Dr Oliver Mathematics
Mathematics
Coordinates Part 1
Past Examination Questions

This booklet consists of 24 questions across a variety of examination topics. The total number of marks available is 213. Calculators may not be used.

1. The points $A(1, 7)$, $B(20, 7)$ and $C(p, q)$ form the vertices of a triangle ABC, as shown in Figure 1.

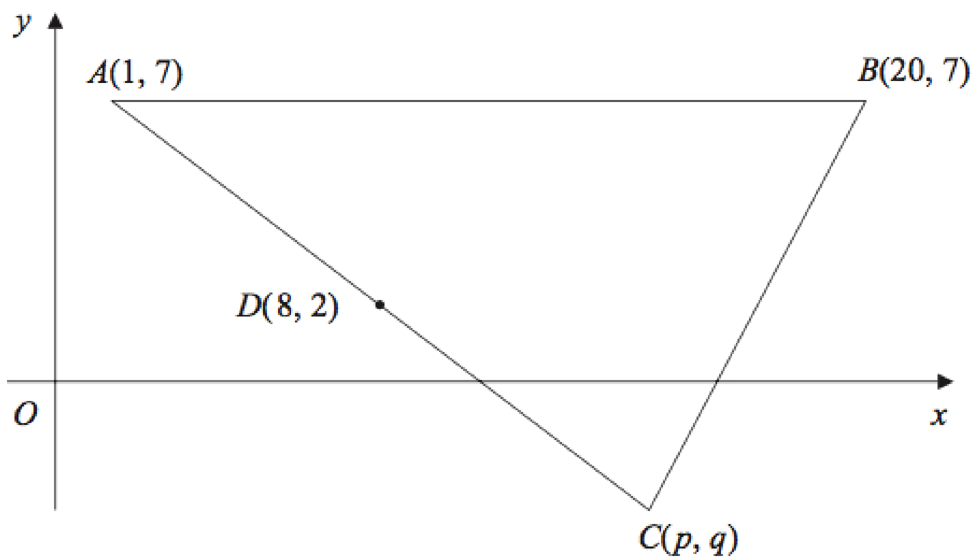

Figure 1: ABC

The point $D(8, 2)$ is the mid-point of AC.

(a) Find the value of p and the value of q. (2)

The line l, which passes through D and is perpendicular to AC, intersects AB at E.

(b) Find an equation for l, in the form $ax + by + c = 0$, where a, b, and c are integers. (5)

(c) Find the exact x-coordinate of E. (2)

2. The line l_1 passes through the point $(9, -4)$ and has gradient $\frac{1}{3}$.

(a) Find an equation for l_1, in the form $ax + by + c = 0$, where a, b, and c are integers. (3)

The line l_2 passes through the origin O and has gradient -2. The lines l_1 and l_2 intersect at the point P.

(b) Calculate the coordinates of P. (4)

Given that l_1 crosses the y-axis at the point C,

(c) calculate the exact area of $\triangle OCP$. (3)

3. The line L has equation $y = 5 - 2x$.

   (a) Show that the point $P(3, -1)$ lies on L. (1)

   (b) Find an equation of the line perpendicular to L, which passes through P. Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

4. The line l_1 passes through the points $P(-1, 2)$ and $Q(11, 8)$.

   (a) Find an equation for l_1 in the form $y = mx + c$, where m and c are constants. (4)

   The line l_2 passes through the point $R(10, 0)$ and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S.

   (b) Calculate the coordinates of S. (5)

   (c) Show that the length of RS is $3\sqrt{5}$. (2)

   (d) Hence, or otherwise, find the exact area of triangle PQR. (4)

5. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.

   (a) Show that the point $P(4, 8)$ lies on C. (1)

   You are given, when $x = 4$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -3$.

   (b) Show that an equation of the normal to C at the point P is $3y = x + 20$. (4)

   The normal to C at P cuts the x-axis at the point Q.

