Dr Oliver Mathematics
Statistics
Normal Distribution
Past Examination Questions

This booklet consists of 30 questions across a variety of examination topics.
The total number of marks available is 287.

1. A scientist found that the time taken M minutes, to carry out an experiment can be modelled by a normal random variable with mean 155 minutes and standard deviation 3.5 minutes. Find

   (a) $\mathrm{P}(M > 160)$, (3)

Solution

$$\begin{aligned}
\mathrm{P}(M > 160) &= \mathrm{P}\left(Z > \tfrac{160-155}{3.5}\right) \\
&= \mathrm{P}(Z > 1.428\ldots) \\
&= 1 - \mathrm{P}(Z < 1.43) \\
&= 1 - \Phi(1.43) \\
&= 1 - 0.9236 \\
&= \underline{\underline{0.0764}}.
\end{aligned}$$

   (b) $\mathrm{P}(150 \leqslant M \leqslant 157)$, (4)

Solution

$$\begin{aligned}
\mathrm{P}(150 \leqslant M \leqslant 157) &= \mathrm{P}\left(\tfrac{150-155}{3.5} \leqslant Z \leqslant \tfrac{157-155}{3.5}\right) \\
&= \mathrm{P}(-1.428\ldots \leqslant Z \leqslant 0.571\ldots) \\
&= \Phi(0.57) - (1 - \Phi(1.43)) \\
&= 0.7157 - (1 - 0.9236) \\
&= \underline{\underline{0.6393}}.
\end{aligned}$$

   (c) the value of m, 1 decimal place, such that $\mathrm{P}(M \leqslant m) = 0.30$. (4)

Solution

$$\begin{aligned} \mathrm{P}(M \leqslant m) = 0.30 &\Rightarrow \tfrac{m-155}{3.5} = -0.5244 \\ &\Rightarrow m - 155 = -1.8354 \\ &\Rightarrow m = 153.1646 \\ &\Rightarrow \underline{\underline{m = 153.2 \text{ (1 dp)}}}. \end{aligned}$$

2. The heights of a group of athletes are modelled by a normal distribution with mean 180 cm and standard deviation 5.2 cm. The weights of this group of athletes are modelled by a normal distribution with mean 85 kg and standard deviation 7.1 kg. Find the probability that a randomly chosen athlete,

   (a) is taller than 188 cm, (3)

Solution

$$\begin{aligned} \mathrm{P}(H > 188) &= \mathrm{P}\left(Z > \tfrac{188-180}{5.2}\right) \\ &= \mathrm{P}(Z > 1.538\ldots) \\ &= 1 - \mathrm{P}(Z < 1.54) \\ &= 1 - \Phi(1.54) \\ &= 1 - 0.9382 \\ &= \underline{\underline{0.0618}}. \end{aligned}$$

   (b) weighs less than 97 kg. (2)

Solution

$$\begin{aligned} \mathrm{P}(W < 97) &= \mathrm{P}\left(W < \tfrac{97-85}{7.1}\right) \\ &= \mathrm{P}(Z < 1.690\ldots) \\ &= \mathrm{P}(Z < 1.69) \\ &= \Phi(1.69) \\ &= \underline{\underline{0.9545}}. \end{aligned}$$

   (c) Assuming that for these athletes height and weight are independent, find the probability that a randomly chosen athlete is taller than 188 cm and weighs more than 97 kg. (3)

Solution

$$
\begin{aligned}
\mathrm{P}(H > 188 \text{ and } W > 95) &= 0.0618 \times (1 - 0.9545) \\
&= 0.0618 \times 0.0455 \\
&= \underline{\underline{0.002\,811\,9}}.
\end{aligned}
$$

(d) Comment on the assumption that height and weight are independent. (1)

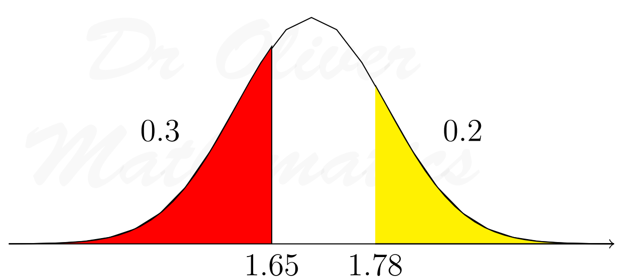
Solution

The evidence suggests that heights and weights are positively correlated meaning that the assumption of independence is not sensible.

3. From experience a high-jumper knows that he can clear a height of at least 1.78 m once in 5 attempts. He also knows that he can clear a height of at least 1.65 m on 7 out of 10 attempts. Assuming that the heights the high-jumper can reach follow a normal distribution,

(a) draw a sketch to illustrate the above information, (3)

Solution



(b) find, to 3 decimal places, the mean and the standard deviation of the heights the high-jumper can reach, (6)

Solution

$H \sim \mathrm{N}(\mu, \sigma^2)$ and we have

$$1.78 - \mu = 0.8416\sigma \text{ and } 1.65 - \mu = -0.5244\sigma :$$

$$0.13 = 1.366\sigma \Rightarrow \sigma = 0.095\,168\,374\,82 \text{ (FCD)} = \underline{\underline{0.095 \text{ (3 dp)}}}$$

and

$$\mu = 1.78 - 0.8416\sigma = 1.699\,906\,296 = \underline{\underline{1.700 \text{ (3 dp)}}}.$$

(c) calculate the probability that he can jump at least 1.74 m. (3)

Solution

$$\begin{aligned}
\mathrm{P}(H > 1.74) &= \mathrm{P}\left(Z > \tfrac{1.74-1.699...}{0.095...}\right) \\
&= \mathrm{P}(Z > 0.421\ldots) \\
&= 1 - \mathrm{P}(Z < 0.42) \\
&= 1 - \Phi(0.42) \\
&= 1 - 0.6628 \\
&= \underline{\underline{0.3372}}.
\end{aligned}$$

4. The measure of intelligence, IQ, of group of students is assumed to be normally distributed with mean 100 and standard deviation 15.

(a) Find the probability that a student selected at random has an IQ less than 91. (4)

Solution

We have $M \sim \mathrm{N}(100, 15^2)$ and

$$\begin{aligned}
\mathrm{P}(M < 91) &= \mathrm{P}\left(Z < \tfrac{91-100}{15}\right) \\
&= \mathrm{P}(Z < -0.6) \\
&= \mathrm{P}(Z > 0.6) \\
&= 1 - \mathrm{P}(Z < 0.6) \\
&= 1 - \Phi(0.6) \\
&= 1 - 0.7257 \\
&= \underline{\underline{0.2743}}.
\end{aligned}$$

The probability that a random selected student has an IQ of at least $(100 + k)$ is 0.2090.

