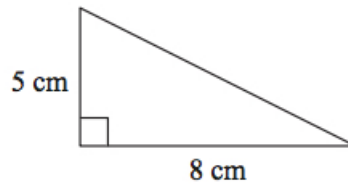


Dr Oliver Mathematics
GCSE Mathematics
2010 November Paper 4H: Calculator
1 hour 45 minutes

The total number of marks available is 100.

You must write down all the stages in your working.

1. Work out the area of this right-angled triangle. (2)



Solution

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 5 \times 8 \\ &= \underline{\underline{20 \text{ cm}^2}}.\end{aligned}$$

2. A spinner can land on red or blue or pink. (2)

The table shows the probabilities that the spinner will land on red or on blue.

Colour	Red	Blue	Pink
Probability	0.58	0.30	

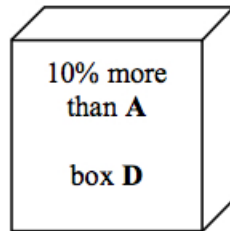
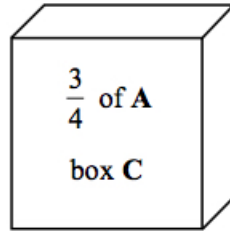
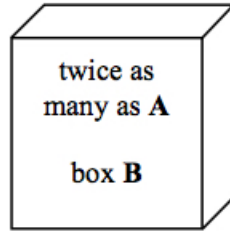
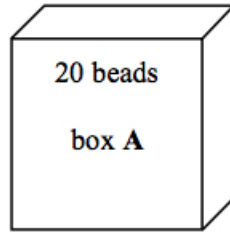
Work out the probability that the spinner will land on pink.

Solution

$$\begin{aligned}\text{Probability} &= 1 - 0.58 - 0.30 \\ &= \underline{\underline{0.12}}.\end{aligned}$$

3. There are 20 beads in box **A**.

(4)



In box **B** there are twice as many beads as in box **A**.
In box **C** there are $\frac{3}{4}$ of the number of beads as in box **A**.
In box **D** there are 10% more beads than in box **A**.
Work out the total number of beads in the four boxes.

Solution

There are 40 beads in box **B**.

There are $20 \times \frac{3}{4} = 15$ beads in box **C**.

There are $20 \times 1.1 = 22$ beads in box **D**.

Hence, there are

$$20 + 40 + 15 + 22 = \underline{\underline{97 \text{ beads}}}.$$

4. Here is a list of ingredients to make melon sorbet for 6 people.

Melon Sorbet	
for 6 people	
800 g	melon
4	egg whites
$\frac{1}{2}$	lime
100 g	caster sugar

Terry makes melon sorbet for 18 people.

- (a) Work out how much caster sugar he uses.

(2)

Solution

$$\frac{18}{6} \times 100 = \underline{\underline{300 \text{ g}}}.$$

Hedley makes melon sorbet.

He uses 2 limes.

- (b) Work out how many people he makes melon sorbet for.

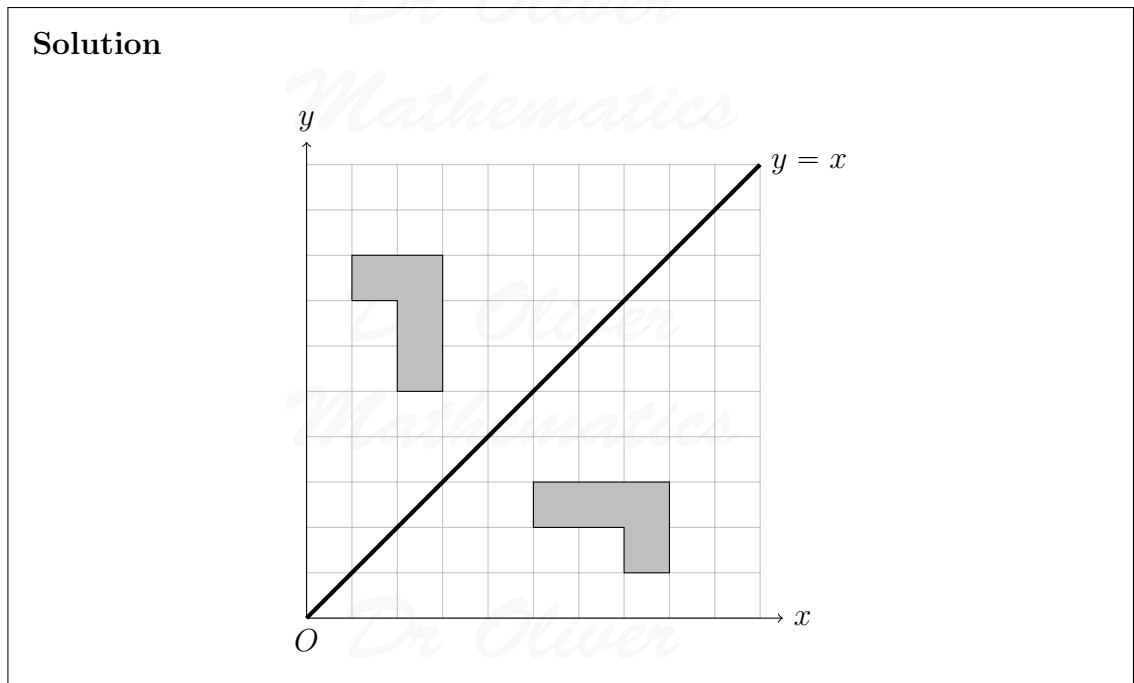
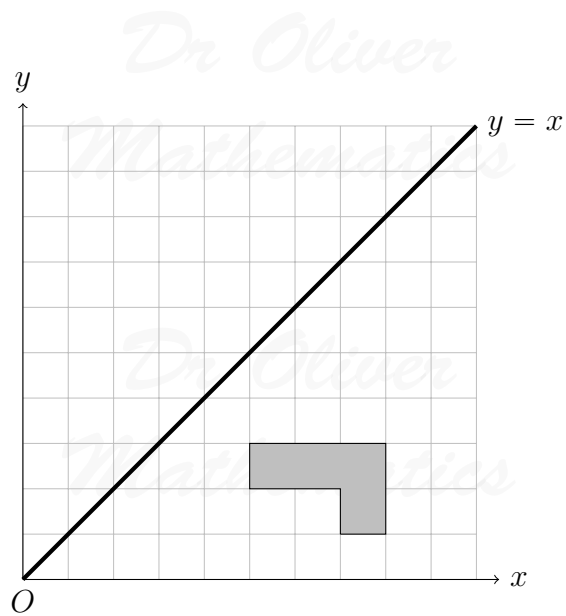
(2)

