Dr Oliver Mathematics
GCSE Mathematics
2021 November Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. (a) Work out (3)

$$3.67 \times 4.2.$$

Solution

$$\begin{array}{ccccc}
 & & 3^{12} & 6^{12} & 7 \\
\times & & & 4 & 2 \\
\hline
 & & 7 & 3 & 4 \\
1 & 4_1 & 6_1 & 8 & \\
\hline
1 & 5 & 4 & 1 & 4 \\
\hline
\end{array}$$

After the decimal point: *two* digits in the first number (67) and *one* number in the second (2), we need *three*. Hence,

$$3.67 \times 4.2 = \underline{\underline{15.414}}.$$

(b) Work out (3)

$$59.84 \div 1.6.$$

Solution

$$\begin{array}{r|cccc}
 & & 3 & 7 & .4 \\
\hline
1.6 & 5 & 9 & 8 & .4 \\
 & 4 & 8 & & \\
\hline
 & 1 & 1 & 8 & .4 \\
 & 1 & 1 & 2 & \\
\hline
 & & & 6 & .4 \\
 & & & 6 & .4 \\
\hline
 & & & & 0
\end{array}$$

Hence,

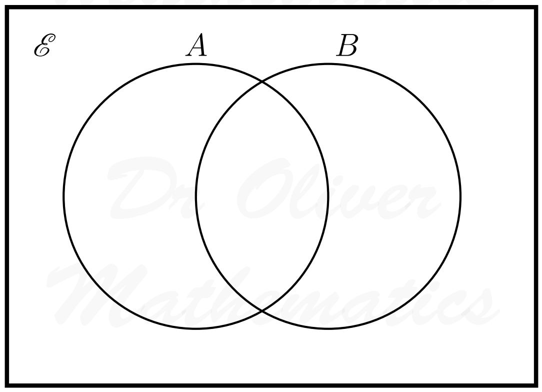
$$59.84 \div 1.6 = \underline{\underline{37.4}}.$$

2. $\mathscr{E} = \{\text{even numbers less than } 19\}$. (3)

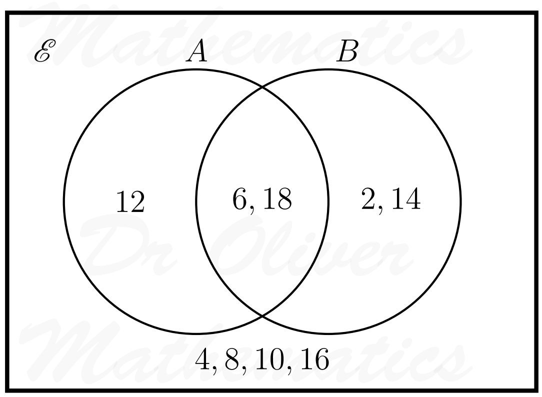
   $A = \{6, 12, 18\}$.

   $B = \{2, 6, 14, 18\}$.

   Complete the Venn diagram for this information.



Solution



3. Work out (3)

$$4\tfrac{1}{5} - 2\tfrac{2}{3}.$$

   Give your answer as a mixed number.

Solution

$$
\begin{aligned}
4\tfrac{1}{5} - 2\tfrac{2}{3} &= 2 + \tfrac{1}{5} - \tfrac{2}{3} \\
&= 2 + \tfrac{3}{15} - \tfrac{10}{15} \\
&= 2 - \tfrac{7}{15} \\
&= \underline{\underline{1\tfrac{8}{15}}}.
\end{aligned}
$$

4. At the end of 2017, (4)

- the value of Tamara's house was £220 000 and
- the value of Rahim's house was £160 000.

At the end of 2019,

- the value of Tamara's house had decreased by 20% and
- at the value of Rahim's house had increased by 30%.

At the end of 2019, whose house had the greater value?
You must show how you get your answer.

Solution

Tamara's house is now worth

$$
\begin{aligned}
220\,000 - (220\,000 \times 0.2) &= 220\,000 - 44\,000 \\
&= 176\,000
\end{aligned}
$$

and Rahim's house is now worth

$$
\begin{aligned}
160\,000 + (160\,000 \times 0.3) &= 160\,000 + 48\,000 \\
&= 208\,000.
\end{aligned}
$$

Hence, Rahim's house is now more.

5. Rosie, Matilda and Ibrahim collect stickers: (3)

$$\text{Rosie} : \text{Matilda} : \text{Ibrahim} = 4 : 7 : 15.$$

Ibrahim has 24 more stickers than Matilda.

Ibrahim has more stickers than Rosie.
How many more?