(b) Find, to the nearest integer, the value of k. (6)

Solution

$$
\begin{aligned}
\mathrm{P}(M > 100 + k) = 0.2090 &\Rightarrow \mathrm{P}(M < 100 + k) = 0.7910 \\
&\Rightarrow \frac{(100 + k) - 100}{15} = 0.7910 \\
&\Rightarrow k = 11.865 \\
&\Rightarrow \underline{\underline{k = 12 \text{ (nearest integer)}}}.
\end{aligned}
$$

5. The random variable X has a normal distribution with mean 20 and standard deviation 4.

   (a) Find $\mathrm{P}(X > 25)$. (3)

Solution

$X \sim \mathrm{N}(20, 4^2)$ and we have

$$
\begin{aligned}
\mathrm{P}(X > 25) &= \mathrm{P}\left(Z > \tfrac{25-20}{4}\right) \\
&= \mathrm{P}(Z > 1.25) \\
&= 1 - \mathrm{P}(Z < 1.25) \\
&= 1 - \Phi(1.25) \\
&= 1 - 0.8944 \\
&= \underline{\underline{0.1056}}.
\end{aligned}
$$

   (b) Find the value of d such that $\mathrm{P}(20 < X < d) = 0.4641$. (4)

Solution

$$
\begin{aligned}
\mathrm{P}(20 < X < d) = 0.4641 &\Rightarrow \mathrm{P}(X < d) = 0.9641 \\
&\Rightarrow \frac{d - 20}{4} = 1.80 \\
&\Rightarrow d - 20 = 7.2 \\
&\Rightarrow \underline{\underline{d = 27.2}}.
\end{aligned}
$$

6. The masses of bags of popcorn are normally distributed with a mean of 200 g and 60% of all bags have a mass between 190 g and 210 g.

   (a) Write down the median mass of the bags of popcorn. (1)

Solution

$\underline{\underline{200 \text{ g}}}$.

(b) Find the standard deviation of the masses of the bags of popcorn. (5)

Solution

$M \sim \text{N}(200, \sigma^2)$ and we have $\text{P}(M < 210) = 0.8$ (why?). Now,

$$\begin{aligned} \text{P}(M < 210) = 0.8 &\Rightarrow \text{P}\left(Z < \tfrac{210-200}{\sigma}\right) = 0.8 \\ &\Rightarrow \text{P}\left(Z < \tfrac{10}{\sigma}\right) = 0.8 \\ &\Rightarrow \frac{10}{\sigma} = 0.8416 \\ &\Rightarrow \sigma = \underline{\underline{11\tfrac{232}{263} \text{ g}}}. \end{aligned}$$

A shopkeeper finds that customers will complain if their bag of popcorn has a mass of less than 180 g.

(c) Find the probability that a customer will complain. (3)

Solution

$$\begin{aligned} \text{P}(M < 180) &= \text{P}\left(Z < \frac{180 - 200}{11\frac{232}{263}}\right) \\ &= \text{P}(Z < -1.6832) \\ &= \text{P}(Z > 1.68) \\ &= 1 - \text{P}(Z < 1.68) \\ &= 1 - \Phi(1.68) \\ &= 1 - 0.9535 \\ &= \underline{\underline{0.0465}}. \end{aligned}$$

7. A packing plant fills bags with cement. The weight X kg of a bag of cement can be modelled by a normal distribution with mean 50 kg and standard deviation 2 kg.

(a) Find $\text{P}(X > 53)$. (3)

Solution

$X \sim \mathrm{N}(50, 2^2)$ and we have

$$\begin{aligned} \mathrm{P}(X > 53) &= \mathrm{P}\left(Z > \frac{53-50}{2}\right) \\ &= \mathrm{P}(Z > 1.5) \\ &= 1 - \mathrm{P}(Z < 1.5) \\ &= 1 - \Phi(1.5) \\ &= 1 - 0.9332 \\ &= \underline{\underline{0.0668}}. \end{aligned}$$

(b) Find the weight that is exceeded by 99% of the bags. (5)

Solution

$$\begin{aligned} \mathrm{P}(X < x) = 0.01 &\Rightarrow \frac{x-50}{2} = -2.3263 \\ &\Rightarrow x - 50 = -4.6526 \\ &= \underline{\underline{x = 43.3474 \text{ kg}}}. \end{aligned}$$

Three bags are selected at random.

(c) Find the probability that two weigh more than 53 kg and one weighs less than 53 kg. (4)

Solution

$$\begin{aligned} \mathrm{P}(\text{two} > 53, \text{one} < 53) &= 3 \times 0.0668^2 \times 0.9332 \\ &= \underline{\underline{0.012\,492\,487\,1 \text{ (FCD)}}}. \end{aligned}$$

8. The random variable has a normal distribution with mean 30 and standard deviation 5.

(a) Find $\mathrm{P}(X < 39)$. (2)

Solution

We need to standardise to get the problem in terms of the standard normal distribution, Z. So

$$\mathrm{P}(X < 39) = \mathrm{P}\left(Z < \frac{39-30}{5}\right) = \mathrm{P}(Z < 1.8) = \Phi(1.8) = \underline{\underline{0.9641}}.$$

(b) Find the value of d such that $\mathrm{P}(X < d) = 0.1151$. (4)

Solution

We want to find the value of z such that $\Phi(z) = 0.1151$. But this does not appear in our tables! So the value of z that we want has to be negative and we will use the relationship that $\Phi(-z) = 1 - \Phi(z)$. So

$$\begin{aligned}\Phi(-z) &= 1 - \Phi(z)\\ &= 1 - 0.1151\\ &= 0.8849\\ &\Rightarrow -z = 1.20\\ &\Rightarrow z = -1.20.\end{aligned}$$

Hence the standardised value of d is -1.20. So

$$\frac{d-30}{5} = -1.20 \Rightarrow \underline{\underline{d = 24}}.$$

(c) Find the value of e such that $\mathrm{P}(X > e) = 0.1151$. (2)

Solution

Given the symmetry of the normal distribution about the mean,

$$\frac{e-30}{5} = 1.20 \Rightarrow \underline{\underline{e = 36}}.$$

(d) Find $\mathrm{P}(d < X < e)$. (2)

Solution

$$\mathrm{P}(d < X < e) = 1 - \mathrm{P}(X < d) - \mathrm{P}(X > e) = 1 - 0.1151 - 0.1151 = \underline{\underline{0.7698}}.$$

9. The lifetimes of bulbs used in a lamp are normally distributed. A company X sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.

(a) Find the probability of a bulb, from company X having a lifetime of less than 830 hours. (3)

Solution

$X \sim \mathrm{N}(850, 50^2)$ and so we have

$$\begin{aligned}\mathrm{P}(X < 830) &= \mathrm{P}\left(Z < \frac{830 - 850}{50}\right) \\ &= \mathrm{P}(Z < -0.4) \\ &= \mathrm{P}(Z > 0.4) \\ &= 1 - \Phi(0.4) \\ &= 1 - 0.6554 \\ &= \underline{\underline{0.3446}}.\end{aligned}$$

(b) In a box of 500 bulls, from company X, find the expected number having a lifetime of less that 830 hours. (2)

Solution

$$500 \times 0.3446 = 172.3$$

and so we'll take $\underline{\underline{\text{172 or 173 bulbs}}}$.