Solution

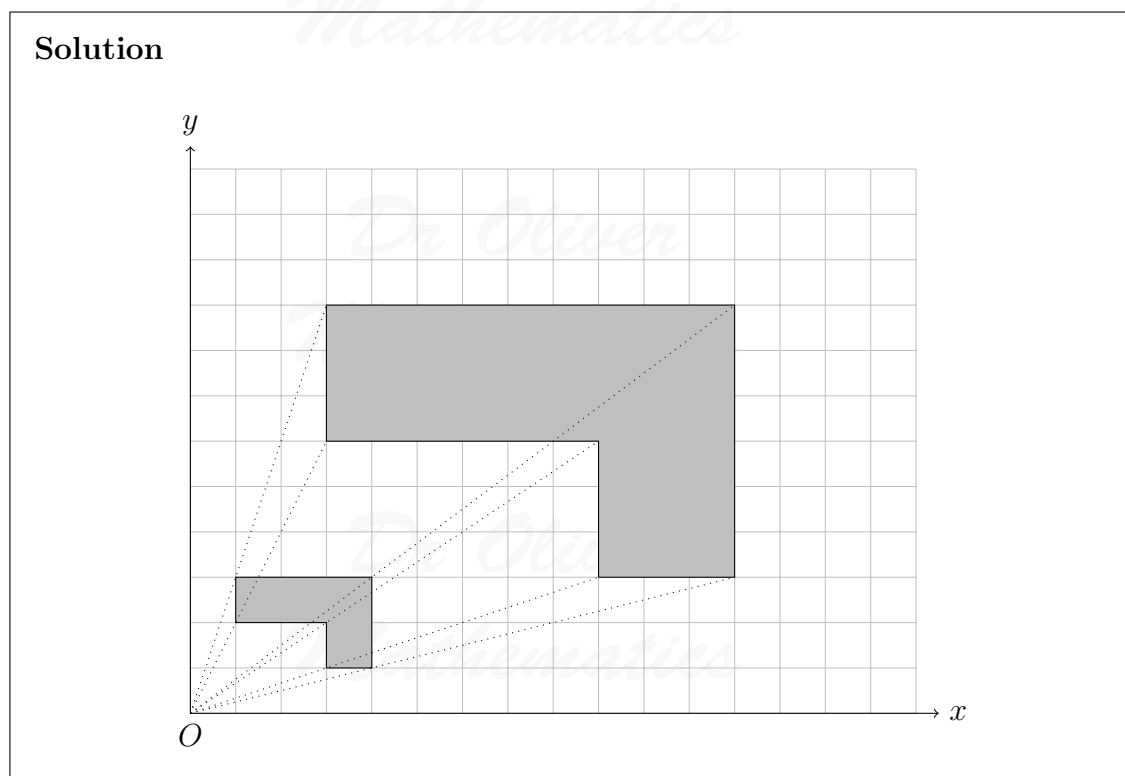
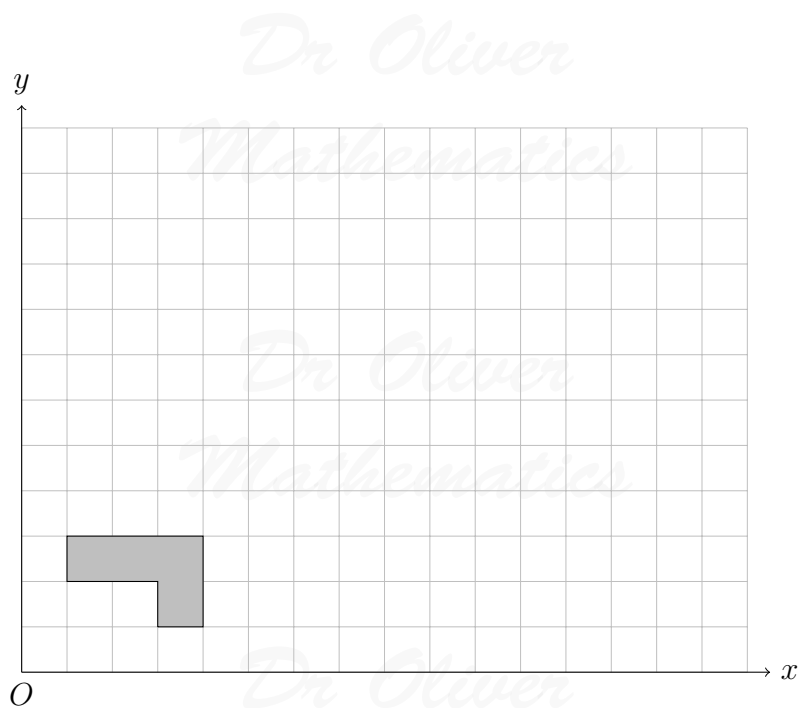
$$\frac{2}{0.5} \times 6 = \underline{\underline{24 \text{ persons}}}.$$

5. (a) Reflect the shaded shape in the line $y = x$.

(2)



- (b) On the grid, enlarge the shaded shape by a scale factor of 3, centre O . (3)



6. (a) Simplify

$$7x + 2y - x + 3y.$$

(2)

Solution

$$7x + 2y - x + 3y = \underline{\underline{6x + 5y.}}$$

(b) Solve

$$2x + 3 = 10.$$

(2)

Solution

$$\begin{aligned} 2x + 3 = 10 &\Rightarrow 2x = 7 \\ &\Rightarrow x = \underline{\underline{3\frac{1}{2}}}. \end{aligned}$$

(c) Simplify

(i) $c^5 \times c^6$,

Solution

$$c^5 \times c^6 = \underline{\underline{c^{11}}}.$$

(ii) $e^{12} \div e^4$.