Solution

Well, suppose there are s stickers. Now,

$$4 + 7 + 15 = 26$$

and

$$\begin{aligned}\left(\frac{15-7}{26}\right) \times s = 24 &\Rightarrow \frac{8}{26} \times s = 24 \\ &\Rightarrow 8s = 624 \\ &\Rightarrow s = \frac{624}{8} \\ &\Rightarrow s = 78.\end{aligned}$$

So,

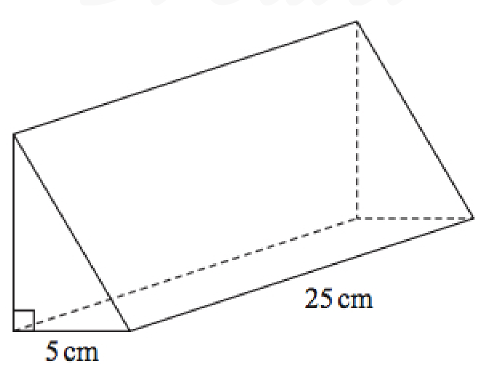
$$\begin{aligned}\text{Rosie has } &= \frac{4}{26} \times 78 = 4 \times 3 = 12, \\ \text{Matilda has } &= \frac{7}{26} \times 78 = 7 \times 3 = 21, \\ \text{Ibrahim has } &= \frac{13}{26} \times 78 = 15 \times 3 = 45.\end{aligned}$$

Ibrahim has

$$45 - 12 = \underline{\underline{33 \text{ stickers}}}$$

than Rosie.

6. The diagram shows a prism. (3)



The cross section of the prism is a right-angled triangle.
The base of the triangle has length 5 cm.

The prism has length 25 cm.
The prism has volume 750 cm^3.

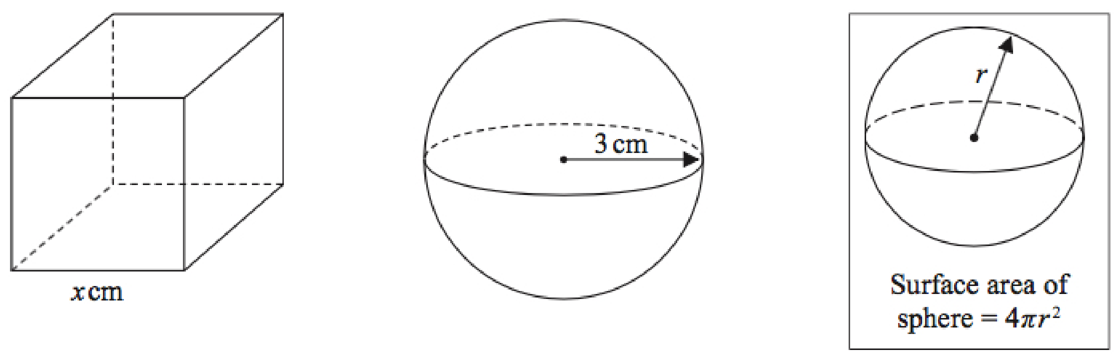
Work out the height of the prism.

Solution

Let h cm be the height of the prism. Then

$$\begin{aligned}(\tfrac{1}{2} \times 5 \times h) \times 25 = 750 &\Rightarrow \tfrac{1}{2} \times 5 \times h = 30 \\ &\Rightarrow \tfrac{1}{2} \times h = 6 \\ &\Rightarrow \underline{\underline{h = 12}}.\end{aligned}$$

7. The diagram shows a cube with edges of length x cm and a sphere of radius 3 cm. (4)



The surface area of the cube is equal to the surface area of the sphere.

Show that

$$x = \sqrt{k\pi},$$

where k is an integer.

Solution

Well,

$$6x^2 = 4 \times \pi \times 3^2 \Rightarrow 6x^2 = 36\pi$$
$$\Rightarrow x^2 = 6\pi$$
$$\Rightarrow \underline{\underline{x = \sqrt{6\pi}}};$$

hence, $\underline{\underline{k = 6}}$.

8. Solve (3)

$$x^2 = 5x + 24.$$

Solution

$$x^2 = 5x + 24 \Rightarrow x^2 - 5x - 24 = 0$$

$$\left.\begin{array}{ll}\text{add to:} & -5 \\ \text{multiply to:} & -24\end{array}\right\} -8,\ +3$$

$$\Rightarrow (x-8)(x+3) = 0$$
$$\Rightarrow x - 8 = 0 \text{ or } x + 3 = 0$$
$$\Rightarrow \underline{\underline{x = 8 \text{ or } x = -3}}.$$

9. (a) Write down the value of (1)

$$7^0.$$

Solution

$$7^0 = \underline{\underline{1}}.$$

(b) Find the value of (1)

$$3 \times 3^6 \times 3^{-6}.$$

Solution

$$3 \times 3^6 \times 3^{-6} = \underline{\underline{3}}.$$

(c) Find the value of (1)

$$2^{-4}.$$

Solution

$$2^{-4} = \frac{1}{2^4} = \underline{\underline{\tfrac{1}{16}}}.$$

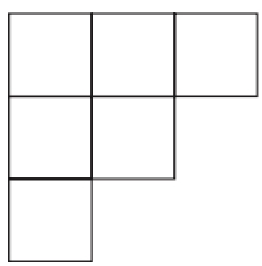
(d) Find the value of (1)

$$27^{\frac{1}{3}}.$$

Solution

$$27^{\frac{1}{3}} = \sqrt[3]{27} = \underline{\underline{3}}.$$

10. The diagram shows a shape made from 6 identical squares.



The total area of the shape is $5\,406 \text{ cm}^2$.