A rival company, Y, sells bulbs with a mean lifetime of 860 hours and 20% of these bulbs have a lifetime of less the 818 hours.

(c) Find the standard deviation of the lifetime of bulbs from company Y. (4)

Solution

$Y \sim \mathrm{N}(860, \sigma^2)$ and so we have

$$\begin{aligned}\mathrm{P}(Y < 818) = 0.20 &\Rightarrow \mathrm{P}\left(Z < \frac{818 - 860}{\sigma}\right) = 0.20 \\ &\Rightarrow \mathrm{P}\left(Z < \frac{-42}{\sigma}\right) = 0.20 \\ &\Rightarrow \frac{-42}{\sigma} = -0.8416 \\ &\Rightarrow \underline{\underline{\sigma = 49\tfrac{238}{263} \text{ hours}}}.\end{aligned}$$

Both companies sell the bulbs for the same price.

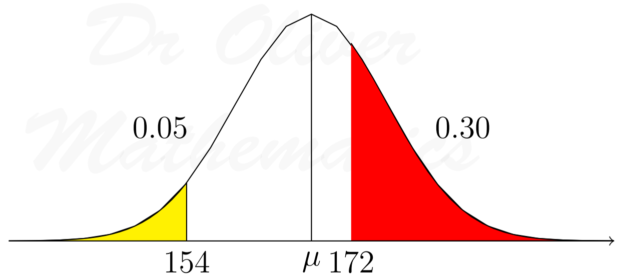
(d) State which company you would recommend. Give reasons for your answer. (2)

Solution
I would recommend company Y: although their standard deviations are almost the same, company Y's light bulbs last 10 hours longer.

10. The heights of a population of women are normally distributed with a mean μ cm and a standard deviation σ cm. It is known that 30% of the women are taller than 172 cm and 5% are shorter than 154 cm.

(a) Sketch a diagram to show the distribution of heights represented by this information. (3)

Solution



(b) Show that $\mu = 154 + 1.6449\sigma$. (3)

Solution
$H \sim \mathrm{N}(\mu, \sigma^2)$ and so we have

$$\begin{aligned} \mathrm{P}(H < 154) = 0.05 &\Rightarrow \mathrm{P}\left(Z < \frac{154-\mu}{\sigma}\right) = 0.05 \\ &\Rightarrow \frac{154-\mu}{\sigma} = -1.6449 \\ &\Rightarrow 154 - \mu = -1.6449\sigma \\ &\Rightarrow \mu = 154 + 1.6449\sigma. \end{aligned}$$

(c) Obtain a second equation and hence find the value of μ and the value of σ. (4)

Solution

$$\begin{aligned}
\mathrm{P}(H > 172) = 0.30 \Rightarrow \mathrm{P}\left(Z > \frac{172-\mu}{\sigma}\right) &= 0.30 \\
\Rightarrow \frac{172-\mu}{\sigma} &= 0.5244 \\
\Rightarrow 172 - \mu &= 0.5244\sigma \\
\Rightarrow \mu &= 172 - 0.5244\sigma.
\end{aligned}$$

Now,

$$\begin{aligned}
154 + 1.6449\sigma = 172 - 0.5244\sigma &\Rightarrow 2.1693\sigma = 18 \\
&\Rightarrow \underline{\underline{\sigma = 8.297\,607\,523 \text{ cm (FCD)}}}
\end{aligned}$$

and

$$\mu = 154 + 1.6449\sigma = \underline{\underline{167.648\,734\,6 \text{ cm (FCD)}}}.$$

A woman is chosen at random from the population.

(d) Find the probability that she is taller than 160 cm. (3)

Solution

$$\begin{aligned}
\mathrm{P}(H > 160) &= \mathrm{P}\left(Z > \frac{160-\mu}{\sigma}\right) \\
&= \mathrm{P}(Z > -0.921\ldots) \\
&= \mathrm{P}(Z < 0.92) \\
&= \underline{\underline{0.8212}}.
\end{aligned}$$

11. The distances travelled to work, D km, by the employees at a large company are normally distributed with $D \sim \mathrm{N}(30, 8^2)$.

(a) Find the probability that a randomly selected employee has a journey to work of more than 20 km. (3)

Solution

$$\begin{aligned}
P(D > 20) &= P\left(Z > \frac{20-30}{8}\right) \\
&= P(Z > -1.25) \\
&= P(Z < 1.25) \\
&= \Phi(1.25) \\
&= \underline{\underline{0.8944}}.
\end{aligned}$$

(b) Find the upper quartile, Q_3, of D. (3)

Solution

We want $\Phi(0.75) = 0.6745$.

$$\begin{aligned}
P(D < Q_3) = 0.75 &\Rightarrow \frac{Q_3 - 30}{8} = 0.6745 \\
&\Rightarrow Q_3 - 30 = 5.396 \\
&\Rightarrow \underline{\underline{Q_3 = 35.396}}.
\end{aligned}$$

(c) Write down the lower quartile, Q_1, of D. (1)

Solution

$30 - 5.396 = \underline{\underline{24.604}}$.

An outlier is defined as any value of D such that $D < h$ or $D > k$ where

$$h = Q_1 - 1.5(Q_3 - Q_1) \text{ and } k = Q_3 + 1.5(Q_3 - Q_1).$$

(d) Find the value of h and the value of k. (2)

Solution

$h = 24.64 - 1.5 \times 10.792 = \underline{\underline{8.452}}$ and $k = 35.36 + 1.5 \times 10.792 = \underline{\underline{51.548}}$.

An employee is selected at random.

(e) Find the probability that the distance travelled to work by this employee is an outlier. (3)

Solution

This is $2\,\mathrm{P}(D > 51.548)$ (why?):

$$\begin{aligned} 2\,\mathrm{P}(D > 51.44) &= 2\,\mathrm{P}\left(Z > \frac{51.548 - 30}{8}\right) \\ &= 2\,\mathrm{P}(Z > 2.6935) \\ &= 2(1 - \mathrm{P}(Z < 2.70)) \\ &= 2(1 - 0.9965) \\ &= \underline{\underline{0.007}}. \end{aligned}$$

12. The weight, X grams, of soup put in a tin by a machine A is normally distributed with a mean of 160 grams and a standard deviation of 5 grams. A tin is selected at random.

(a) Find the probability that this tin contains more than 168 grams. (3)

Solution

$X \sim \mathrm{N}\left(160, 5^2\right)$ and we have

$$\begin{aligned} \mathrm{P}(X > 168) &= \mathrm{P}\left(Z > \frac{168 - 160}{5}\right) \\ &= \mathrm{P}(Z > 1.6) \\ &= 1 - \mathrm{P}(Z < 1.6) \\ &= 1 - \Phi(1.6) \\ &= 1 - 0.9452 \\ &= \underline{\underline{0.0548}}. \end{aligned}$$

The weight stated on the tin is w grams.

