Dr Oliver Mathematics
AQA Further Maths Level 2
January 2013 Paper 1
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.
You are **not** permitted to use a scientific or graphical calculator in this paper.

1. The line $y = mx + c$ passes through the point $(4, 3)$. (3)
   It is parallel to the line $y = 5x + 6$.

   Work out the values of m and c.

Solution

Clearly, $\underline{\underline{m = 5}}$. Finally,

$$\begin{aligned} 3 = (5 \times 4) + c &\Rightarrow 3 = 20 + c \\ &\Rightarrow \underline{\underline{c = -17}}. \end{aligned}$$

2. The matrix (4)

$$\begin{pmatrix} 5 & b \\ 4 & -1 \end{pmatrix}$$

maps the point $(a, 2)$ onto the point $(28, 18)$ such that

$$\begin{pmatrix} 5 & b \\ 4 & -1 \end{pmatrix} \begin{pmatrix} a \\ 2 \end{pmatrix} = \begin{pmatrix} 28 \\ 18 \end{pmatrix}.$$

Work out the values of a and b.

Solution

$$\begin{pmatrix} 5 & b \\ 4 & -1 \end{pmatrix} \begin{pmatrix} a \\ 2 \end{pmatrix} = \begin{pmatrix} 5a + 2b \\ 4a - 2 \end{pmatrix}.$$

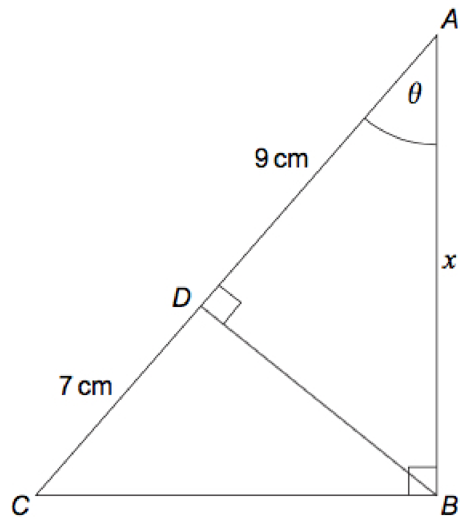
Now,

$$\begin{aligned} 4a - 2 = 18 &\Rightarrow 4a = 20 \\ &\Rightarrow \underline{\underline{a = 5}} \end{aligned}$$

and

$$5(5) + 2b = 28 \Rightarrow 2b = 3$$
$$\Rightarrow \underline{\underline{b = 1\tfrac{1}{2}}}.$$

3. ABC is a right-angled triangle.
   D is a point on AC.
   BD is perpendicular to AC.



Not drawn accurately

(a) Use triangle ABC to write $\cos\theta$ in terms of x. (1)

Solution
In $\triangle ABC$,

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \underline{\underline{\cos\theta = \frac{x}{16}}}.$$

(b) By writing another expression for $\cos\theta$ in terms of x, or otherwise, work out the value of x. (2)

Solution
In $\triangle ABD$,

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos\theta = \frac{9}{x}$$

and, hence,

$$\frac{9}{x} = \frac{x}{16} \Rightarrow x^2 = 144$$
$$\Rightarrow \underline{\underline{x = 12 \text{ cm}}}.$$

4. $w \blacktriangledown h$ is defined as

$$5w^2 - 8w + h^2 - 2h.$$

For example,

$$\begin{aligned} 1 \blacktriangledown 6 &= (5 \times 1^2) - (8 \times 1) + 6^2 - (2 \times 6) \\ &= 5 - 8 + 36 - 12 \\ &= 21. \end{aligned}$$

(a) Work out $2 \blacktriangledown 4$. (2)

Solution

$$\begin{aligned} 2 \blacktriangledown 4 &= (5 \times 2^2) - (8 \times 2) + 4^2 - (2 \times 4) \\ &= 20 - 16 + 16 - 8 \\ &= \underline{\underline{12}}. \end{aligned}$$

(b) Solve $x \blacktriangledown 3 = 0$. (4)

Solution

$$\begin{aligned} x \blacktriangledown 3 = 0 &\Rightarrow (5 \times x^2) - (8 \times x) + 3^2 - (2 \times 3) = 0 \\ &\Rightarrow 5x^2 - 8x + 3 = 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & -8 \\ \text{multiply to:} & (+5) \times (+3) = +15 \end{array}\right\} -5, -3$$

$$\begin{aligned} &\Rightarrow 5x^2 - 5x - 3x + 3 = 0 \\ &\Rightarrow 5x(x - 1) - 3(x - 1) = 0 \\ &\Rightarrow (5x - 3)(x - 1) = 0 \\ &\Rightarrow 5x - 3 = 0 \text{ or } x - 1 = 0 \\ &\Rightarrow \underline{\underline{x = \tfrac{3}{5} \text{ or } x = 1}}. \end{aligned}$$

5. n is a positive integer.