(a) Find an estimate for the length of one side of each square. (3)
Give your answer correct to the nearest whole number.

Solution

Let x cm be the length of one side. Now, the area of one small square is

$$\frac{5\,406}{6} = 901$$

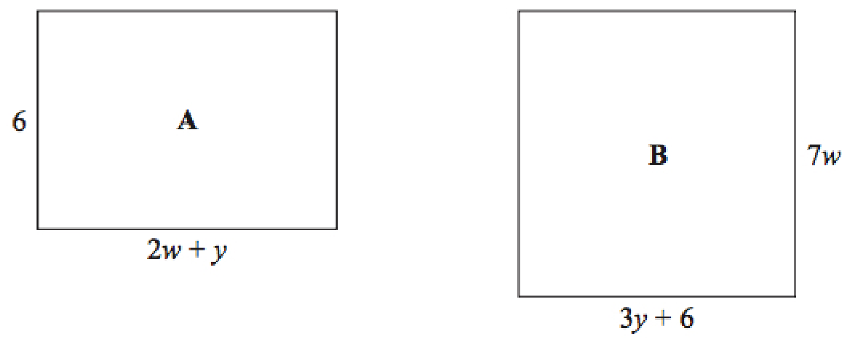
and the length is

$$x^2 = 901 \Rightarrow x^2 \approx 900$$
$$\Rightarrow \underline{\underline{x = 30}}.$$

(b) Is your answer to part (a) an underestimate or an overestimate? (1)
You must give a reason for your answer.

Solution
It is an underestimate as we went from $x^2 = 901$ to $x^2 \approx 900$.

11. The diagram shows two rectangles, **A** and **B**. (4)



All measurements are in centimetres.

The area of rectangle **A** is equal to the area of rectangle **A**.

Find an expression for y in terms of w.

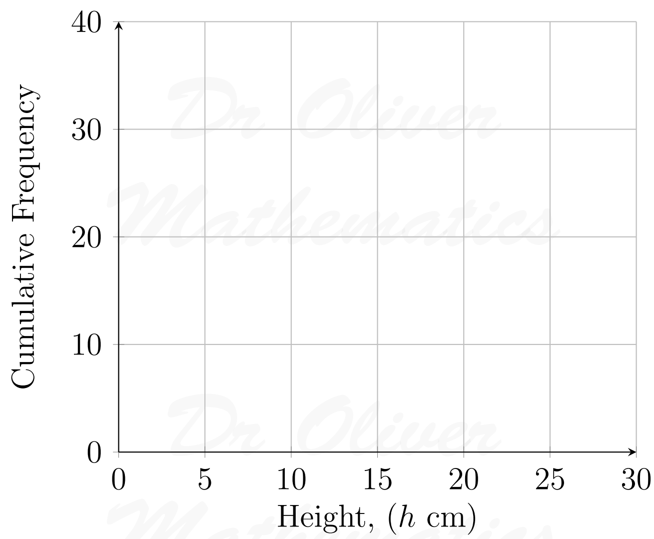
Solution
The areas are equal:

$$\begin{aligned} 6(2w + y) = 7w(3y + 6) &\Rightarrow 12w + 6y = 21wy + 42w \\ &\Rightarrow 6y - 21wy = 30w \\ &\Rightarrow 3y(2 - 7w) = 30w \\ &\Rightarrow = \underline{\underline{\frac{30w}{3(2 - 7w)}}}. \end{aligned}$$

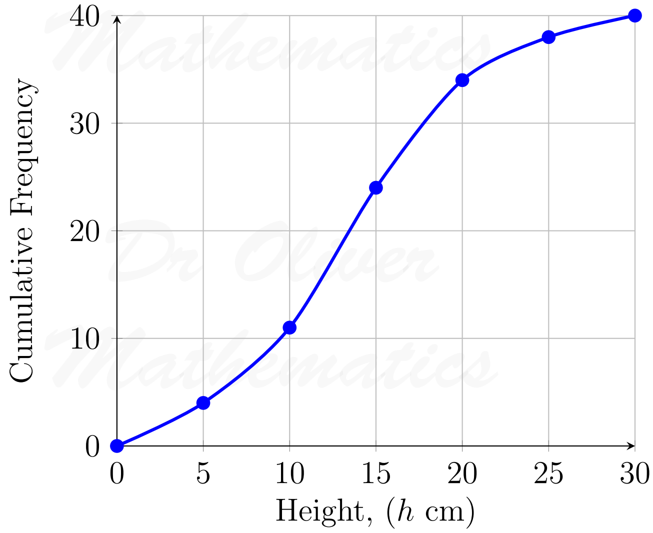
12. The cumulative frequency table gives information about the heights, in cm, of 40 plants.