(b) Find w such that $\mathrm{P}(X < w) = 0.01$. (3)

Solution

$$\begin{aligned} \mathrm{P}(X < w) = 0.01 &\Rightarrow \frac{w - 160}{5} = -2.3263 \\ &\Rightarrow w - 160 = -10.6315 \\ &\Rightarrow \underline{\underline{w = 148.3685 \text{ g}}}. \end{aligned}$$

The weight, Y grams, of soup put in a carton by a machine B is normally distributed with a mean of μ grams and a standard deviation of σ grams.

(c) Given that $\mathrm{P}(Y < 160) = 0.99$ and $\mathrm{P}(Y > 152) = 0.90$, find the value of μ and find the value of σ. (6)

Solution

First,

$$\frac{160-\mu}{\sigma} = 2.3263 \Rightarrow 160 - \mu = 2.3263\sigma$$

and

$$\frac{152-\mu}{\sigma} = -1.2816 \Rightarrow 152 - \mu = 1.2816\sigma.$$

Combining,

$$8 = 3.6079\sigma \Rightarrow \underline{\underline{\sigma = 2.217\,356\,357 \text{ g (FCD)}}}$$

and

$$\mu = 160 - 2.3263\sigma \Rightarrow \underline{\underline{\mu = 154.841\,763\,9 \text{ g (FCD)}}}$$

13. The random variable $X \sim \mathrm{N}(\mu, 5^2)$ and $\mathrm{P}(X < 23) = 0.9192$.

(a) Find the value of μ. (4)

Solution

Using the tables, $\Phi(z) = 0.9192 \Rightarrow z = 1.40$. So, by standardisation,

$$\frac{23-\mu}{5} = 1.40 \Rightarrow \underline{\underline{\mu = 16}}.$$

(b) Write down the value of $\mathrm{P}(\mu < X < 23) = 0.9192$. (1)

Solution

One of the key properties of the normal distribution is that it is symmetrical about the mean and so

$$\mathrm{P}(\mu < X < 23) = \mathrm{P}(X < 23) - \mathrm{P}(X < \mu) = 0.9192 - 0.5 = \underline{\underline{0.4192}}.$$

14. Past records show that the time, in seconds, taken to run 100 m by children at a school can be modelled by a normal distribution with a mean of 16.12 s and a standard deviation of 1.60 s. A child from the school is selected at random.

(a) Find the probability that this child runs 100 m in less than 15 s. (3)

Solution

$X \sim \mathrm{N}(16.12, 1.60^2)$ and we have

$$\begin{aligned}
\mathrm{P}(X < 15) &= \mathrm{P}\left(Z < \frac{15 - 16.12}{1.60}\right) \\
&= \mathrm{P}(Z < -0.7) \\
&= \mathrm{P}(Z > 0.7) \\
&= 1 - \mathrm{P}(Z < 0.7) \\
&= 1 - \Phi(0.7) \\
&= 1 - 0.7580 \\
&= \underline{\underline{0.2420}}.
\end{aligned}$$

On sports day the school awards certificates to the tastes 30% of the children in the 100 m race.

(b) Estimate, to 2 decimal places, the slowest time taken to run 100 m for which a child will be awarded a certificate. (4)

Solution

$$\begin{aligned}
\mathrm{P}(\text{fastest } 30\%) = 0.30 &\Rightarrow \frac{x - 16.12}{1.60} = -0.5244 \\
&\Rightarrow x - 16.12 = -0.83904 \\
&\Rightarrow x = 15.28096 \\
&\Rightarrow \underline{\underline{x = 15.28 \text{ s (2 dp)}}}.
\end{aligned}$$

15. A manufacturer fills jars with coffee. The mass of the coffee, M grams, in a jar can be modelled by a normal distribution with mean 232 grams and standard deviation 5 grams.

(a) Find $\mathrm{P}(M < 224)$. (3)

Solution

$W \sim \mathrm{N}(232, 5^2)$ and we have

$$\begin{aligned} \mathrm{P}(W < 224) &= \mathrm{P}\left(Z < \frac{224 - 232}{5}\right) \\ &= \mathrm{P}(Z < -1.6) \\ &= \mathrm{P}(Z > 1.6) \\ &= 1 - \mathrm{P}(Z < 1.6) \\ &= 1 - \Phi(1.6) \\ &= 1 - 0.9452 \\ &= \underline{\underline{0.0548}}. \end{aligned}$$

(b) Find the value of m such that $\mathrm{P}(232 < M < m) = 0.20$. (4)

Solution

First, $\mathrm{P}(M < m) = 0.70$, and we have

$$\begin{aligned} \mathrm{P}(M < m) = 0.70 &\Rightarrow \frac{m - 232}{5} = 0.5244 \\ &\Rightarrow m - 232 = 2.622 \\ &\Rightarrow \underline{\underline{m = 234.622 \text{ g}}}. \end{aligned}$$

Two jars of coffee are selected at random.

(c) Find the probability that only one of the jars contains between 232 grams and m grams of coffee. (3)

Solution

P(other doesn't) $= 1 - 0.2 = 0.8$ and

$$\mathrm{P}(232 < W < w, \text{ other doesn't}) = 2 \times 0.2 \times 0.8 = \underline{\underline{0.16}}.$$

16. The heights of an adult female population are normally distributed with mean 162 cm and standard deviation 7.5 cm.

(a) Find the probability that a randomly chosen adult female is taller than 150 cm. (3)

Solution

$W \sim \mathrm{N}\left(162, 7.5^2\right)$ and we have

$$\begin{aligned} \mathrm{P}(W > 150) &= \mathrm{P}\left(Z > \frac{150 - 162}{7.5}\right) \\ &= \mathrm{P}(Z > -1.6) \\ &= \mathrm{P}(Z < 1.6) \\ &= \Phi(1.6) \\ &= \underline{\underline{0.9452}}. \end{aligned}$$

Sarah is a young girl. She visits her doctor and is told that she is at the 60th percentile for height.

