Dr Oliver Mathematics
Mathematics: National Qualifications N5
2014 Paper 1: Non-Calculator
1 hour

The total number of marks available is 40.
You must write down all the stages in your working.

1. Evaluate (2)

$$\tfrac{5}{12} \times 2\tfrac{2}{9}.$$

Give the answer in simplest form.

Solution

$$\begin{aligned} \tfrac{5}{12} \times 2\tfrac{2}{9} &= \tfrac{5}{12} \times \tfrac{20}{9} \\ &= \tfrac{5}{3} \times \tfrac{5}{9} \\ &= \underline{\underline{\tfrac{25}{27}}}. \end{aligned}$$

2. Multiply out the brackets and collect like terms: (2)

$$(2x - 5)(3x + 1).$$

Solution

$\times$	$2x$	-5
$3x$	$6x^2$	$-15x$
$+1$	$+2x$	-2

$$(2x - 5)(3x + 1) = \underline{\underline{6x^2 - 13x - 5}}.$$

3. Express (2)

$$x^2 - 14x + 44$$

in the form

$$(x-a)^2 + b.$$

Solution

$$\begin{aligned} x^2 - 14x + 44 &= (x^2 - 14x + 49) - 5 \\ &= \underline{\underline{(x-7)^2 - 5}}; \end{aligned}$$

hence, $\underline{\underline{a = 7}}$ and $\underline{\underline{b = -5}}$.

4. Find the resultant vector $2\mathbf{u} - \mathbf{v}$ when (2)

$$\mathbf{u} = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} 0 \\ -4 \\ 7 \end{pmatrix}.$$

Express your answer in component form.

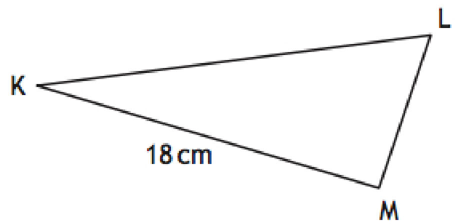
Solution

$$\begin{aligned} 2\mathbf{u} - \mathbf{v} &= 2\begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ -4 \\ 7 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} -4 \\ 10 \\ 3 \end{pmatrix}}}. \end{aligned}$$

5. In triangle KLM, (3)

- $KM = 18$ centimetres
- $\sin K = 0.4$, and

- $\sin L = 0.9$.



Calculate the length of LM.

Solution

$$\frac{LM}{\sin K} = \frac{KM}{\sin L} \Rightarrow \frac{LM}{0.4} = \frac{18}{0.9}$$
$$\Rightarrow LM = 0.4 \times 20$$
$$\Rightarrow \underline{\underline{LM = 8 \text{ cm}}}.$$

6. McGregor's Burgers sells fast food.

   The graph shows the relationship between the amount of fat, F grams, and the number of calories, C, in some of their sandwiches.

A line of best fit has been drawn.
Point A represents a sandwich which has 5 grams of fat and 200 calories.
Point B represents a sandwich which has 25 grams of fat and 500 calories.

(a) Find the equation of the line of best fit in terms of F and C. (3)

Solution

$$\begin{aligned}\text{Gradient} &= \frac{500-200}{25-5}\\ &= \frac{300}{20}\\ &= 15\end{aligned}$$

and the equation of the line of best fit is

$$\begin{aligned}C-200 = 15(F-5) &\Rightarrow C-200 = 15F-75\\ &\Rightarrow \underline{\underline{C = 15F+125}}.\end{aligned}$$

