2x = \tfrac{1}{3} \times \pi \times x^2 \times h \Rightarrow 2x^3 = \tfrac{1}{3}x^2h$$
$$\Rightarrow h = 6x.$$

27. The diagram shows part of the curve with equation $y = \mathrm{f}(x)$.



The coordinates of the maximum point of this curve are $(2, 3)$.
Write down the coordinates of the maximum point of the curve with equation

(a) $y = \mathrm{f}(x - 2)$, (1)

Solution
$\underline{\underline{(4, 3)}}$.

(b) $y = 2\,\mathrm{f}(x)$. (1)

Solution
$\underline{\underline{(2, 6)}}$.

28. Simplify fully (3)

$$\frac{x^2 + x - 6}{x^2 - 7x + 10}.$$

Solution

$$\frac{x^2 + x - 6}{x^2 - 7x + 10} = \frac{(x + 3)(x - 2)}{(x - 5)(x - 2)}$$
$$= \underline{\underline{\frac{x + 3}{x - 5}}}.$$