(b) Assuming that Sarah remain at the 60th percentile, estimate her height as a adult. (3)

Solution

Because she is at 60th percentile there are 40th percentiles above her and

$$\begin{aligned} \mathrm{P}(M < m) = 0.40 \Rightarrow \frac{w - 162}{7.5} &= 0.2533 \\ \Rightarrow w - 162 &= 1.899\,75 \\ \Rightarrow \underline{\underline{w = 163.899\,75 \text{ cm}}}. \end{aligned}$$

The heights of an adult male population are normally distributed with standard deviation 9.0 cm. Given that 90% of adult males are taller than the mean height of adult females,

(c) find the mean height an adult male. (4)

Solution

Let μ cm be the mean height of adult male. Then $M \sim \mathrm{N}\left(\mu, 9.0^2\right)$ and

$$\begin{aligned} \frac{m - 162}{9.0} = 1.2816 \Rightarrow m - 162 &= 11.534\,4 \\ \Rightarrow \underline{\underline{m = 173.534\,4 \text{ cm}}}. \end{aligned}$$

17. The length of time, L hours, that a mobile phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

(a) Find $\mathrm{P}(L > 127)$. (3)

Solution
$L \sim \mathrm{N}(100, 15^2)$ and we have

$$\begin{aligned}\mathrm{P}(L > 127) &= \mathrm{P}\left(Z > \frac{127-100}{15}\right)\\ &= \mathrm{P}(Z > 1.8)\\ &= 1 - \mathrm{P}(Z < 1.8)\\ &= 1 - \Phi(1.8)\\ &= 1 - 0.9641\\ &= \underline{\underline{0.0359}}.\end{aligned}$$

(b) Find the value of d such that $\mathrm{P}(L < d) = 0.10$. (3)

Solution

$$\begin{aligned}\mathrm{P}(L < d) = 0.10 &\Rightarrow \frac{d-100}{15} = -1.2816\\ &\Rightarrow d - 100 = -19.224\\ &\Rightarrow \underline{\underline{d = 80.776 \text{ hours}}}.\end{aligned}$$

Alice is about to go on a 6-hour journey. Given that it is 127 hours since Alice last charged her mobile phone,

(c) find the probability that her mobile phone will not need charging before her journey is completed. (4)

Solution

$$\begin{aligned}\mathrm{P}(L > 133) &= \mathrm{P}\left(Z > \frac{133-100}{15}\right)\\ &= \mathrm{P}(Z > 2.2)\\ &= 1 - \mathrm{P}(Z < 2.2)\\ &= 1 - \Phi(2.2)\\ &= 1 - 0.9861\\ &= \underline{\underline{0.0139}}\end{aligned}$$

and

$$\begin{aligned}\mathrm{P}(L > 133 | L > 127) &= \frac{\mathrm{P}(L > 133)}{\mathrm{P}(L > 127)} \\ &= \frac{0.0139}{0.0359} \\ &= \underline{\underline{0.387\,186\,629\,5 \text{ (FCD)}}}.\end{aligned}$$

18. Katie suggests using the random variable X which has a normal distribution with mean 320 and standard deviation 150 to model the weekly income for these data.

    Find $\mathrm{P}(240 < X < 400)$.

Solution

$$\begin{aligned}\mathrm{P}(240 < X < 400) &= \mathrm{P}\left(\frac{240-320}{150} < Z < \frac{400-320}{150}\right) \\ &= \mathrm{P}(-0.5\dot{3} < Z < 0.5\dot{3}) \\ &= \mathrm{P}(Z < 0.53) - \mathrm{P}(Z < -0.53) \\ &= \mathrm{P}(Z < 0.53) + \mathrm{P}(Z > 0.53) \\ &= 2\,\mathrm{P}(Z < 0.53) - 1 \\ &= 2\Phi(0.53) - 1 \\ &= 2 \times 0.7019 - 1 \\ &= \underline{\underline{0.4038}}.\end{aligned}$$

19. The mass, in grams, of beans in a tin is normally distributed with mean μ and standard deviation 7.8. Given that 10% of tins contain less than 200 g, find

    (a) the value of μ, (3)

Solution

$M_1 \sim \mathrm{N}(\mu, 7.8^2)$ and we have

$$\begin{aligned}\mathrm{P}(M_1 < 200) = 0.1 &\Rightarrow \frac{200-\mu}{7.8} = -1.2816 \\ &\Rightarrow 200 - \mu = -9.996\,48 \\ &\Rightarrow \underline{\underline{\mu = 209.996\,48 \text{ g}}}.\end{aligned}$$

(b) the percentage of tins that contain more than 225 g of beans. (3)

Solution

$$\begin{aligned}
\mathrm{P}(M_1 > 225) &= \mathrm{P}\left(Z > \frac{225 - 209.996\,48}{5}\right) \\
&= \mathrm{P}(Z > 1.923\ldots) \\
&= \mathrm{P}(Z > 1.92) \\
&= 1 - \mathrm{P}(Z < 1.92) \\
&= 1 - \Phi(1.92) \\
&= 1 - 0.9726 \\
&= 0.0274 \\
&= \underline{\underline{2.74\%}}.
\end{aligned}$$

The machine settings are adjusted so that the mass, in grams, of beans in a tin is normally distributed with mean 205 and standard deviation σ.

(c) Given that 98% of the tins contain between 200 g and 210 g find the value of σ. (4)

Solution

$M_2 \sim \mathrm{N}\left(205, \sigma^2\right)$ and we have

$$\begin{aligned}
\mathrm{P}(200 < M_2 < 210) = 0.98 &\Rightarrow \mathrm{P}(M_2 < 210) = 0.99 \\
&\Rightarrow \frac{210 - 205}{\sigma} = 2.3263 \\
&\Rightarrow \frac{5}{\sigma} = 2.3263 \\
&\Rightarrow \underline{\underline{\sigma = 2.149\,335\,855 \text{ g (FCD)}}}.
\end{aligned}$$

20. The time, in minutes, taken to fly from London to Malaga has a normal distribution with mean 150 minutes and standard deviation 10 minutes.

(a) Find the probability that the next flight from London to Malaga takes less than 145 minutes. (3)

Solution

$A \sim \mathrm{N}(150, 10^2)$ and we have

$$\begin{aligned} \mathrm{P}(A < 145) &= \mathrm{P}\left(Z < \frac{145 - 150}{10}\right) \\ &= \mathrm{P}(Z < -0.5) \\ &= \mathrm{P}(Z > 0.5) \\ &= 1 - \mathrm{P}(Z < 0.5) \\ &= 1 - \Phi(0.5) \\ &= 1 - 0.6915 \\ &= \underline{\underline{0.3085}}. \end{aligned}$$

The time taken to fly from London to Berlin has a normal distribution with mean 100 minutes and standard deviation d minutes. Given that 15% of flights from London to Berlin take longer than 115 minutes,

(b) find the value of the standard deviation d. (4)

Solution

$B \sim \mathrm{N}(100, d^2)$ and we have

$$\begin{aligned} \mathrm{P}(B > 115) = 0.15 &\Rightarrow \mathrm{P}(B < 115) = 0.85 \\ &\Rightarrow \mathrm{P}\left(Z < \frac{115 - 100}{d}\right) = 0.85 \\ &\Rightarrow \frac{115 - 100}{d} = 1.0364 \\ &\Rightarrow \frac{15}{d} = 1.0364 \\ &\Rightarrow \underline{\underline{d = 14.473\,176\,38 \text{ mins (FCD)}}}. \end{aligned}$$

The time, X minutes, taken to fly from London to another city has a normal distribution with mean μ minutes. Given that $\mathrm{P}(X < \mu - 15) = 0.35$,