Height, (h cm)	Cumulative Frequency
$0 < h \leqslant 5$	4
$0 < h \leqslant 10$	11
$0 < h \leqslant 15$	24
$0 < h \leqslant 20$	34
$0 < h \leqslant 25$	38
$0 < h \leqslant 30$	40

(a) On the grid, draw a cumulative frequency graph for this information. (2)



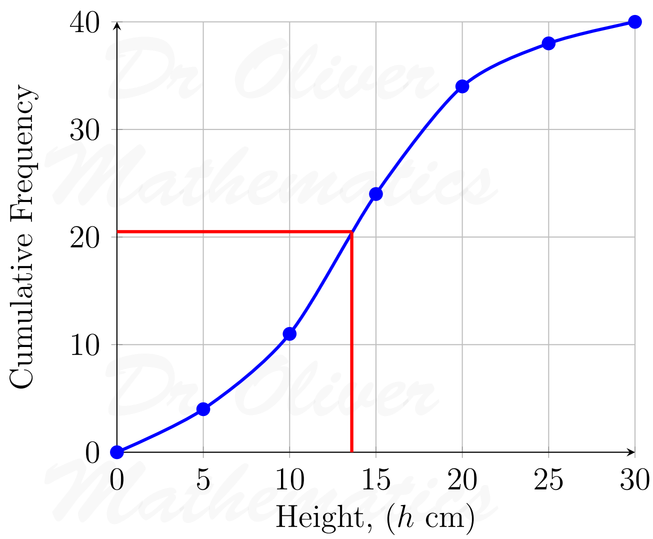
Solution



(b) **Solution** (1)

The median is at the

$$\frac{40+1}{2} = 20\tfrac{1}{2} \text{ place :}$$



Correct read-off: about 13.6 cm.

13. Ted is trying to change (1)

$$0.\dot{4}\dot{3}$$

to a fraction.

Here is the start of his method.

$$x = 0.\dot{4}\dot{3}$$
$$10x = 4.\dot{3}\dot{4}$$
$$10x - x = 4.\dot{3}\dot{4} - 0.\dot{4}\dot{3}$$

Evaluate Ted's method so far.

Solution

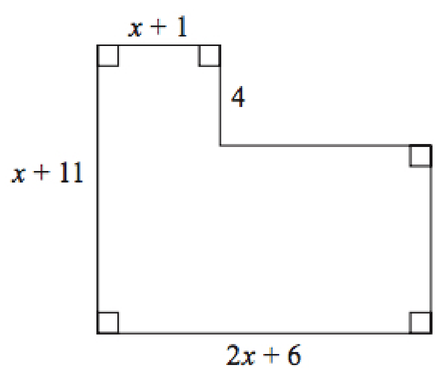
Not bad — but he has made a mistake: rather than

$$10x = 4.\dot{3}\dot{4},$$

he should have used

$$100x = 43.\dot{4}\dot{3}.$$

14. Here is a shape with all its measurements in centimetres. (3)



The area of the shape is A cm^2.

Show that

$$A = 2x^2 + 24x + 46.$$

Solution

Well,

$$(2x+6)-(x+1)=x+5$$

so we can look at a rectangle $(x+11)\times(2x+6)$ minus $4\times(x+5)$.

$\times$	x	$+11$
$2x$	$2x^2$	$+22x$
$+6$	$+6x$	$+66$

So

$$\begin{aligned} A &= (x+11)(2x+6)-4(x+5) \\ &= (2x^2+28x+66)-(4x+20) \\ &= \underline{\underline{2x^2+24x+46}}, \end{aligned}$$

as required.

15. Show that (3)

$$\frac{4x+3}{2x}+\frac{3}{5}$$

can be written in the form

$$\frac{ax+b}{cx},$$

where a, b, and c are integers.

Solution

$$\begin{aligned} \frac{4x+3}{2x}+\frac{3}{5} &= \frac{20x+15}{10x}+\frac{6x}{10x} \\ &= \frac{(20x+15)+6x}{10x} \\ &= \underline{\underline{\frac{26x+15}{10x}}}; \end{aligned}$$

hence,

$$\underline{\underline{a=26,\ b=15,\ \text{and}\ c=10}}.$$

16. There are only 3 red counters and 5 yellow counters in a bag. (4)

Jude takes at random 3 counters from the bag.

Work out the probability that he takes exactly one red counter.