   (c) Find the length PQ, giving your answer in a simplified surd form. (3)

6. The curve C has equation

$$y = x^2(x - 6) + \frac{4}{x},\ x > 0.$$

   The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

   (a) Show that the length of PQ is $\sqrt{170}$. (4)

   (b) Show that the tangents to C at P and Q are parallel. (5)

   (c) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

7. The point $A(-6, 4)$ and the point $B(8, -3)$ lie on the line L.

   (a) Find an equation for L in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

(b) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer. (3)

8. The points $Q(1, 3)$ and $R(7, 0)$ lie on the line l_1, as shown in Figure 2.

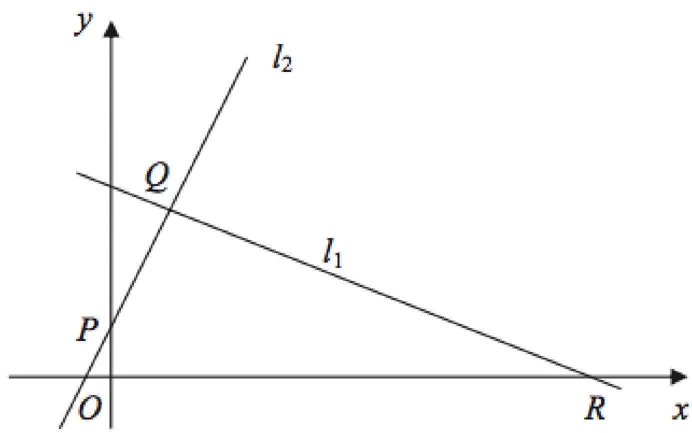

Figure 2: PQR

The length of QR is $a\sqrt{5}$.

(a) Find the value of a. (3)

The line l_2 is perpendicular to l_1, passes through Q and crosses the y-axis at the point P, as shown in Figure 2.

Find

(b) an equation for l_2, (5)

(c) the coordinates of P, (1)

(d) the area of $\triangle PQR$. (4)

9. The line l_1 passes through the point $A(2, 5)$ and has gradient $-\frac{1}{2}$.

(a) Find an equation of l_1, giving your answer in the form $y = mx + c$. (3)

The point B has coordinates $(-2, 7)$.

(b) Show that B lies on l_1. (1)

(c) Find the length of AB, giving your answer in the form $k\sqrt{5}$, where k is an integer. (3)

The point C lies on l_1 and has x-coordinate equal to p. The length of AC is 5 units.

(d) Show that p satisfies (4)

$$p^2 - 4p - 16 = 0.$$

10. The points A and B have coordinates $(6, 7)$ and $(8, 2)$ respectively. The line l passes through the point A and is perpendicular to the line AB, as shown in Figure 3.

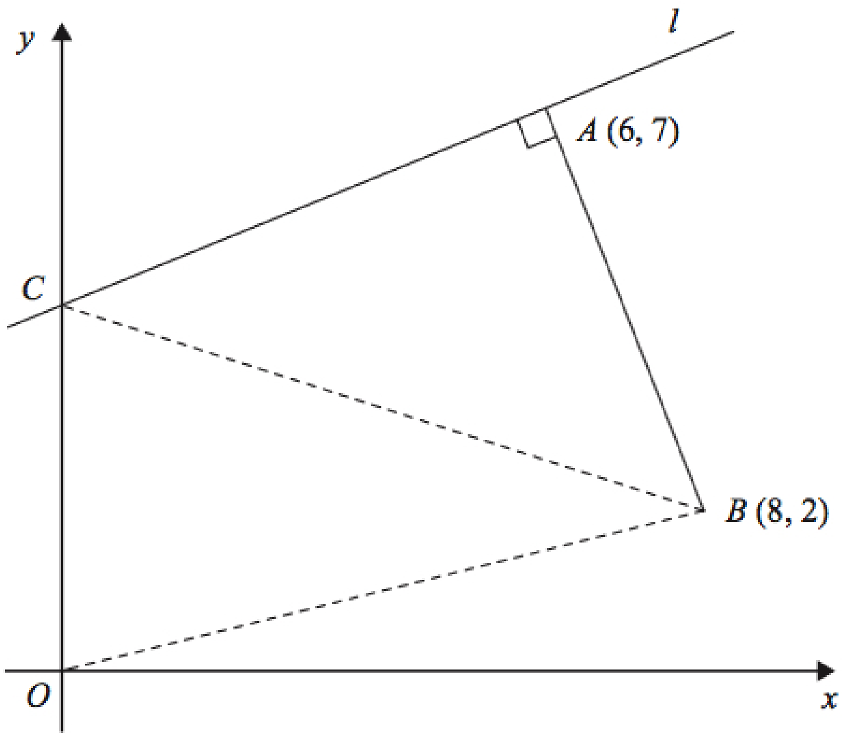

Figure 3: $\triangle OCB$

(a) Find an equation for l in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

Given that l intersects the y-axis at the point C, find

(b) the coordinates of C, (2)

(c) the area of $\triangle OCB$, where O is the origin. (2)

11. The line l_1 has equation $3x + 5y - 2 = 0$.

(a) Find the gradient of l_1. (2)

The line l_2 is perpendicular to l_1 and passes through the point $(3, 1)$.