(c) find $\mathrm{P}(X > \mu + 15 | X > \mu - 15)$. (3)

Solution

We have $X \sim \mathrm{N}(\mu, \sigma^2)$. Now,

$$\mathrm{P}(X > \mu + 15) = \mathrm{P}(X < \mu - 15) = 0.35,$$

$$P(X > \mu - 15) = 0.65,$$

and hence

$$\begin{aligned} P(X > \mu + 15 | X > \mu - 15) &= \frac{P(X > \mu + 15)}{P(X > \mu - 15)} \\ &= \frac{0.35}{0.65} \\ &= \underline{\underline{\tfrac{7}{13}}}. \end{aligned}$$

21. The heights of adult females are normally distributed with mean 160 cm and standard deviation 8 cm.

(a) Find the probability that a random selected adult female has a height greater than 170 cm. (3)

Solution

$H \sim N(160, 8^2)$ and we have

$$\begin{aligned} P(H > 170) &= P\left(Z > \frac{170 - 160}{8}\right) \\ &= P(Z > 1.25) \\ &= 1 - P(Z < 1.25) \\ &= 1 - \Phi(1.25) \\ &= 1 - 0.8944 \\ &= \underline{\underline{0.1056}}. \end{aligned}$$

Any adult female whose height is greater than 170 cm is defined as tall. An adult female is chosen at random. Given that she is tall,

(b) find the probability that she has a height greater than 180 cm. (4)

Solution

$$\begin{aligned}
\mathrm{P}(H > 180) &= \mathrm{P}\left(Z > \frac{180-160}{8}\right) \\
&= \mathrm{P}(Z > 2.5) \\
&= 1 - \mathrm{P}(Z < 2.5) \\
&= 1 - \Phi(2.5) \\
&= 1 - 0.9938 \\
&= 0.0062
\end{aligned}$$

and

$$\begin{aligned}
\mathrm{P}(H > 180 | H > 170) &= \frac{\mathrm{P}(H > 180)}{\mathrm{P}(H > 170)} \\
&= \frac{0.0062}{0.1056} \\
&= \underline{\underline{\tfrac{31}{528}}}.
\end{aligned}$$

Half of tall adult females have a height greater that h cm.

(c) Find the value of h. (5)

Solution

$$\mathrm{P}(H > h | H > 170) = 0.5 \Rightarrow \mathrm{P}(H > h) = 0.5 \times 0.1056 = 0.0528$$

and we have

$$\mathrm{P}(H < h) = 0.9472.$$

Now,

$$\begin{aligned}
\frac{h-160}{8} &= 1.62 \Rightarrow h - 160 = 12.96 \\
&\Rightarrow \underline{\underline{h = 172.96 \text{ cm}}}.
\end{aligned}$$

22. The continuous random variable Y has the normal distribution $\mathrm{N}(10, 2^2)$. Write down the value of

(a) $\mathrm{P}(Y = 10)$, (1)

Solution

$\underline{\underline{0}}$.

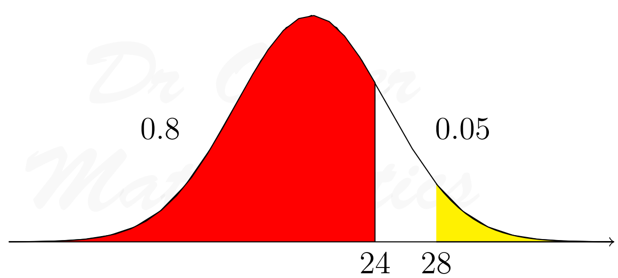
(b) P($Y < 10$). (1)

Solution
0.5.

23. The time taken, in minutes, by children to complete a mathematical puzzle is assumed to be normally distributed with mean μ and standard deviation σ. The puzzle can be completed in less than 24 minutes by 80% of the children. For 5% of the children it takes more than 28 minutes to complete the puzzle.

(a) Show this information on the normal curve. (2)

Solution



(b) Write down the percentage of children who take between 24 minutes and 28 minutes to complete the puzzle. (1)

Solution
0.15.

(i) Find two equation in μ and σ. (4)

Solution

$$\frac{24-\mu}{\sigma} = 0.8416 \Rightarrow 24-\mu = 0.8416\sigma$$

and

$$\frac{28-\mu}{\sigma} = 1.6449 \Rightarrow 28-\mu = 1.6449\sigma.$$

(ii) Hence find, to 3 significant figures, the value of μ and the value of σ. (3)

Solution

$$\begin{aligned}24 - 0.8416\sigma = 28 - 1.6449\sigma &\Rightarrow 0.8033\sigma = 4\\ &\Rightarrow \sigma = 4.979\,459\,729 \text{ (FCD)}\\ &\Rightarrow \underline{\underline{\sigma = 4.98 \text{ (3 sf)}}}\end{aligned}$$

and

$$\begin{aligned}\mu &= 24 - 0.8416\sigma\\ &= 19.809\,286\,69 \text{ (FCD)}\\ &= \underline{\underline{19.8 \text{ (3 sf)}}}.\end{aligned}$$

A child is selected at random.

(c) Find the probability that the child takes less than 12 minutes to complete the puzzle. (3)

Solution

$X \sim \mathrm{N}\left(19.809\ldots, 4.979\ldots^2\right)$ and we have

$$\begin{aligned}\mathrm{P}(X < 12) &= \mathrm{P}\left(Z < \frac{12 - 19.809\ldots}{4.979\ldots}\right)\\ &= \mathrm{P}(Z < -1.5683)\\ &= \mathrm{P}(Z < -1.57)\\ &= \mathrm{P}(Z > 1.57)\\ &= 1 - \mathrm{P}(Z < 1.57)\\ &= 1 - \Phi(1.57)\\ &= 1 - 0.9418\\ &= \underline{\underline{0.0582}}.\end{aligned}$$

24. The random variable $Z \sim \mathrm{N}(0, 1)$.
A is the event $Z > 1.1$.
B is the event $Z > -1.9$.
C is the event $-1.5 < Z < 1.5$.

(a) Find (6)

(i) $\mathrm{P}(A)$,

Solution

$$P(A) = 1 - \Phi(1.1) = 1 - 0.8665 = \underline{\underline{0.1335}}.$$

(ii) $P(B)$,

Solution

$$P(B) = 1 - \Phi(-1.9) = 1 - [1 - \Phi(1.9)] = \Phi(1.9) = \underline{\underline{0.9713}}.$$

(iii) $P(C)$,

Solution

$$\begin{aligned} P(C) &= \Phi(1.5) - \Phi(-1.5) \\ &= \Phi(1.5) - [1 - \Phi(1.5)] \\ &= 2\Phi(1.5) - 1 \\ &= 2 \times 0.9332 - 1 \\ &= \underline{\underline{0.8664}}. \end{aligned}$$

(iv) $P(A \cup C)$.