(a) Write down the **next** odd number after $2n - 1$. (1)

Solution

$\underline{\underline{2n + 1}}$.

(b) Prove that the product of two consecutive odd numbers is **always** one less than a multiple of 4. (3)

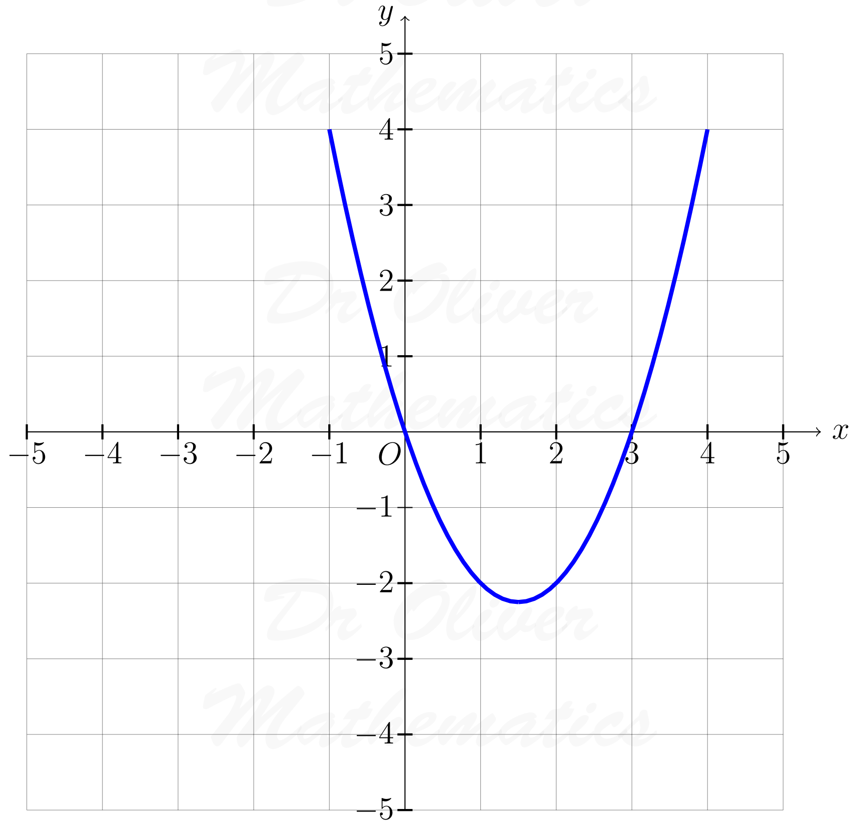
Solution

$\times$	$2n$	-1
$2n$	$4n^2$	$-2n$
$+1$	$+2n$	-1

$$\begin{aligned}(2n - 1)(2n + 1) + 1 &= (4n^2 - 1) + 1 \\ &= 4n^2 \\ &= 4 \times \text{some integer};\end{aligned}$$