Solution

$$\frac{e^{12}}{e^4} = \underline{\underline{e^8}}.$$

7. Noah got 8 out of 20 in a test.

Write 8 out of 20 as a percentage.

(2)

Solution

$$\frac{8}{20} \times 100\% = \underline{\underline{40\%}}.$$

8. The table shows some information about the ages, in years, of 60 people.

Age (in years)	Frequency
0 to 9	6
10 to 19	13
20 to 29	12
30 to 39	9
40 to 49	7
50 to 59	3
60 to 69	10

- (a) Write down the modal class. (1)

Solution

10 to 19.

Luke says, "The median lies in the class 30 to 39."

Luke is wrong.

- (b) Explain why. (1)

Solution

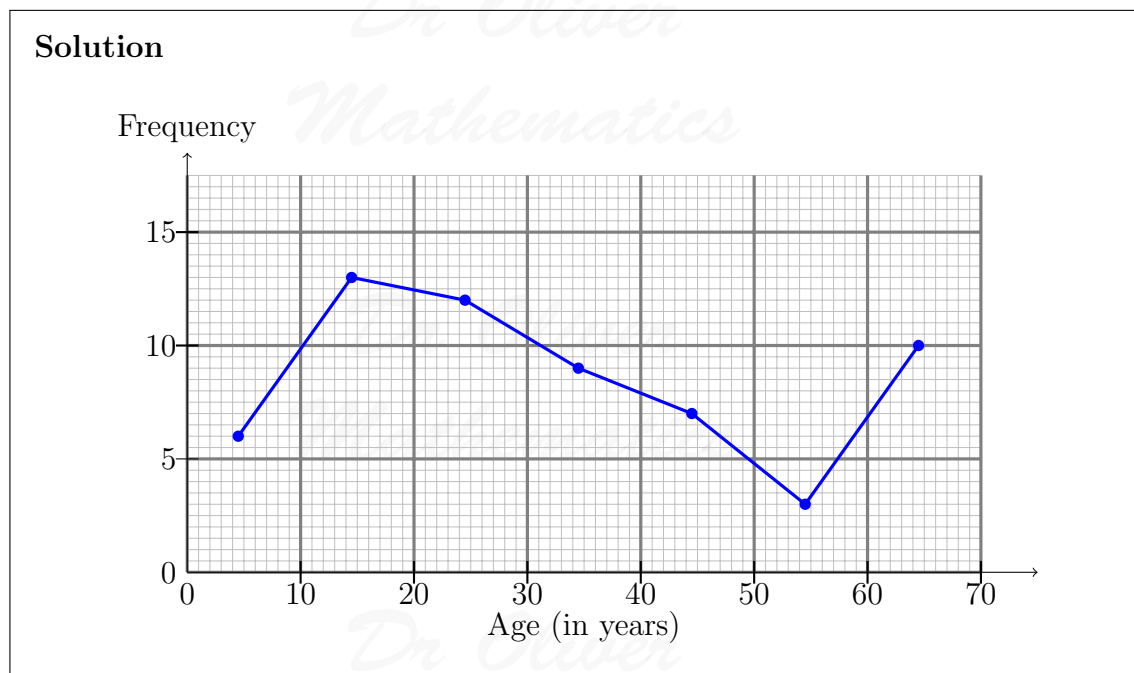
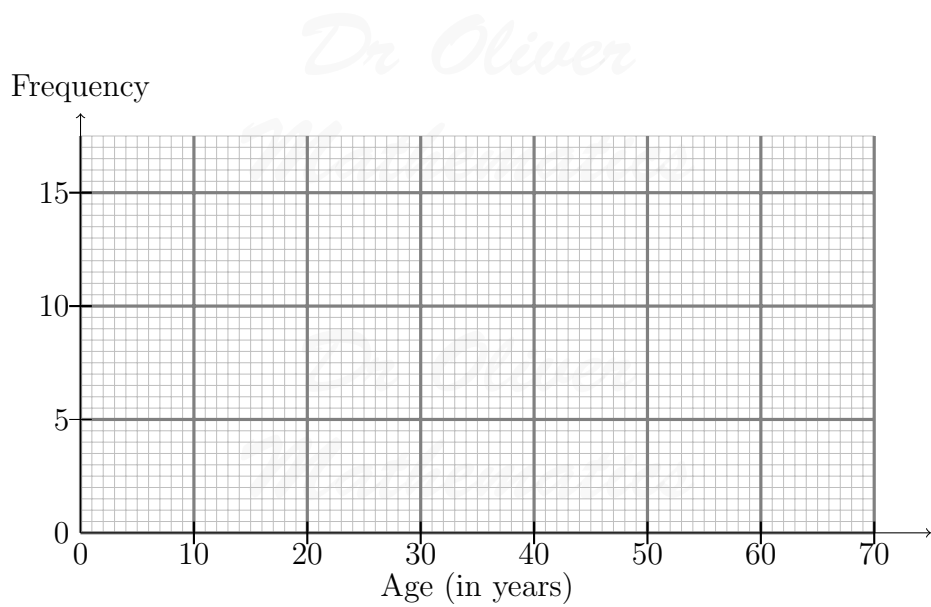
Age (in years)	Frequency	Cumulative frequency
0 to 9	6	6
10 to 19	13	6 + 13 = 19
20 to 29	12	19 + 12 = 31
30 to 39	9	31 + 9 = 40
40 to 49	7	40 + 7 = 47
50 to 59	3	47 + 3 = 50
60 to 69	10	50 + 10 = 60

The median is in the

$$\frac{60 + 1}{2} = 30\frac{1}{2}\text{th}$$

place and the median is located in 20 to 29.