Solution

'Exactly one red counter' means

$$\binom{3}{1} = 3$$

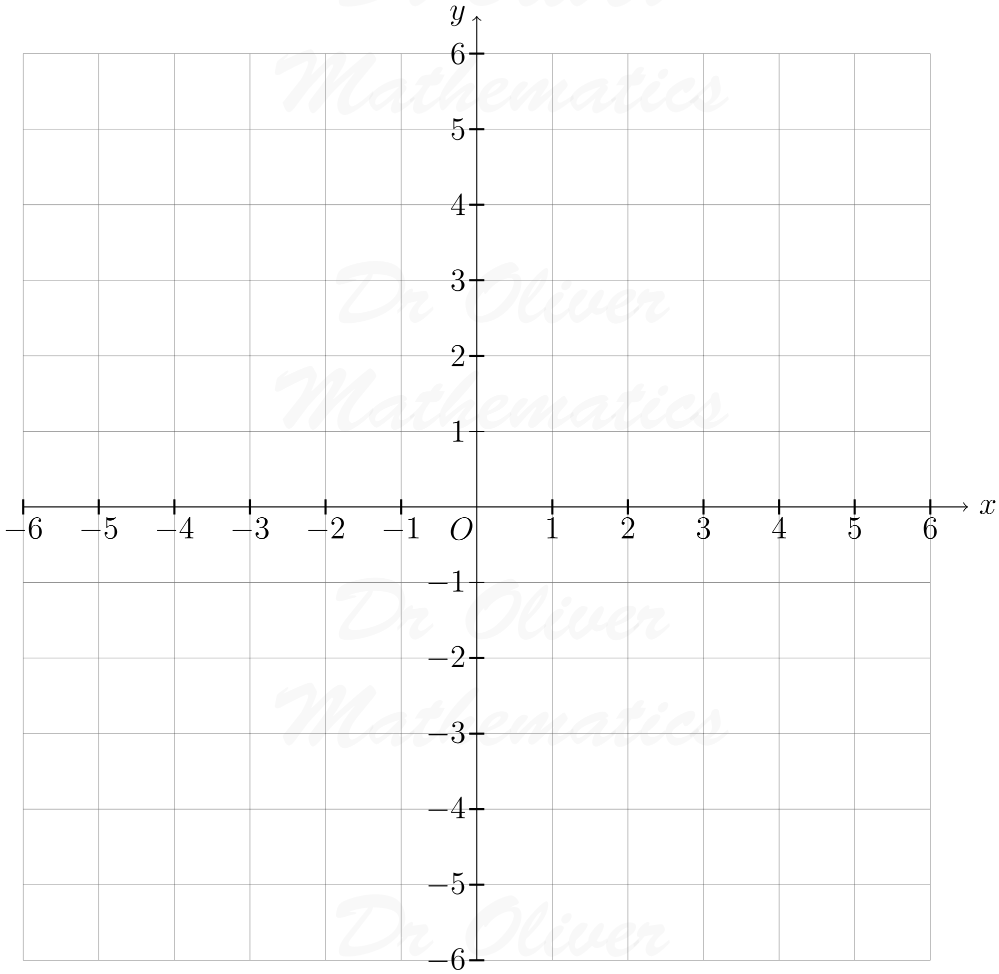
and the

$$\begin{aligned} \text{P(exactly one red counter)} &= 3 \times \tfrac{3}{8} \times \tfrac{5}{7} \times \tfrac{4}{6} \\ &= 3 \times \tfrac{1}{2} \times \tfrac{5}{7} \times \tfrac{1}{2} \\ &= \underline{\underline{\tfrac{15}{28}}}. \end{aligned}$$

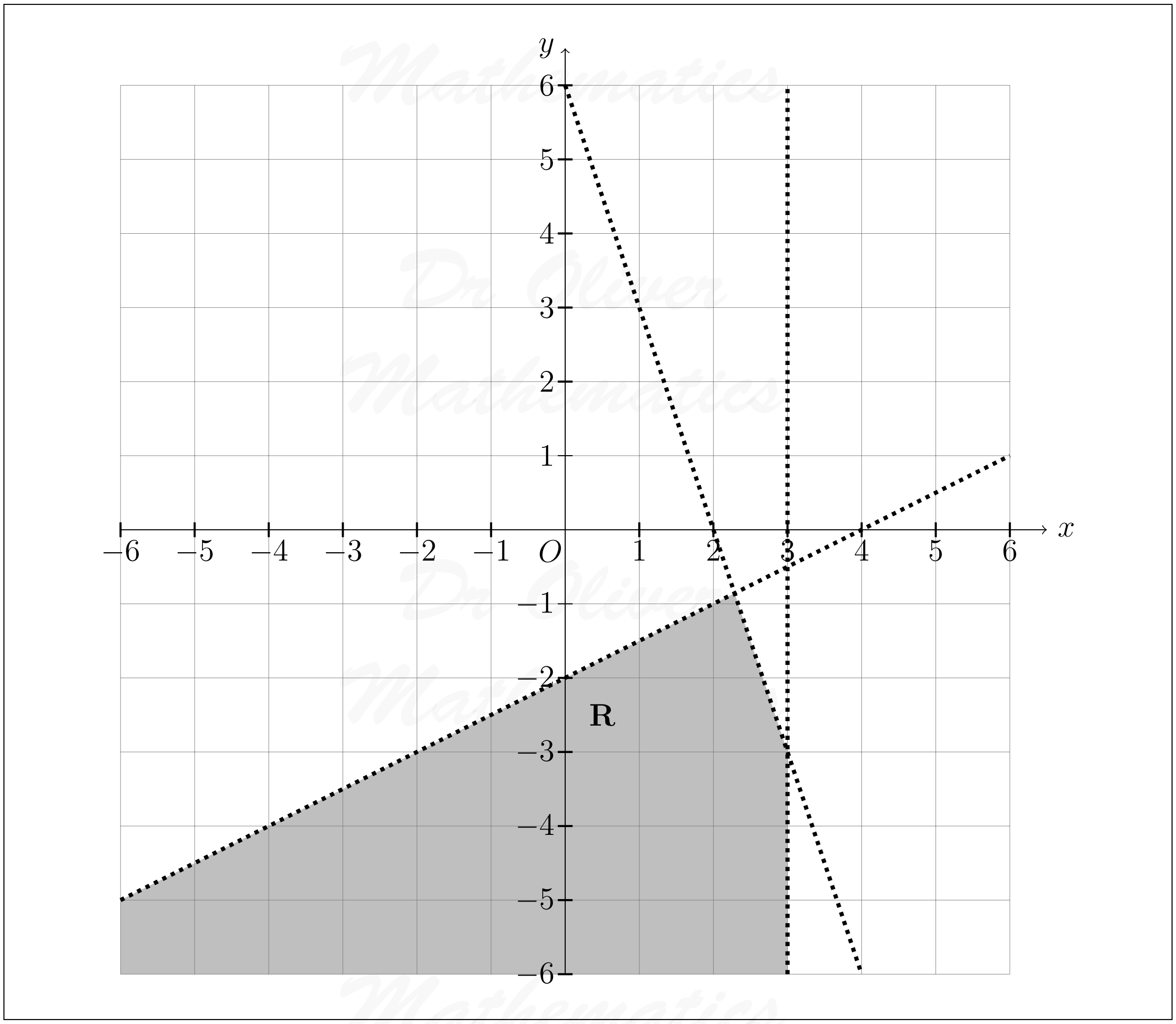
17. On the grid show, by shading, the region that satisfies all of these inequalities: (3)