Solution

$$P(A \cup C) = P(Z > -1.5) = 1 - \Phi(-1.5) = \Phi(1.5) = \underline{\underline{0.9332}}.$$

The random variable X has a normal distribution with mean 21 and standard deviation 5.

(b) Find the value of w such that $P(X > w | X > 28) = 0.625$. (6)

Solution

First we notice that we must have $w > 28$ or else the conditional probability would have to be 1. Hence

$$\begin{aligned} P(X > w | X > 28) = 0.625 &\Rightarrow \frac{P(X > w \cap X > 28)}{P(X > 28)} = 0.625 \\ &\Rightarrow \frac{P(X > w)}{P(X > 28)} = 0.625 \\ &\Rightarrow P(X > w) = 0.625\, P(X > 28). \end{aligned}$$

Now

$$\begin{aligned}
P(X > 28) &= 1 - P(X < 28) \\
&= 1 - \Phi(\tfrac{28-21}{5}) \\
&= 1 - \Phi(1.4) \\
&= 1 - 0.9192 \\
&= 0.0808
\end{aligned}$$

and hence $P(X > w) = 0.0505$. So

$$\begin{aligned}
P(X > w) = 0.0505 &\Rightarrow P(X < w) = 0.9495 \\
&\Rightarrow \frac{w-21}{5} = 1.64 \text{ (since } \Phi(1.64) = 0.9495) \\
&\Rightarrow \underline{\underline{w = 21.2}}.
\end{aligned}$$

25. Shyam decides to model the weight of babies born at the hospital by the random variable W, where $W \sim N(3.43, 0.65^2)$. Find $P(W < 3)$.

Solution

$$\begin{aligned}
P(W < 3) &= P\left(Z < \frac{3-3.43}{0.65}\right) \\
&= P(Z < -0.661\ldots) \\
&= P(Z < -0.66) \\
&= P(Z > 0.66) \\
&= 1 - P(Z < 0.66) \\
&= 1 - \Phi(0.66) \\
&= 1 - 0.7454 \\
&= \underline{\underline{0.2546}}.
\end{aligned}$$

26. The time, in minutes, taken by men to run a marathon is modelled by a normal distribution with mean 240 minutes and standard deviation 40 minutes.

(a) Find the proportion on men that take longer than 300 minutes to a run marathon. (3)

Solution

$M \sim \mathrm{N}\left(240, 40^2\right)$ and we have

$$\begin{aligned}
\mathrm{P}(M < 300) &= \mathrm{P}\left(Z < \frac{240 - 300}{40}\right) \\
&= \mathrm{P}(Z < -1.5) \\
&= \mathrm{P}(Z > 1.5) \\
&= 1 - \mathrm{P}(Z < 1.5) \\
&= 1 - \Phi(1.5) \\
&= 1 - 0.9332 \\
&= \underline{\underline{0.0668}}.
\end{aligned}$$

Nathaniel is preparing to a run marathon. He aims to finish in the first 20% of male runners.

(b) Using the above model estimate the longest time that Nathaniel can take to a run marathon and achieve his aim. (3)

Solution

$$\begin{aligned}
\mathrm{P}(20\%) = 0.2 &\Rightarrow \frac{m - 240}{40} = -0.8416 \\
&\Rightarrow m - 240 = -33.664 \\
&\Rightarrow \underline{\underline{m = 206.336 \text{ minutes}}}.
\end{aligned}$$

The time, in minutes, taken by men to run a marathon is modelled by a normal distribution with mean μ minutes. Given that $\mathrm{P}(W < \mu + 30) = 0.82$,

(c) find $\mathrm{P}(W < \mu - 30 | W < \mu)$. (3)

Solution

$\mathrm{P}(W < \mu - 30) = 0.18$ (why?) and we have

$$\begin{aligned}
\mathrm{P}(W < \mu - 30 | W < \mu) &= \frac{\mathrm{P}(W < \mu - 30)}{\mathrm{P}(W < \mu)} \\
&= \frac{0.18}{0.5} \\
&= \underline{\underline{0.36}}.
\end{aligned}$$

27. An athlete believes that her times for running 200 metres in races are normally distributed with a mean of 22.8 seconds.

(a) Given that her time is over 23.3 seconds in 20% of her races, calculate the variance of her times. (5)

Solution

$X \sim \mathrm{N}\left(22.8, \sigma^2\right)$ and we have

$$\begin{aligned} \mathrm{P}(X > 23.2) = 0.2 \Rightarrow \frac{23.3 - 22.8}{\sigma} &= 0.8416 \\ \Rightarrow \frac{0.5}{\sigma} &= 0.8416 \\ \Rightarrow \sigma &= \frac{0.5}{0.8416} \\ \Rightarrow \underline{\underline{\sigma^2 = 0.352\,962\,490\,4 \text{ (FCD)}}}. \end{aligned}$$

(b) The record over this distance for women at her club is 21.82 seconds. According to her model, what is the probability that she will beat this record in her next race? (3)

Solution

$$\begin{aligned} \mathrm{P}(X < 21.82) &= \mathrm{P}\left(Z < \frac{21.28 - 22.8}{0.594\ldots}\right) \\ &= \mathrm{P}(Z < -1.649\ldots) \\ &= \mathrm{P}(Z < -1.65) \\ &= \mathrm{P}(Z > 1.65) \\ &= 1 - \mathrm{P}(Z < 1.65) \\ &= 1 - \Phi(1.65) \\ &= 1 - 0.9505 \\ &= \underline{\underline{0.0495}}. \end{aligned}$$

28. The volume of liquid in bottles of sparkling water from one producer is believed to be normally distributed with a mean of 704 ml and a variance of 3.2 ml^2. Calculate the probability that a randomly chosen bottle from this producer contains

(a) more than 706 ml, (3)

Solution

$X \sim \mathrm{N}\,(704, 3.2)$ and we have

$$\begin{aligned}
\mathrm{P}(X > 706) &= \mathrm{P}\left(Z > \frac{706 - 704}{\sqrt{3.2}}\right) \\
&= \mathrm{P}(Z > 1.118\ldots) \\
&= \mathrm{P}(Z > 1.12) \\
&= 1 - \mathrm{P}(Z < 1.12) \\
&= 1 - \Phi(1.12) \\
&= 1 - 0.8686 \\
&= \underline{\underline{0.1314}}.
\end{aligned}$$

(b) between 703 and 708 ml. (4)

Solution

$$\begin{aligned}
\mathrm{P}(703 < X < 708) &= \mathrm{P}\left(\frac{703 - 704}{\sqrt{3.2}} Z < \frac{708 - 704}{\sqrt{3.2}}\right) \\
&= \mathrm{P}(-0.559\ldots < Z < 2.236\ldots) \\
&= \mathrm{P}(-0.56 < Z < 2.24) \\
&= \mathrm{P}(Z < 2.24) - \mathrm{P}(Z < -0.56) \\
&= \mathrm{P}(Z < 2.24) - \mathrm{P}(Z > 0.56) \\
&= \mathrm{P}(Z < 2.24) - (1 - \mathrm{P}(Z < 0.56)) \\
&= \mathrm{P}(Z < 2.24) - 1 + \mathrm{P}(Z < 0.56)) \\
&= \Phi(2.24) - 1 + \Phi(0.56) \\
&= 0.9875 - 1 + 0.7123 \\
&= \underline{\underline{0.6998}}.
\end{aligned}$$

The bottles are labelled as containing 700 ml.