- (c) On the grid, draw a frequency polygon for the information in the table. (2)



9. Use your calculator to work out

$$\frac{13.7 + 5.86}{2.54 \times 3.17}$$

(2)

Write down all the figures on your calculator display.
You must give your answer as a decimal.

Solution

$$\frac{13.7 + 5.86}{2.54 \times 3.17} = \frac{19.56}{8.0518} \\ = \underline{\underline{2.429270474}} \text{ (FCD).}$$

10. $-3 < k \leq 2$.

k is an integer.

(a) Write down all the possible values of k .

(2)

Solution

$-2, -1, 0, 1, 2$

(b) Solve the inequality

$$\frac{2x}{3} < 10.$$

(2)

Solution

$$\frac{2x}{3} < 10 \Rightarrow \underline{\underline{x < 15}}.$$

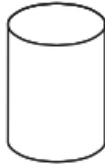
11. Here are four containers.

(2)

Water is poured into each container at a constant rate.



1



2



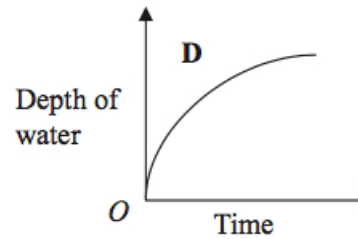
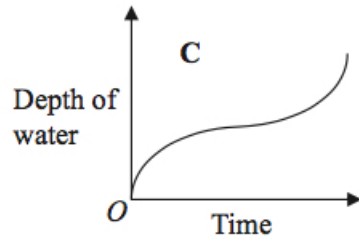
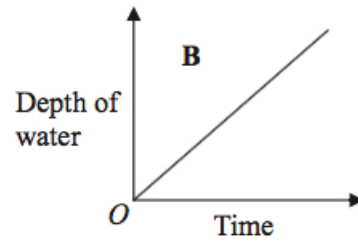
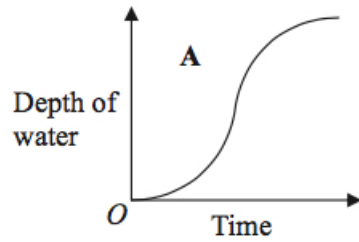
3



4

Here are four graphs.

The graphs show how the depth of the water in each container changes with time.



Match each graph with the correct container.

Solution

A and 3

B and 2

C and 4

D and 1

12. A shop sells small boxes and large boxes for storing CDs. (3)

A small box stores x CDs.

A large box stores y CDs.

Ethan buys 7 small boxes.

He also buys 5 large boxes.

Ethan can store a total of T CDs in these boxes.

Write down a formula for T in terms of x and y .

Solution

$T = 7x + 5y$

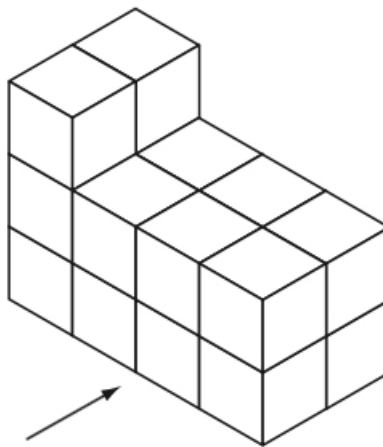
13. A family went on holiday to Miami. (2)
They travelled from London by plane.

The distance from London to Miami is 7 120 km.
The plane journey took 8 hours.
Calculate the average speed of the plane.

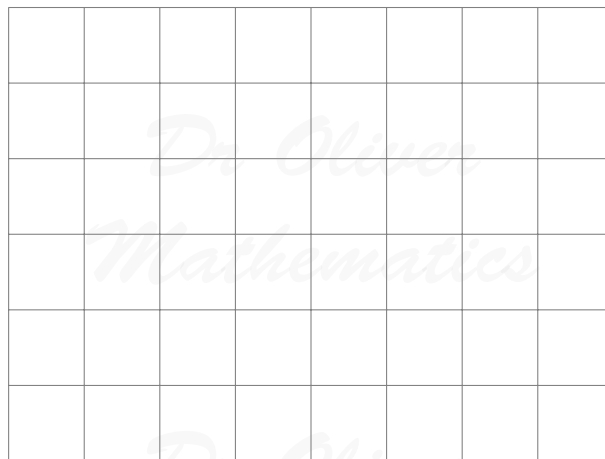
Solution

$$\frac{7\,120}{8} = \underline{\underline{890 \text{ km/h.}}}$$

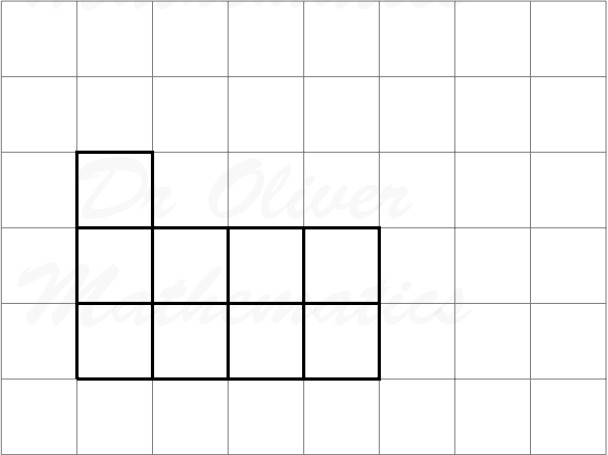
14. The diagram shows a solid prism made from centimetre cubes.