$$2y + 4 < x \qquad x < 3 \qquad y < 6 - 3x.$$

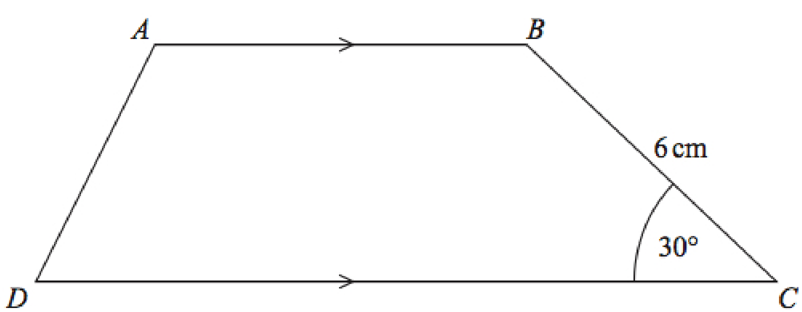
Label the region **R**.



Solution



18. Here is trapezium $ABCD$. (5)



The area of the trapezium is 66 cm^2.

The length of AB : the length of $CD = 2 : 3$.

Find the length of AB.

Solution

Let E be the foot of the perpendicular from B down to CD. Then

$$\sin = \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin 30^\circ = \frac{BE}{6}$$
$$\Rightarrow BE = 6 \sin 30^\circ$$
$$\Rightarrow BE = 3 \text{ cm}.$$

Now,

$$\text{the length of } AB : \text{the length of } CD = 2 : 3$$

which means

$$\text{the length of } AB : \text{the length of } CD = 1 : 1.5.$$

Now,

$$\tfrac{1}{2} \times 3 \times (AB + CD) = 66 \Rightarrow 3 \times (AB + \tfrac{3}{2}AB) = 132$$
$$\Rightarrow \tfrac{5}{2}AB = 44$$
$$\Rightarrow 5AB = 88$$
$$\Rightarrow \underline{\underline{AB = 17.6 \text{ cm}}}$$

19. Show that (4)

$$\frac{8 + \sqrt{12}}{5 + \sqrt{3}}$$

can be written in the form

$$\frac{a + \sqrt{3}}{b},$$

where a and b are integers.

Solution

Now,

$$\sqrt{12} = \sqrt{4 \times 3}$$
$$= \sqrt{4} \times \sqrt{3}$$
$$= 2\sqrt{3}$$

and

$$\frac{8+\sqrt{12}}{5+\sqrt{3}} = \frac{8+2\sqrt{3}}{5+\sqrt{3}}$$
$$= \frac{8+2\sqrt{3}}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$$