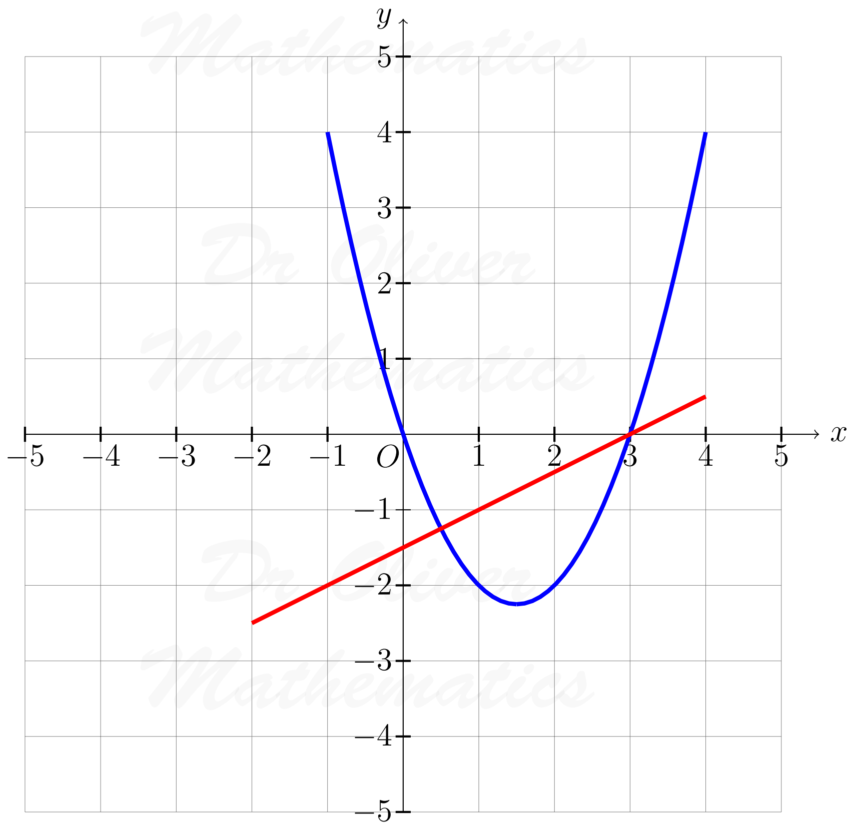
hence, the product of two consecutive odd numbers is always <u>one less</u> than a multiple of 4.

6. The diagram shows a sketch of $y = x^2 - 3x$.



(a) Sketch the line $y = \frac{1}{2}(x - 3)$ on the diagram. (2)
Mark the value where this line crosses the y-axis.

Solution



$\underline{\underline{(0, -1\frac{1}{2})}}$.

(b) By factorising $x^2 - 3x$, or otherwise, work out the smaller solution of (2)

$$x^2 - 3x = \tfrac{1}{2}(x - 3).$$

Solution

$$\begin{aligned} x^2 - 3x = \tfrac{1}{2}(x-3) &\Rightarrow x(x-3) = \tfrac{1}{2}(x-3) \\ &\Rightarrow x(x-3) - \tfrac{1}{2}(x-3) = 0 \\ &\Rightarrow \tfrac{1}{2}(x-3)(2x-1) = 0 \\ &\Rightarrow x - 3 \text{ or } 2x - 1 = 0 \\ &\Rightarrow x = 3 \text{ or } \underline{\underline{x = \tfrac{1}{2}}}. \end{aligned}$$

7. (4)

$$y = \frac{2x^2(3x^3 - 7x)}{x}.$$

Work out $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Solution

$$\begin{aligned} y = \frac{2x^2(3x^3 - 7x)}{x} &\Rightarrow y = \frac{6x^5 - 14x^3}{x} \\ &\Rightarrow y = 6x^4 - 14x^2 \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 24x^3 - 28x}}. \end{aligned}$$

8. $\mathrm{f}(x)$ is a decreasing function. (4)
$\mathrm{f}(x) = b - ax$ for $4 \leqslant x < 8$.
The range of $\mathrm{f}(x)$ is $5 < \mathrm{f}(x) \leqslant 7$.

Work out the values of a and b.

Solution

$$\begin{aligned} \mathrm{f}(4) = 7 \Rightarrow 7 = b - 4a \quad (1) \\ \mathrm{f}(8) = 5 \Rightarrow 5 = b - 8a \quad (2). \end{aligned}$$

Do (1) − (2):

$$\begin{aligned} 2 = 4a &\Rightarrow \underline{\underline{a = \tfrac{1}{2}}} \\ &\Rightarrow 7 = b - 4(\tfrac{1}{2}) \\ &\Rightarrow 7 = b - 2 \\ &\Rightarrow \underline{\underline{b = 9}}. \end{aligned}$$

9. Bag A contains $7x$ counters.
Bag B contains $2x$ counters.
Five counters are taken from bag A and put in bag B.

(a) Write an expression, in terms of x, for the number of counters now in bag B. (1)

Solution

$\underline{\underline{2x+5}}$.

The ratio of counters in bag A to bag B is now $8:3$.