(a) On the centimetre square grid, draw the front elevation of the solid prism from the direction shown by the arrow. (2)



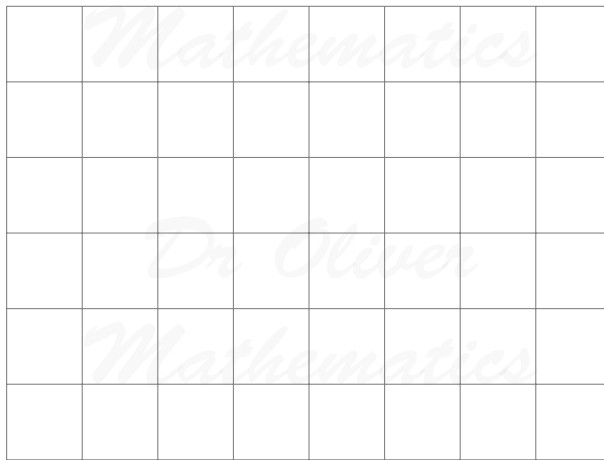
Solution



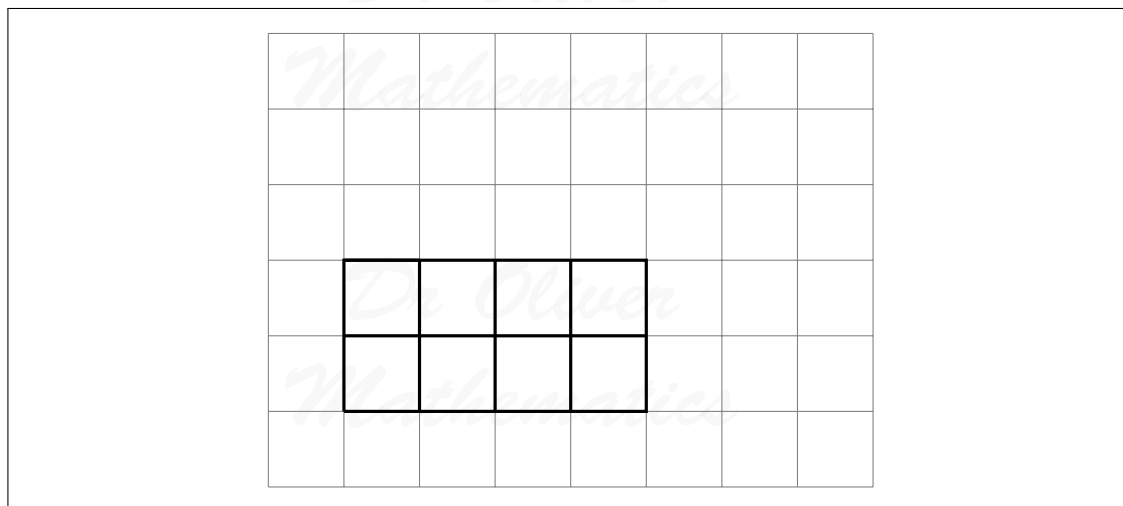
A 6x8 grid of squares. A 3x3 square is drawn in the center, with its top-left corner at the intersection of the 3rd vertical line and the 3rd horizontal line from the top-left. The 3x3 square is drawn with a thicker border than the grid lines.

(b) On the centimetre square grid below, draw the plan of the solid prism.

(2)



Solution



15. 200 students in Year 11 took a mathematics test.
 Kamini wants to find out whether students in Year 11 like mathematics.
 For her sample she asks the 20 students who got the highest marks in the test.
 This is **not** a good sample to use.

(a) Write down **one** reason why.

(1)

Solution

E.g., only the best students were asked, sample size is too small, etc.

She uses this question on her questionnaire.

What do you think of mathematics?		
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Excellent	Very good	Good

(b) Write down **one** thing that is wrong with this question.

(1)

Solution

E.g., the choices are all positive, question does not reference liking, etc.

Kamini also wants to find out how many hours students spend on their mathematics homework.

(c) Design a suitable question that Kamini could use on her questionnaire.
 You must include some response boxes.

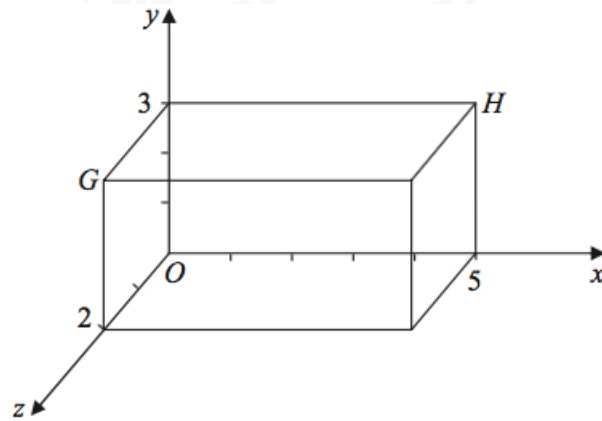
(2)

Solution

A suitable question with a time frame, e.g., “Did you do the mathematics homework last night/last week/last month? Tick the appropriate box.”

At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, $0 \text{ mins} \leq t < 15 \text{ mins}$, $15 \text{ mins} \leq t < 45 \text{ mins}$, $45 \text{ mins} \leq t < 75 \text{ mins}$, $t \geq 75 \text{ mins}$.

16. G and H are vertices of a cuboid.



- (a) Write down the coordinates of point G . (1)

Solution

$G(0, 3, 2)$.

- (b) Write down the coordinates of point H . (1)

Solution

$H(5, 3, 0)$.

17. (a) Write 82 500 000 in standard form. (1)

Solution

$$82\,500\,000 = \underline{\underline{8.25 \times 10^7}}.$$

- (b) Work out (2)

$$(5.2 \times 10^{-7}) \times (2.8 \times 10^{-9}).$$

Give your answer in standard form.

Solution

$$\begin{aligned}(5.2 \times 10^{-7}) \times (2.8 \times 10^{-9}) &= 14.56 \times 10^{-16} \\ &= \underline{\underline{1.456 \times 10^{-15}}}.\end{aligned}$$

18. A water container has 19.5 litres of water in it. (3)
A cup holds 210 ml of water.
At most 92 cups can be filled completely from the water container.
Explain why.
You must show all your working.

Solution

$$\begin{aligned}\frac{19.5 \text{ litres}}{210 \text{ ml}} &= \frac{19.5 \text{ litres}}{0.21 \text{ ml}} \\ &= 92\frac{6}{7};\end{aligned}$$

hence, 92 cups can be filled and there is $\frac{6}{7}$ cups left over.