$\times$	8	$+2\sqrt{3}$
5	40	$+10\sqrt{3}$
$-\sqrt{3}$	$-8\sqrt{3}$	-6

$\times$	5	$+\sqrt{3}$
5	25	$+5\sqrt{3}$
$-\sqrt{3}$	$-5\sqrt{3}$	-3

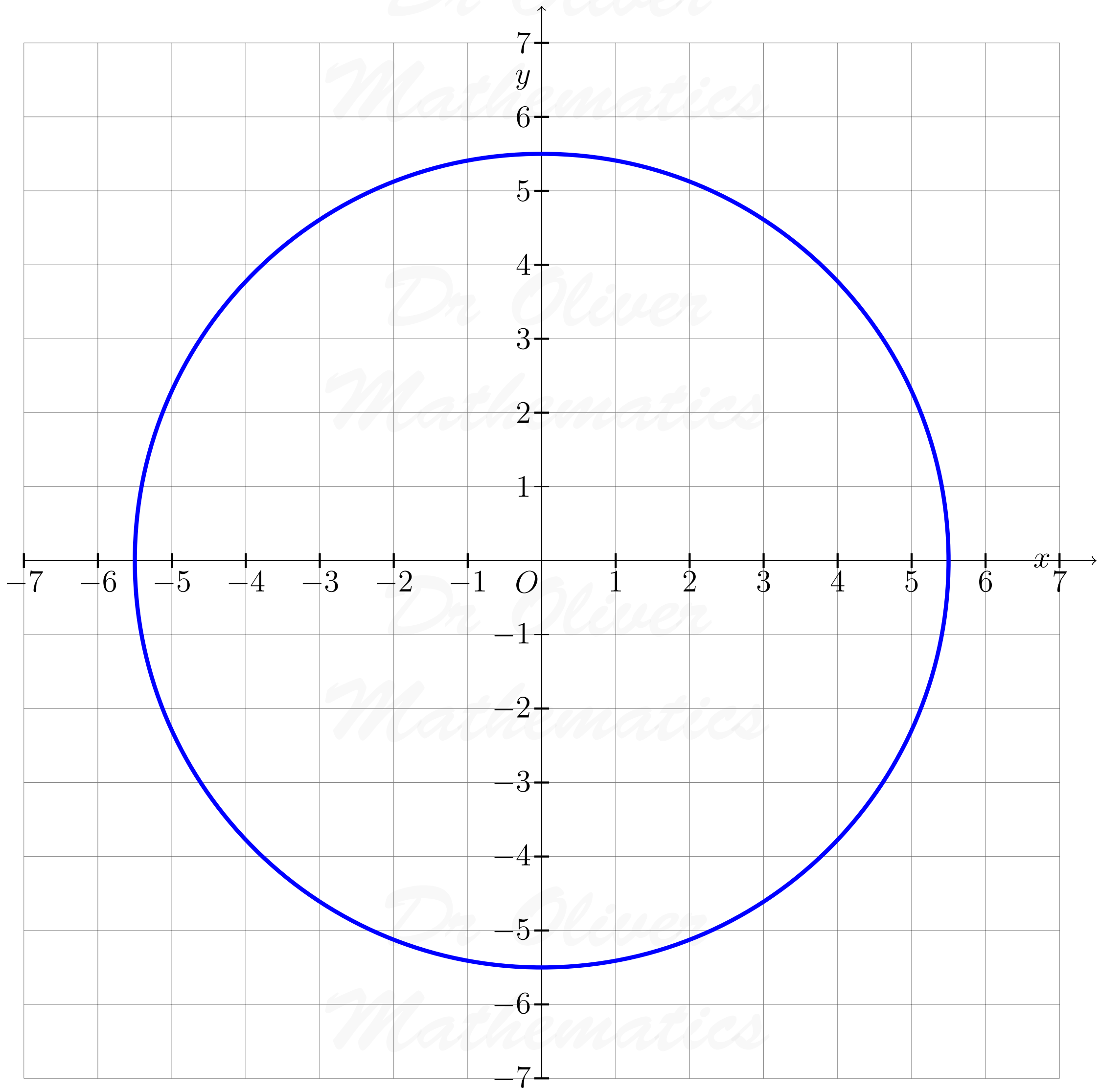
$$= \frac{34+2\sqrt{3}}{22}$$
$$= \underline{\underline{\frac{17+\sqrt{3}}{11}}};$$