(b) Find the equation of l_2 in the form $y = mx + c$, where m and c are constants. (3)

12. (a) Find an equation of the line joining $A(7, 4)$ and $B(2, 0)$, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (3)

(b) Find the length of AB, leaving your answer in surd form. (2)

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(c) Find the value of t. (1)

(d) Find the area of triangle ABC. (2)

13. The line L_1 has equation $2y - 3x - k = 0$, where k is a constant. Given that the point $A(1, 4)$ lies on L_1, find

(a) the value of k, (1)

(b) the gradient of L_1. (2)

The line L_2 passes through A and is perpendicular to L_1.

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

The line L_2 crosses the x-axis at the point B.

(d) Find the coordinates of B. (2)

(e) Find the exact length of AB. (2)

14. The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively. The line l is perpendicular to PQ and passes through the mid-point of PQ. Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (5)

15. The line l_1 has equation $2x - 3y + 12 = 0$.

(a) Find the gradient of l_1. (1)

The line l_1 crosses the x-axis at the point A and the y-axis at the point B, as shown in Figure 4.

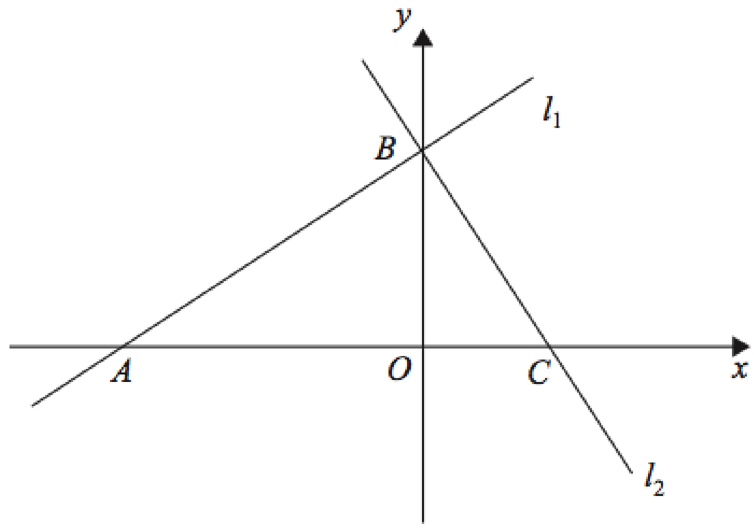

Figure 4: $\triangle ABC$

The line l_2 is perpendicular to l_1 and passes through B.

(b) Find an equation of l_2. (3)

The line l_2 crosses the x-axis at the point C.

(c) Find the area of triangle ABC. (4)

16. The line L_1 has equation $4y + 3 = 2x$. The point $A(p, 4)$ lies on L_1.

(a) Find the value of the constant p. (1)

The line L_2 passes through the point $C(2, 4)$ and is perpendicular to L_1.

(b) Find an equation for L_2 giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (5)

The line L_1 and the line L_2 intersect at the point D.

(c) Find the coordinates of the point D. (3)

(d) Show that the length of CD is $\frac{3}{2}\sqrt{5}$. (3)

A point B lies on L_1 and the length of $AB = \sqrt{80}$. The point E lies on L_2 such that the

length of the line $CDE = 3$ times the length of CD.

(e) Find the area of the quadrilateral $ACBE$. (3)

17. The line l_1 has equation $y = -2x + 3$. The line l_2 is perpendicular to l_1 and passes through the point $(5, 6)$.

 (a) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b, and c are integers. (3)

 The line l_2 crosses the x-axis at the point A and the y-axis at the point B.

 (b) Find the x-coordinate of A and the y-coordinate of B. (2)

 Given that O is the origin,

 (c) find the area of the triangle OAB. (2)

18. The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

 (a) Find an equation for L_1 in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

 The line L_2 has equation $3y + 4x - 30 = 0$.

 (b) Find the coordinates of the point of intersection of L_1 and L_2. (3)

19. The line L_1 has equation $4x + 2y - 3 = 0$.

 (a) Find an gradient of L_1 (2)

 The line L_2 is perpendicular to L_1 and passes through the point $(2, 5)$.