19. There are 100 teachers at Maria's school.
Maria found out the age of each teacher.
The table gives information about her results.

Age (A years)	Frequency
$20 < A \leq 30$	26
$30 < A \leq 40$	35
$40 < A \leq 50$	21
$50 < A \leq 60$	12
$60 < A \leq 70$	6

- (a) Complete the cumulative frequency table. (1)

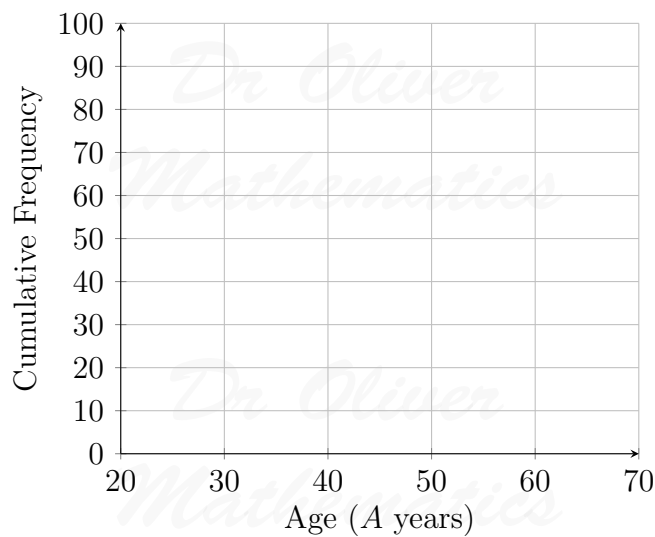
Age (A years)	Cumulative Frequency
$20 < A \leq 30$	26
$20 < A \leq 40$	
$20 < A \leq 50$	
$20 < A \leq 60$	
$20 < A \leq 70$	

Solution

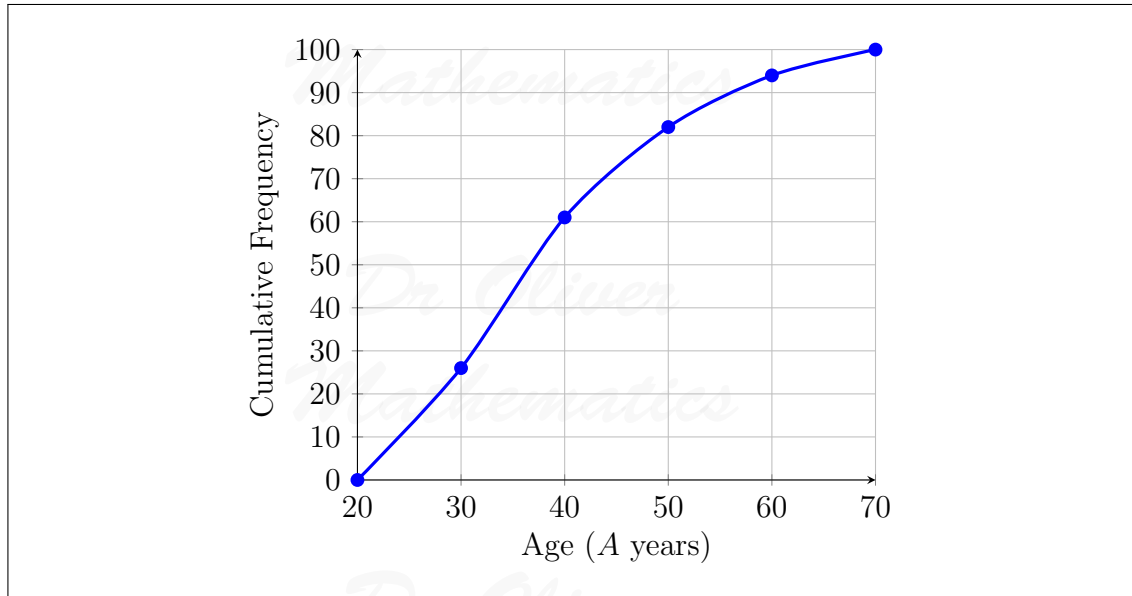
Age (A years)	Cumulative Frequency
$20 < A \leq 30$	26
$20 < A \leq 40$	$26 + 35 = \underline{61}$
$20 < A \leq 50$	$61 + 21 = \underline{82}$
$20 < A \leq 60$	$82 + 12 = \underline{94}$
$20 < A \leq 70$	$94 + 6 = \underline{100}$

(b) On the grid below, draw a cumulative frequency graph for your table.

(2)