hence,

$$\underline{\underline{a = 17,\ b = 1,\ \text{and}\ c = 11}}.$$

20. The diagram shows the graph of (3)

$$x^2 + y^2 = 30.25.$$

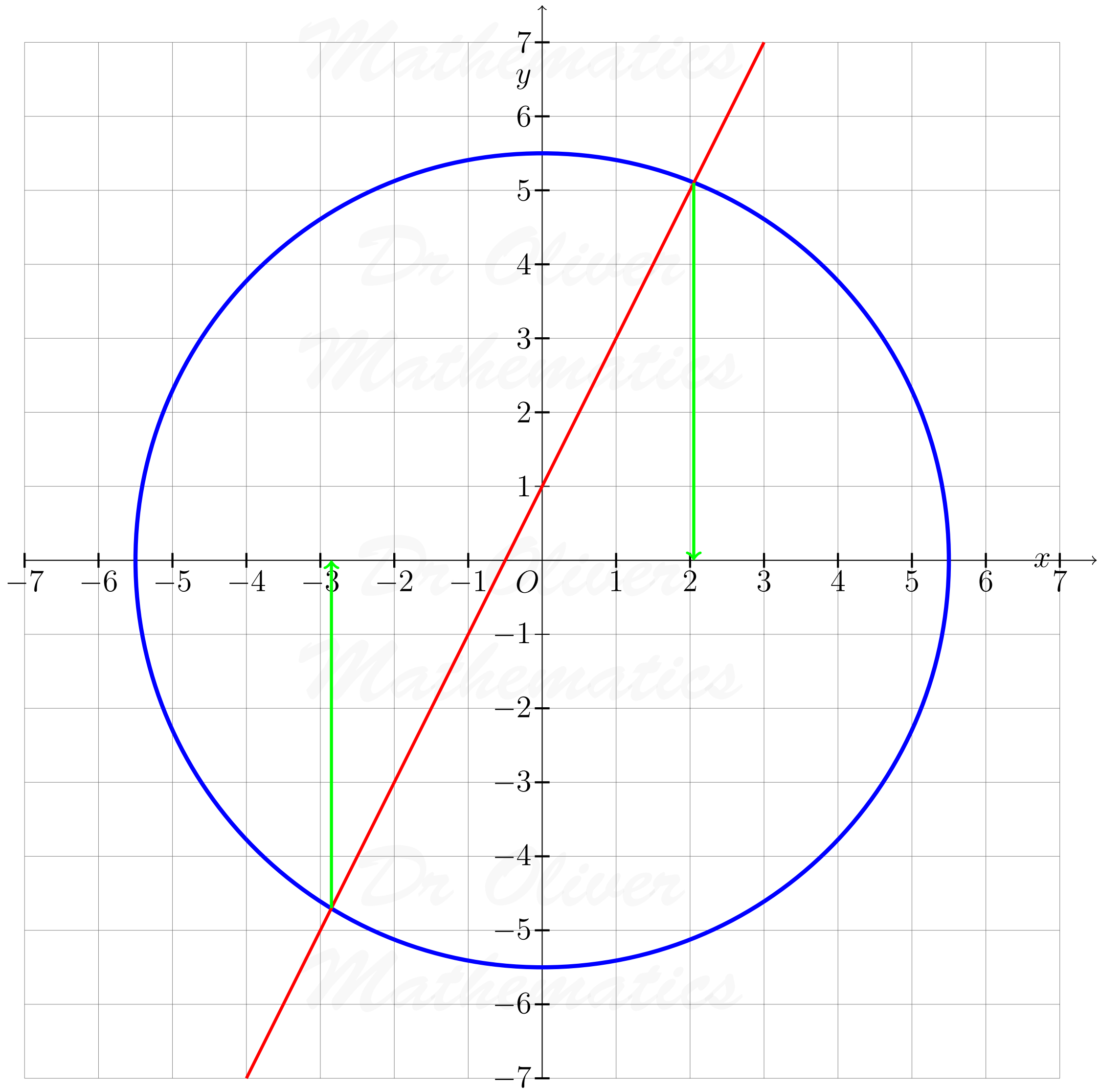


Use the graph to find estimates for the solutions of the simultaneous equations

$$x^2 + y^2 = 30.25$$
$$y - 2x = 1.$$

Solution

$$y - 2x = 1 \Rightarrow y = 2x + 1 :$$



Read-off the intersections between the curve and the line:

$$\underline{\underline{x = -2.85,\ y = -4.7 \text{ or } x = 2.05,\ y = 5.1.}}$$

21. The functions f and g are such that

$$\mathrm{f}(x) = 3x^2 + 1 \text{ for } x > 0$$

and

$$\mathrm{g}(x) = \frac{4}{x^2} \text{ for } x > 0.$$

(a) Work out g f(1). (2)

Solution

$$\begin{aligned} \mathrm{g\,f}(1) &= \mathrm{g}(\mathrm{f}(1)) \\ &= \mathrm{g}(4) \\ &= \frac{4}{4^2} \\ &= \underline{\underline{\tfrac{1}{4}}}. \end{aligned}$$

The function h is such that

$$h = (\mathrm{f\,g})^{-1}.$$

(b) Find $\mathrm{h}(x)$. (4)

Solution

Well,

$$\begin{aligned} \mathrm{f\,g}(x) &= \mathrm{f}(\mathrm{g}(x)) \\ &= \mathrm{f}\left(\frac{4}{x^2}\right) \\ &= 3\left(\frac{4}{x^2}\right)^2 + 1 \\ &= \frac{48}{x^4} + 1. \end{aligned}$$

Now,

$$\begin{aligned} y = \frac{48}{x^4} + 1 \Rightarrow y - 1 &= \frac{48}{x^4} \\ \Rightarrow \frac{1}{y-1} &= \frac{x^4}{48} \\ \Rightarrow \frac{148}{y-1} &= x^4 \\ \Rightarrow \sqrt[4]{\frac{48}{y-1}} &= x; \end{aligned}$$

hence,

$$\underline{\underline{\mathrm{h}(x) = \sqrt[4]{\frac{48}{x-1}}}}.$$

22. Find the coordinates of the turning point on the curve with equation (4)

$$y = 9 + 18x - 3x^2.$$

You must show all your working.

Solution

Well,

$$\begin{aligned} y &= 9 + 18x - 3x^2 \\ &= 9 - 3(x^2 - 6x) \\ &= 9 - 3[(x^2 - 6x + 9) - 9] \\ &= 9 - 3[(x-3)^2 - 9] \\ &= 9 - 3(x-3)^2 + 27 \\ &= 36 - 3(x-3)^2; \end{aligned}$$

hence, the turning point is $\underline{\underline{(3, 36)}}$.