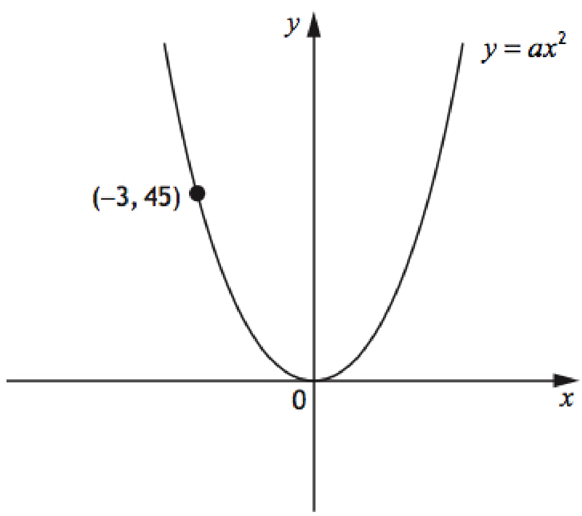
(b) A Super Deluxe sandwich contains 40 grams of fat. (1)
Use your answer to part (a) to estimate the number of calories this sandwich contains.

Show your working.

Solution

$$\begin{aligned} C &= (15 \times 40) + 125 \\ &= 600 + 125 \\ &= \underline{\underline{725 \text{ calories}}}. \end{aligned}$$

7. The diagram below shows part of the graph of $y = ax^2$. (2)



Find the value of a.

Solution

$$\begin{aligned} 45 = a(-3)^2 &\Rightarrow 9a = 45 \\ &\Rightarrow \underline{\underline{a = 5}}. \end{aligned}$$

8. Express (3)

$$\sqrt{40} + 4\sqrt{10} + \sqrt{90}$$

as a surd in its simplest form.

Solution

$$\begin{aligned}\sqrt{40} + 4\sqrt{10} + \sqrt{90} &= \sqrt{4 \times 10} + 4\sqrt{10} + \sqrt{9 \times 10} \\ &= (\sqrt{4} \times \sqrt{10}) + 4\sqrt{10} + (\sqrt{9} \times \sqrt{10}) \\ &= 2\sqrt{10} + 4\sqrt{10} + 3\sqrt{10} \\ &= \underline{\underline{9\sqrt{10}}}.\end{aligned}$$

9. 480 000 tickets were sold for a tennis tournament last year. (3)
This represents 80% of all the available tickets.
Calculate the total number of tickets that were available for this tournament.

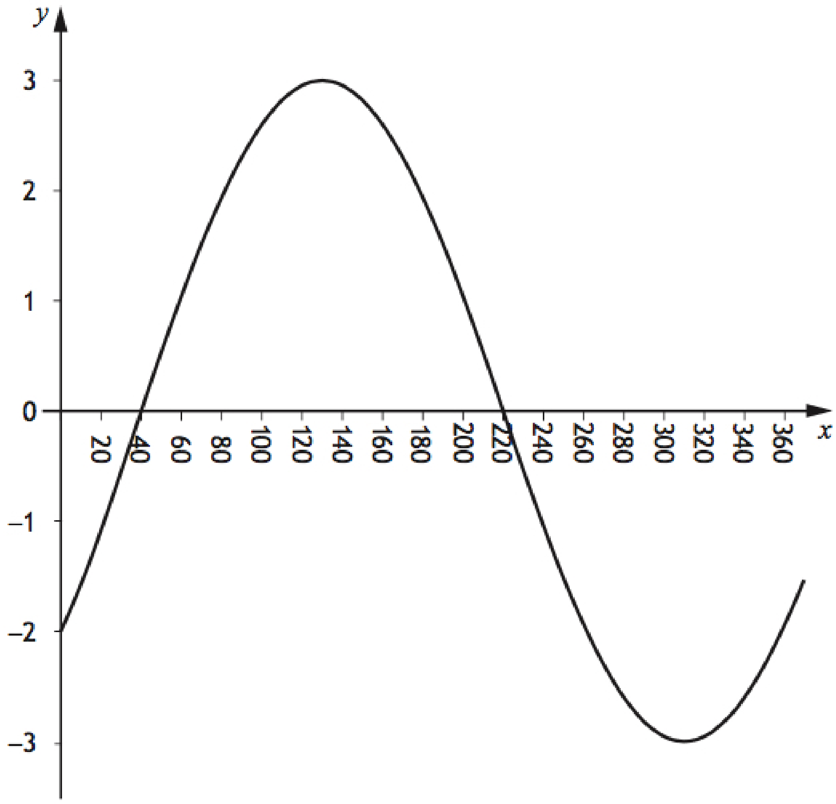
Solution

$$\begin{aligned}80\% &\leftrightarrow 480\,000 \\ 10\% &\leftrightarrow 60\,000 \\ 100\% &\leftrightarrow \underline{\underline{600\,000 \text{ tickets}}}.\end{aligned}$$