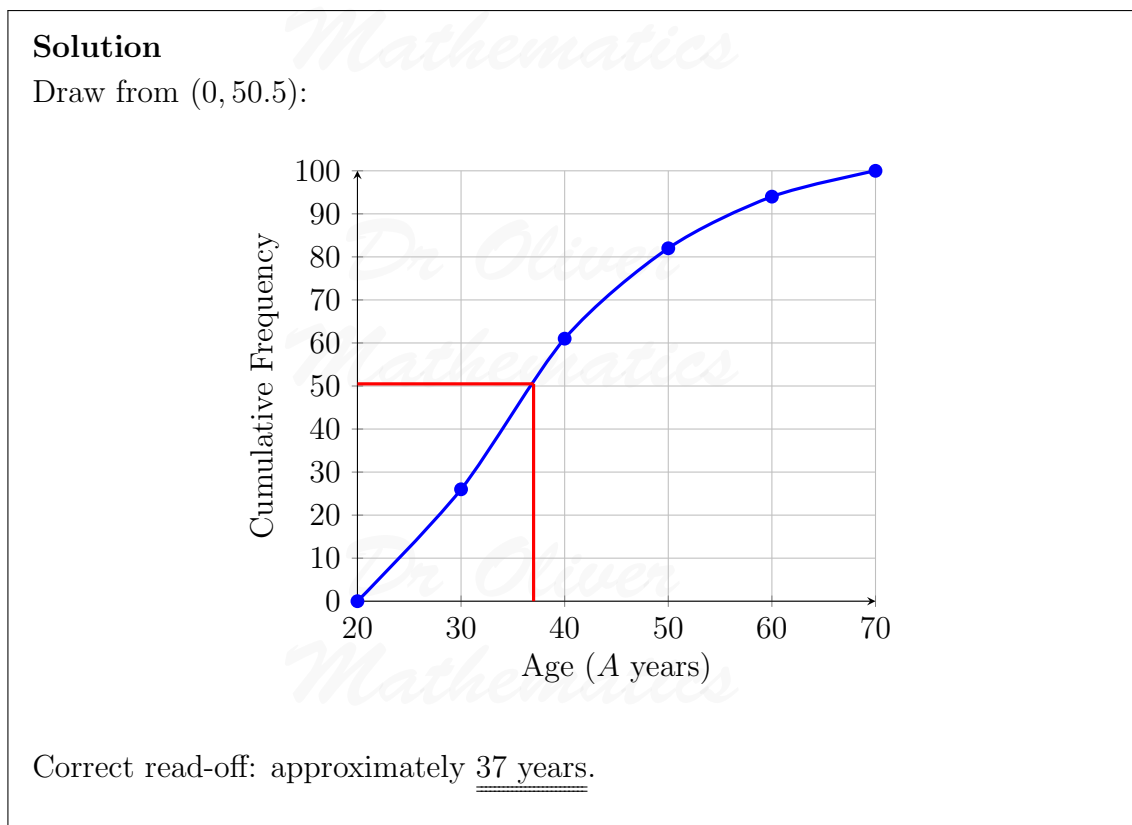


Solution



- (c) Use your graph to find an estimate for the median age.

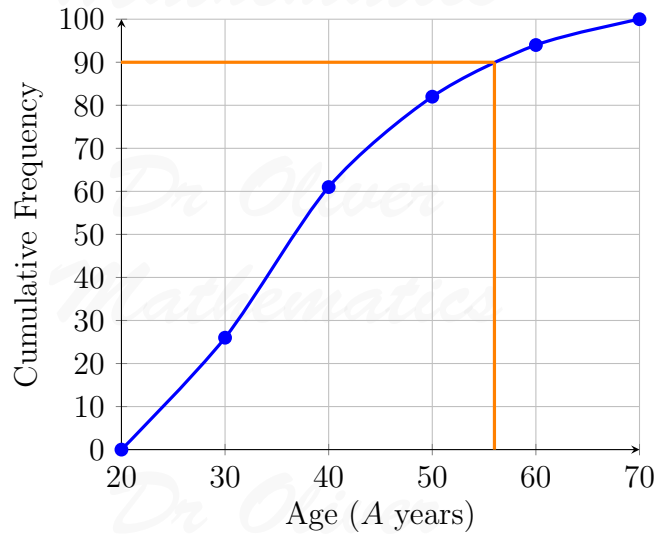
(1)



- (d) Use your graph to find an estimate for the number of these teachers who are **older** than 56 years old.

(2)

Solution



Correct read-off: approximately $100 - 90 = \underline{\underline{10}}$ teachers.

20. (a) Write 56 as a product of its prime factors.

(2)

Solution

$$\begin{array}{r|l} & 56 \\ 2 & 28 \\ 2 & 14 \\ 2 & 7 \\ 7 & 1 \end{array}$$

So

$$56 = 2 \times 2 \times 2 \times 7 = \underline{\underline{2^3 \times 7}}.$$

- (b) Find the Highest Common Factor (HCF) of 56 and 42.

(2)

Solution

Dr Oliver
Mathematics

$$\begin{array}{r|l} & 42 \\ 2 & 21 \\ \hline 3 & 7 \\ \hline 7 & 1 \\ \hline \end{array}$$

So

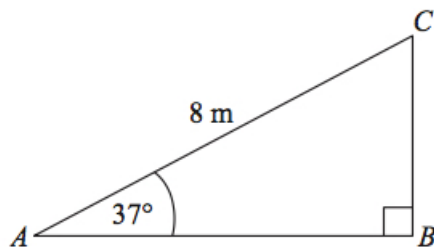
$$42 = 2 \times 3 \times 7.$$

Hence

$$\text{HCF}(56, 42) = 2 \times 7 = \underline{\underline{14}}.$$

21. ABC is a right-angled triangle.

(3)



$AC = 8$ m.

Angle $CAB = 37^\circ$.

Calculate the length of AB .

Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} \text{adj} &= \text{hyp} \times \cos \Rightarrow AB = 8 \cos 37^\circ \\ &= 6.38908408 \text{ (FCD)} \\ &= \underline{\underline{6.39 \text{ m (3 sf)}}}. \end{aligned}$$

22. (a) Complete the table of values for $y = x^3 - 7$.

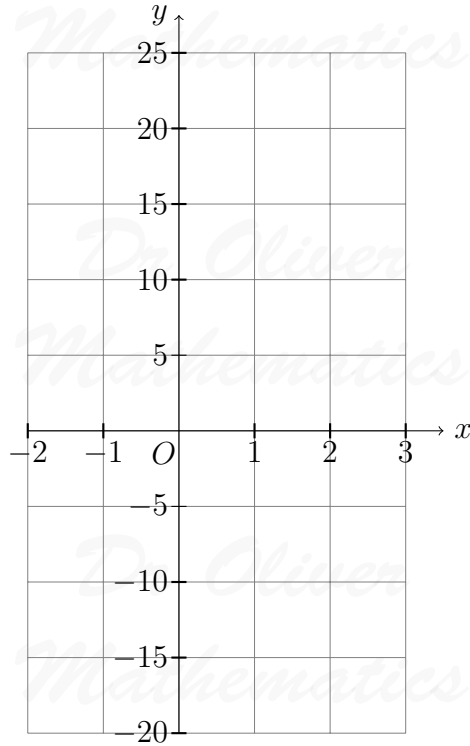
(2)

