Dr Oliver Mathematics
Rational Expressions: Part 1

1. If
$$\mathrm{f}(x) = \frac{4}{x-1} \text{ and } \mathrm{g}(x) = 2x,$$
what is the solution set of
$$\mathrm{f}(\mathrm{g}(x)) = \mathrm{g}(\mathrm{f}(x))?$$

Solution

Well,
$$\begin{aligned}\mathrm{f}(\mathrm{g}(x)) &= \mathrm{f}(2x)\\ &= \frac{4}{(2x)-1}\\ &= \frac{4}{2x-1}\end{aligned}$$
and
$$\begin{aligned}\mathrm{g}(\mathrm{f}(x)) &= \mathrm{g}\left(\frac{4}{x-1}\right)\\ &= \frac{8}{x-1}.\end{aligned}$$
Now,
$$\begin{aligned}\mathrm{f}(\mathrm{g}(x)) = \mathrm{g}(\mathrm{f}(x)) &\Rightarrow \frac{4}{2x-1} = \frac{8}{x-1}\\ &\Rightarrow 4(x-1) = 8(2x-1)\\ &\Rightarrow 4x-4 = 16x-8\\ &\Rightarrow 12x = 4\\ &\Rightarrow \underline{\underline{x = \tfrac{1}{3}}}.\end{aligned}$$