(b) Use algebra to work out the **total** number of counters in the bags. (4)

Solution

$$\begin{aligned} 7x-5 : 2x+5 = 8:3 &\Rightarrow \frac{7x-5}{2x+5} = \frac{8}{3} \\ &\Rightarrow 3(7x-5) = 8(2x+5) \\ &\Rightarrow 21x-15 = 16x+40 \\ &\Rightarrow 5x = 55 \\ &\Rightarrow x = 11. \end{aligned}$$

Now, at the start, there are 77 counters in bag A and 22 counters in bag B. Hence, the total number of counters in the bags is $\underline{\underline{99}}$.

10. Solve the simultaneous equations (5)

$$\begin{aligned} \frac{x-1}{y-2} &= 3 \\ \frac{x+6}{y-1} &= 4. \end{aligned}$$

Do **not** use trial and improvement.
You **must** show your working.

Solution

$$\begin{aligned} \frac{x-1}{y-2} = 3 &\Rightarrow x-1 = 3(y-2) \\ &\Rightarrow x-1 = 3y-6 \\ &\Rightarrow x = 3y-5 \quad (1) \end{aligned}$$

and

$$\frac{x+6}{y-1} = 4 \Rightarrow x + 6 = 4(y-1)$$
$$\Rightarrow x + 6 = 4y - 4$$
$$\Rightarrow x = 4y - 10 \quad (2).$$

Set (1) = (2):

$$3y - 5 = 4y - 10 \Rightarrow \underline{\underline{y = 5}}$$
$$\Rightarrow \underline{\underline{x = 10}}.$$

11. Write (2)

$$\sqrt{500} - 2\sqrt{45}$$

in the form $a\sqrt{5}$ where a is an integer.

Solution

$$\begin{aligned}\sqrt{500} - 2\sqrt{45} &= \sqrt{100 \times 5} - 2\sqrt{9 \times 5} \\ &= \sqrt{100} \times \sqrt{5} - 2\sqrt{9} \times \sqrt{5} \\ &= 10\sqrt{5} - 6\sqrt{5} \\ &= \underline{\underline{4\sqrt{5}}};\end{aligned}$$

hence, $\underline{\underline{a = 4}}$.

12. Simplify fully (5)

$$\frac{4x^2 + 19x - 5}{9x^2 - 16} \div \frac{x+5}{3x-4}.$$

Solution

$$\left.\begin{array}{ll}\text{add to:} & +19 \\ \text{multiply to:} & (+4) \times (-5) = -20\end{array}\right\} + 20, -1$$

$$
\begin{aligned}
4x^2 + 19x - 5 &= 4x^2 + 20x - x - 5 \\
&= 4x(x+5) - (x+5) \\
&= (4x-1)(x+5).
\end{aligned}
$$

$$
\left.\begin{array}{ll}
\text{add to:} & 0 \\
\text{multiply to:} & (+9) \times (-16) = -144
\end{array}\right\} +12, -12
$$

$$
\begin{aligned}
9x^2 - 16 &= 9x^2 + 12x - 12x - 16 \\
&= 3x(3x+4) - 4(3x+4) \\
&= (3x-4)(3x+4).
\end{aligned}
$$

Now,

$$
\begin{aligned}
\frac{4x^2+19x-5}{9x^2-16} \div \frac{x+5}{3x-4} &= \frac{4x^2+19x-5}{9x^2-16} \times \frac{3x-4}{x+5} \\
&= \frac{(4x-1)(x+5)}{(3x-4)(3x+4)} \times \frac{3x-4}{x+5} \\
&= \underline{\underline{\frac{4x-1}{3x+4}}}.
\end{aligned}
$$

13.

$$y = 2x^3 - 12x^2 + 24x - 11.$$

(a) Work out $\frac{dy}{dx}$. (3)

Give your answer in the form $\frac{dy}{dx} = a(x-b)^2$, where a and b are integers.

Solution

$$
\begin{aligned}
y = 2x^3 - 12x^2 + 24x - 11 \Rightarrow \frac{dy}{dx} &= 6x^2 - 24x + 24 \\
\Rightarrow \frac{dy}{dx} &= 6(x^2 - 4x + 4)
\end{aligned}
$$

$$
\left.\begin{array}{ll}
\text{add to:} & -4 \\
\text{multiply to:} & +4
\end{array}\right\} -2, -2
$$

$$\Rightarrow \underline{\underline{\frac{dy}{dx} = 6(x-2)^2}};$$

hence, $\underline{\underline{a = 6}}$ and $\underline{\underline{b = 2}}$.