 (b) Find the equation of L_2 in the form $y = mx + c$, where m and c are constants. (3)

20. The line l_1, shown in Figure 5 has equation $2x + 3y = 26$.

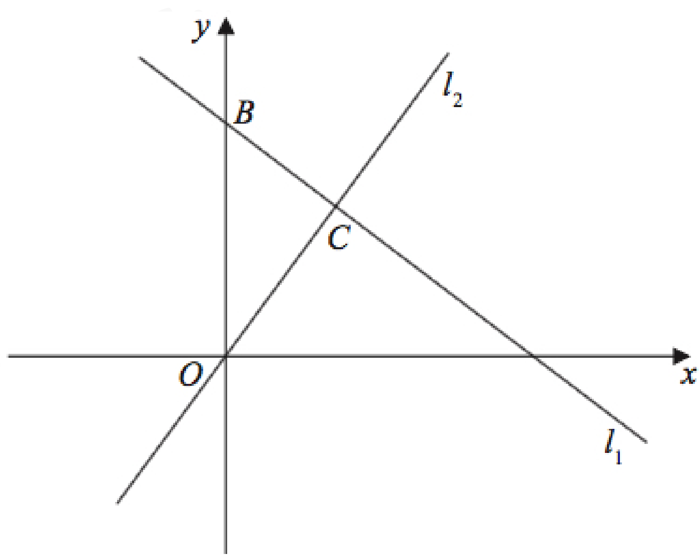

Figure 5: $\triangle OBC$

The line l_2 passes through the origin O and is perpendicular to l_1.

(a) Find an equation for the line l_2. (4)

The line l_2 intersects the line l_1 at the point C. Line l_1 crosses the y-axis at the point B, as shown in Figure 5.

(b) Find the area of triangle OBC. Give your answer in the form $\frac{a}{b}$, where a and b are integers to be determined. (6)

21. Figure 6 shows a right angled triangle LMN.

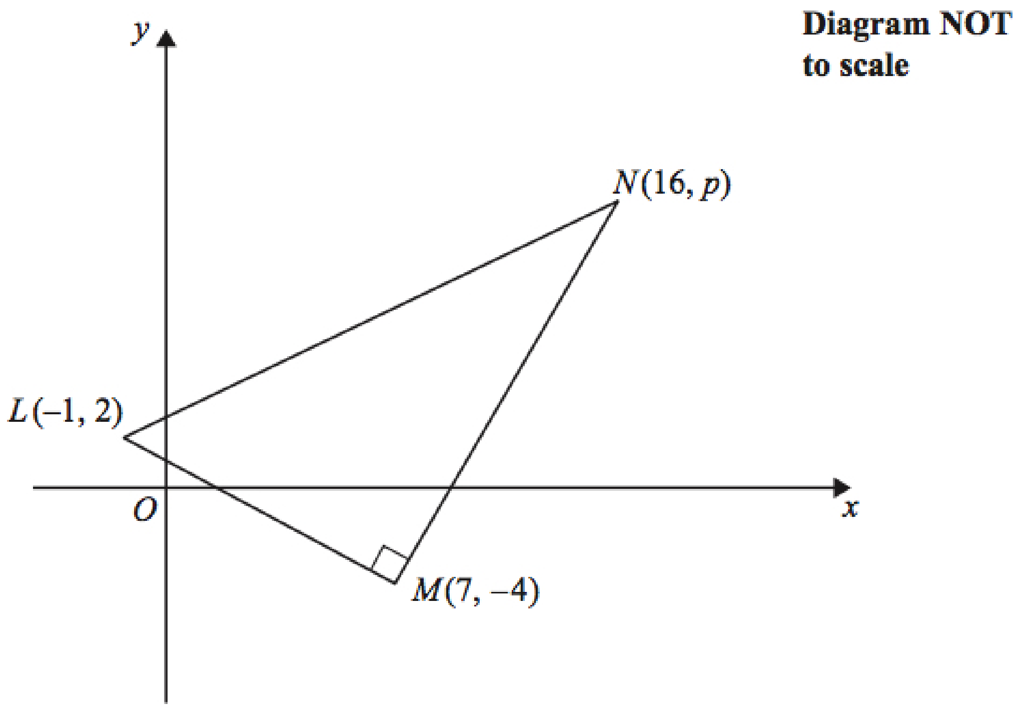

Figure 6: $\triangle LMN$

The points L and M have coordinates $(-1, 2)$ and $(7, -4)$ respectively.