(c) In a delivery of 1200 bottles, how many could be expected to contain less than the stated 700 ml? (4)

Solution

$$
\begin{aligned}
P(X < 700) &= P\left(Z < \frac{700 - 704}{\sqrt{3.2}}\right) \\
&= P(Z < -2.236\ldots) \\
&= P(Z < -2.24) \\
&= P(Z > 2.24) \\
&= 1 - P(Z < 2.24) \\
&= 1 - \Phi(2.24) \\
&= 1 - 0.9875 \\
&= 0.0125
\end{aligned}
$$

and we have

$$1200 \times 0.0125 = \underline{\underline{15 \text{ bottles}}}.$$

The bottling process can be adjusted so that the mean changes but the variance is unchanged.

(d) What should the mean be changed to in order to have only a 0.1% chance of a bottle having less than 700 ml of sparkling water? Give your answer correct to one decimal place. (4)

Solution

$Y \sim \mathrm{N}(\mu, 3.2)$ and we have

$$
\begin{aligned}
P(Y < 700) = 0.001 &\Rightarrow \frac{700 - \mu}{\sqrt{3.2}} = -3.0902 \\
&\Rightarrow 700 - \mu = -5.527\,917\,811 \text{ (FCD)} \\
&\Rightarrow \mu = 705.527\,917\,811 \text{ (FCD)} \\
&\Rightarrow \underline{\underline{\mu = 705.5 \text{ ml (1 dp)}}}.
\end{aligned}
$$

29. A geologist is analysing the size of quartz crystals in a sample of granite. She estimates that the longest diameter of 75% of the crystals is greater than 2 mm but only 10% of the crystals have a longest diameter of more than 6 mm. The geologist believes that the distribution of the longest diameters of the quartz crystals can be modelled by a normal distribution.

(a) Find the mean and variance of this normal distribution. (9)

Solution

$Q \sim \mathrm{N}(\mu, \sigma^2)$, $\Phi(0.25) = -0.6745$,

$$\begin{aligned} \mathrm{P}(Q < 2) = 0.25 &\Rightarrow \frac{2-\mu}{\sigma} = -0.6745 \\ &\Rightarrow 2 - \mu = -0.6745\sigma, \end{aligned}$$

and

$$\begin{aligned} \mathrm{P}(Q > 6) = 0.1 &\Rightarrow \frac{6-\mu}{\sigma} = 1.2816 \\ &\Rightarrow 6 - \mu = 1.2816\sigma. \end{aligned}$$

Now,

$$\begin{aligned} 2 + 0.6745\sigma = 6 - 1.2816\sigma &\Rightarrow 1.9561\sigma = 4 \\ &\Rightarrow \underline{\underline{\sigma = 2.044\,885\,231 \text{ mm (FCD)}}} \end{aligned}$$

and

$$\mu = 2 + 0.6745\sigma = \underline{\underline{3.379\,275\,088 \text{ mm (FCD)}}}.$$

The geologist estimates that only 2% of the longest diameters were smaller than 1 mm.

(b) Calculate the corresponding percentage that would be predicted by a normal distribution with the parameters calculated in part (a). (3)

Solution

$$\begin{aligned} \mathrm{P}(Q < 1) &= \mathrm{P}\left(Z < \frac{1-\mu}{\sigma}\right) \\ &= \mathrm{P}(Z < -1.163\ldots) \\ &= \mathrm{P}(Z < -1.16) \\ &= \mathrm{P}(Z > 1.16) \\ &= 1 - \mathrm{P}(Z < 1.16) \\ &= 1 - \Phi(1.16) \\ &= 1 - 0.8770 \\ &= 0.1230 \end{aligned}$$

and so it is $\underline{\underline{12.3\%}}$.

(c) Hence comment on the suitability of the normal distribution as a model in this situation. (2)

Solution

The discrepancy between the calculated value and the actual value is quite large and so we can say that it is not suitable.

30. Yuto works in the quality control department of a large company. The time, T minutes, it takes Yuto to analyse a sample is normally distributed with mean 18 minutes and standard deviation 5 minutes.

(a) Find the probability that Yuto takes longer than 20 minutes to analyse the next sample. (3)

Solution

$T \sim \mathrm{N}(18, 5^2)$ and

$$\begin{aligned} \mathrm{P}(T > 20) &= \mathrm{P}\left(Z > \frac{20-18}{5}\right) \\ &= \mathrm{P}(Z > 0.4) \\ &= 1 - \mathrm{P}(Z < 0.4) \\ &= 1 - 0.6554 \\ &= \underline{\underline{0.3446}}. \end{aligned}$$

The company has a large store of samples analysed by Yuto with the time taken for each analysis recorded. Serena is investigating the samples that took Yuto longer than 15 minutes to analyse. She selects, at random, one of the samples that took Yuto longer than 15 minutes to analyse.

(b) Find the probability that this sample took Yuto more than 20 minutes to analyse. (4)

Solution

$$\begin{aligned} \mathrm{P}(T > 15) &= \mathrm{P}\left(Z > \frac{15-18}{5}\right) \\ &= \mathrm{P}(Z > -0.6) \\ &= \mathrm{P}(Z < 0.6) \\ &= 0.7257 \end{aligned}$$

and

$$\begin{aligned}
P(T > 20 | T > 15) &= \frac{P(T > 20)}{P(T > 15)} \\
&= \frac{0.3446}{0.7257} \\
&= 0.474\,851\,867\,2 \text{ (FCD)} \\
&= \underline{\underline{0.4749 \text{ (4 dp)}}}.
\end{aligned}$$

Serena can identify, in advance, the samples that Yuto can analyse in under 15 minutes and in future she will assign these to someone else.

(c) Estimate the median time taken by Yuto to analyse samples in future. (5)

Solution

$$\begin{aligned}
P(T > t | T > 15) = 0.5 &\Rightarrow P(T > t) = 0.5\, P(T > 15) \\
&\Rightarrow P(T > t) = 0.362\,85 \\
&\Rightarrow P(T < t) = 0.637\,15 \\
&\Rightarrow P\left(T < \frac{d - 18}{5}\right) = 0.637\,15 \\
&\Rightarrow \frac{d - 18}{5} = 0.35 \text{ (this is as close as we can get)} \\
&\Rightarrow d - 18 = 1.75 \\
&\Rightarrow d = 19.75;
\end{aligned}$$

the median time is $\underline{\underline{\text{19 minutes 45 seconds}}}$.