(b) Hence, or otherwise, work out the coordinates of the stationary point of (2)

$$y = 2x^3 - 12x^2 + 24x - 11.$$

Solution

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = 0 &\Rightarrow 6(x-2)^2 = 0 \\ &\Rightarrow x - 2 = 0 \\ &\Rightarrow x = 2 \\ &\Rightarrow y = 16 - 48 + 48 - 11 \\ &\Rightarrow y = 5;\end{aligned}$$

hence, the coordinates of the stationary point is $\underline{\underline{(2, 5)}}$.

(c) Explain how you know that this stationary point is a point of inflection. (1)

Solution

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 6(x-2)^2 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 12(x-2)$$

and

$$x = 2 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0;$$

hence, this stationary point is a $\underline{\underline{\text{point of inflection}}}$.

14. (5)

$$x^2 - 2x + y^2 - 6y = 0$$

is the equation of a circle.
By writing the equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$

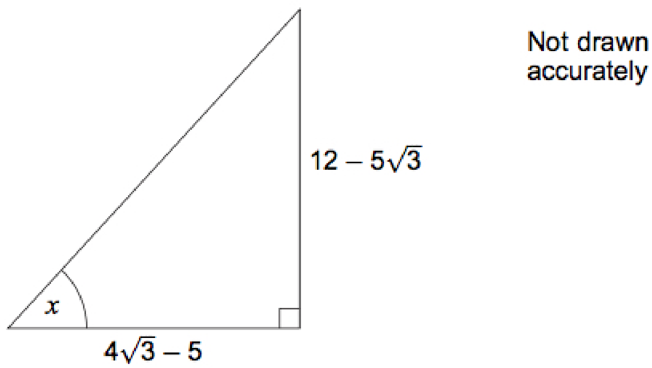
work out the centre and radius of the circle.

Solution

$$x^2 - 2x + y^2 - 6y = 0 \Rightarrow (x^2 - 2x + 1) + (y^2 - 6y + 9) = 1 + 9$$
$$\Rightarrow (x - 1)^2 + (y - 3)^2 = 10.$$

Hence, the centre is $\underline{\underline{(1, 3)}}$ and the radius is $\underline{\underline{\sqrt{10}}}$.

15. Show that angle $x = 60°$. (4)



You **must** show your working.

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan x = \frac{12 - 5\sqrt{3}}{4\sqrt{3} - 5}$$
$$\Rightarrow \tan x = \frac{(12 - 5\sqrt{3})(4\sqrt{3} + 5)}{(4\sqrt{3} - 5)(4\sqrt{3} + 5)}$$