(a) Find an equation for the straight line passing through the points L and M. Give your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

Given that the coordinates of point N are $(16, p)$, where p is a constant, and angle $LMN = 90°$,

(b) find the value of p. (3)

Given that there is a point K such that the points L, M, N, and K form a rectangle,

(c) find the y-coordinate of K. (2)

22. A curve C is given by (4)

$$y = 9x - 4x^3.$$

The points A and B lie on C and have x-coordinates of -2 and 1 respectively. Show that the length of AB is $k\sqrt{10}$ where k is a constant to be found.

23. The points $P(0, 2)$ and $Q(3, 7)$ lie on the line l_1, as shown in Figure 7.

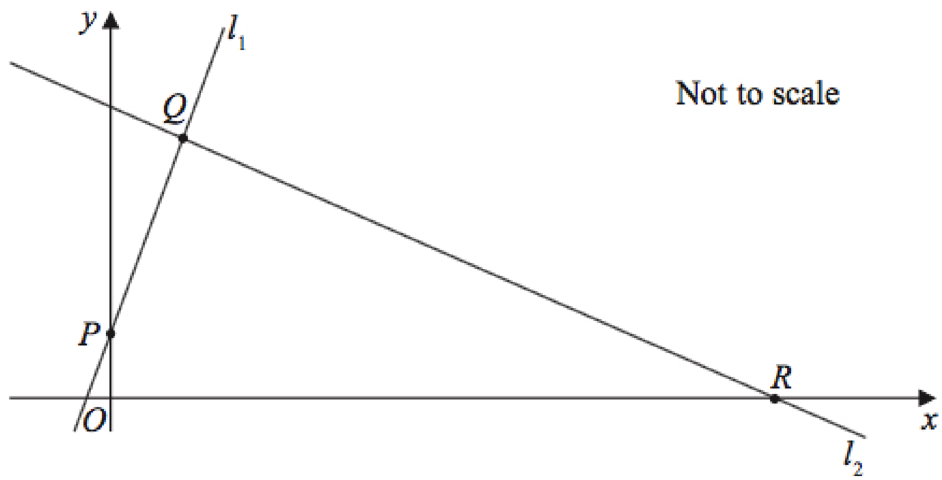

Figure 7: P, Q, and R

The line l_2 is perpendicular to l_1, passes through Q and crosses the x-axis at the point R, as shown in Figure 7. Find

(a) an equation for l_2, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers, (5)

(b) the exact coordinates of R, (2)

(c) the exact area of the quadrilateral $ORQP$, where O is the origin. (5)

24. The straight line l_1, shown in Figure 8, has equation $5y = 4x + 10$.

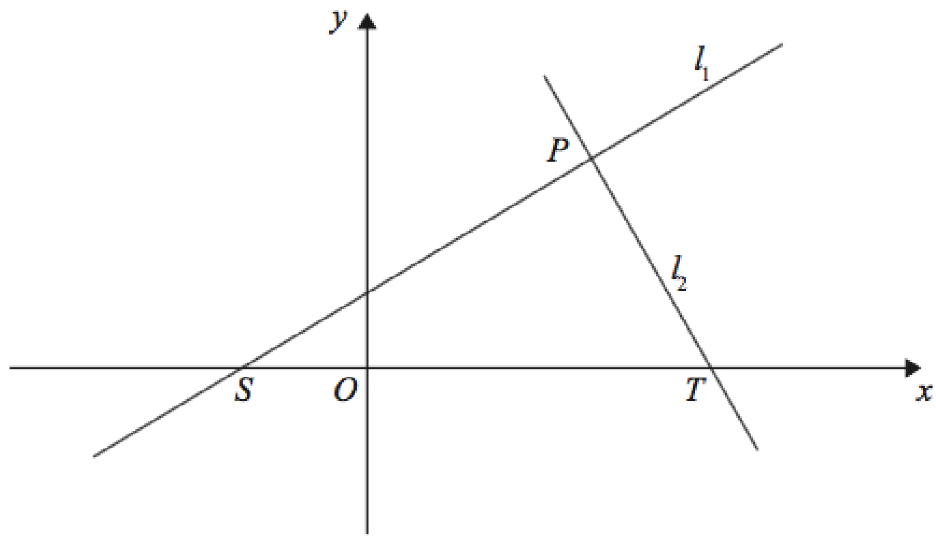

Figure 8: P, S, and T

The point P with x-coordinate 5 lies on l_1. The straight line l_2 is perpendicular to l_1 and passes through P.

(a) Find an equation for l_2, writing your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

The lines l_1 and l_2 cut the x-axis at the points S and T respectively, as shown in Figure 8.

(b) Calculate the area of triangle SPT. (4)