10. The graph of (2)

$$y = a\sin(x + b)^\circ,\ 0 \leqslant x \leqslant 360,$$

is shown below.



Write down the values of a and b.

Solution

$\underline{\underline{a = 3}}$ (the amplitude of the graph) and $\underline{\underline{b = -40}}$ (the graph moves to the right).

11. (a) A straight line has equation (2)

$$4x + 3y = 12.$$

Find the gradient of this line.

Solution

$$\begin{aligned} 4x + 3y = 12 &\Rightarrow 3y = -4x + 12 \\ &\Rightarrow y = -\tfrac{4}{3}x + 4; \end{aligned}$$

hence, the gradient of this line is $\underline{\underline{-\tfrac{4}{3}}}$.

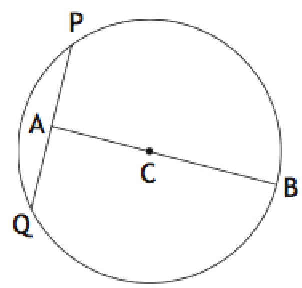
(b) Find the coordinates of the point where this line crosses the x-axis. (2)

Solution

$$y = 0 \Rightarrow 4x = 12 \Rightarrow x = 3;$$

hence, the coordinates of the point where this line crosses the x-axis is $\underline{\underline{(3, 0)}}$.

12. The diagram below shows a circle, centre C. (4)



The radius of the circle is 15 centimetres.
A is the mid-point of chord PQ.
The length of AB is 27 centimetres.
Calculate the length of PQ.

Solution

$$\begin{aligned} AC &= AB - CB \\ &= 27 - 15 \\ &= 12 \end{aligned}$$

and

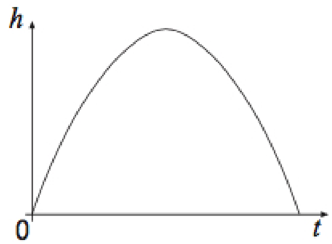
$$\begin{aligned} PA &= \sqrt{CP^2 - AC^2} \\ &= \sqrt{15^2 - 12^2} \\ &= \sqrt{225 - 144} \\ &= \sqrt{81} \\ &= 9. \end{aligned}$$

Hence,

$$PQ = 2PA = \underline{\underline{18 \text{ cm}}}.$$

13. The diagram below shows the path of a small rocket which is fired into the air. The height, h metres, of the rocket after t seconds is given by

$$\mathrm{h}(t) = 16t - t^2.$$



(a) After how many seconds will the rocket first be at a height of 60 metres? (4)

Solution

$$60 = 16t - t^2 \Rightarrow t^2 - 16t + 60 = 0$$

$$\left.\begin{array}{ll}\text{add to:} & -16 \\ \text{multiply to:} & +60\end{array}\right\} -6, -10$$

$$\begin{aligned} &\Rightarrow (t-6)(t-10) = 0 \\ &\Rightarrow t - 6 = 0 \text{ or } t - 10 = 0 \\ &\Rightarrow t = 6 \text{ or } t = 10; \end{aligned}$$

hence, $\underline{\underline{t = 6 \text{ s}}}$.

(b) Will the rocket reach a height of 70 metres? (3)
Justify your answer.

Solution

$$\frac{6 + 10}{2} = 8$$

and

$$\begin{aligned} \mathrm{h}(8) &= (16 \times 8) - 8^2 \\ &= 128 - 64 \\ &= 64 \end{aligned}$$

and hence the rocket <u>will not</u> reach a height of 70 metres.

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