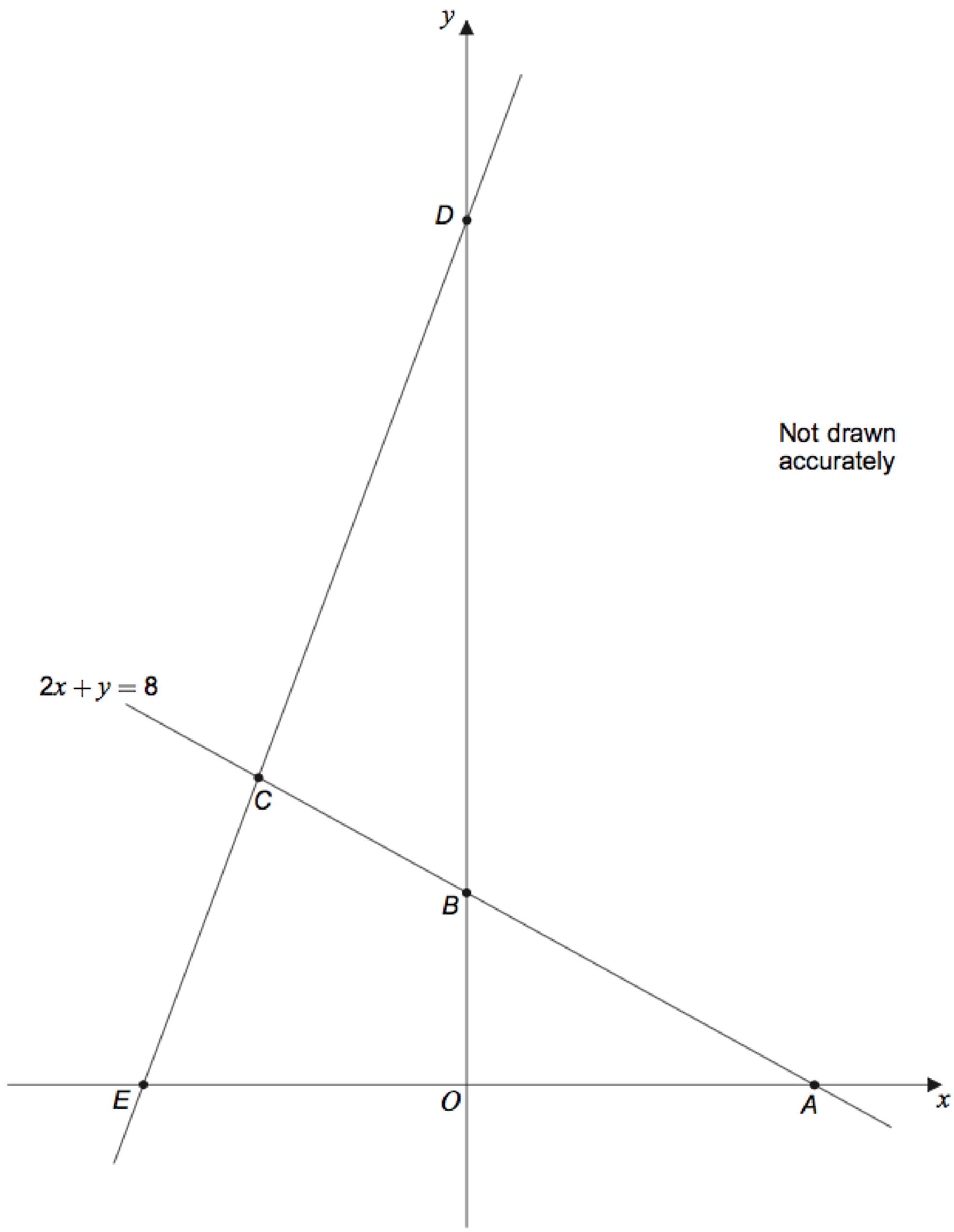
$\times$	12	$-5\sqrt{3}$
$4\sqrt{3}$	$48\sqrt{3}$	-60
$+5$	$+60$	$-25\sqrt{3}$

$\times$	$4\sqrt{3}$	-5
$4\sqrt{3}$	48	$-20\sqrt{3}$
$+5$	$+20\sqrt{3}$	-25

$$\Rightarrow \tan x = \frac{23\sqrt{3}}{23}$$
$$\Rightarrow \tan x = \sqrt{3}$$
$$\Rightarrow \underline{\underline{x = 60^\circ}},$$

as required.

16. A, B, and C are points on the line $2x + y = 8$. (6)
DCE is a straight line.
$AB : BC = 2 : 1$.
$EC : CD = 1 : 2$.



Work out the ratio

$$\text{area of triangle } AEC : \text{area of triangle } BCD.$$

Give your answer in its simplest form.

Solution

So, $A(4,0)$ and $B(0,8)$. Now, $AB : BC = 2 : 1$ which means the x-coordinate of C is $x = -2$ and

$$x = -2 \Rightarrow y = 12;$$

so, $C(-2,12)$. Next, $D(0,y)$ and $E(x,0)$. But $EC : CD = 1 : 2$ which means that $E(-3,0)$ and $D(0,36)$ (why?). Now,

$$\begin{aligned}\text{area of triangle } AEC &= \tfrac{1}{2} \times [(4-(-3)] \times 12 \\ &= \tfrac{1}{2} \times 7 \times 12 \\ &= 42\end{aligned}$$

and

$$\begin{aligned}\text{area of triangle } BCD &= \tfrac{1}{2} \times 2 \times (36-8) \\ &= \tfrac{1}{2} \times 2 \times 28 \\ &= 28.\end{aligned}$$

Finally,

$$\begin{aligned}\text{area of triangle } AEC : \text{area of triangle } BCD &= 42 : 28 \\ &= 14 \times 3 : 14 \times 2 \\ &= \underline{\underline{3 : 2}}.\end{aligned}$$