x	-2	-1	0	1	2	3
y		-8				20

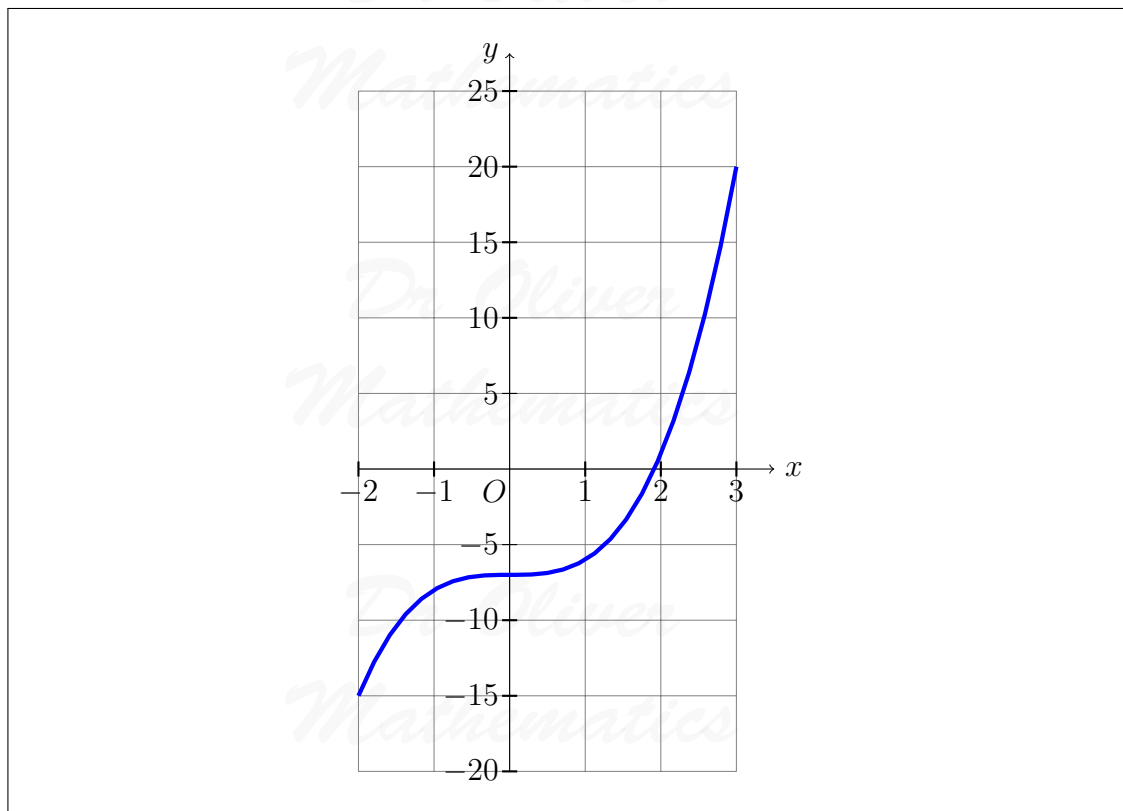
Solution

x	-2	-1	0	1	2	3
y	<u>-15</u>	-8	<u>-7</u>	<u>-6</u>	<u>1</u>	20

(b) On the grid, draw the graph of $y = x^3 - 7$ for values of x from -2 to 3 . (2)

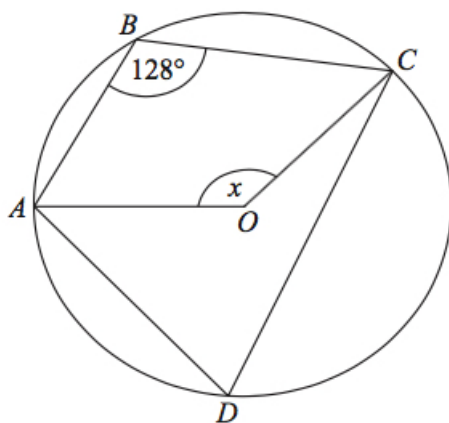


Solution



23. The diagram shows a circle, centre O .

(2)



A , B , C , and D are points on the circumference of the circle.
 Angle $ABC = 128^\circ$.
 Work out the size of the angle marked x .

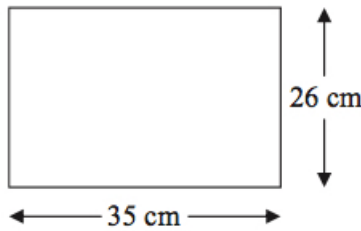
Solution

$\angle ADC = 180 - 128 = 52^\circ$ (opposite angles of a cyclic quadrilateral are supplementary).

$\angle AOC = 2 \times 52 = \underline{104^\circ}$ (angle at the centre is twice the angle at the circumference).

24. Here is a picture of a rectangle.

(3)



The length of the rectangle is 35 cm, correct to the nearest cm.

The width of the rectangle is 26 cm, correct to the nearest cm.

Calculate the upper bound for the area of the rectangle.

Write down all the figures on your calculator display.

Solution

$$34.5 \leq \text{length} < 35.5$$

and

$$25.5 \leq \text{width} < 26.5.$$

Now, the upper bound is

$$35.5 \times 26.5 = \underline{940.75 \text{ cm}^2}.$$

25. (a) Expand and simplify

(2)

$$(2x + 4y)(4x - 5y).$$

Solution

×		2x	+4y
4x		8x ²	+16xy
-5y		-10xy	-20y ²

$$(2x + 4y)(4x - 5y) = \underline{\underline{8x^2 + 6xy - 20y^2}}.$$

(b) Simplify fully

$$\frac{(x + 10)^5}{(x + 10)^4}.$$

(1)

Solution

$$\frac{(x + 10)^5}{(x + 10)^4} = \underline{\underline{x + 10}}.$$

(c) Simplify fully

$$\frac{x^2 - 25}{x^2 + 7x + 10}.$$

(3)

Solution

We will do the numerator first:

$$\left. \begin{array}{l} \text{add to: } 0 \\ \text{multiply to: } -25 \end{array} \right\} -5, +5$$

and the denominator second:

$$\left. \begin{array}{l} \text{add to: } +7 \\ \text{multiply to: } +10 \end{array} \right\} +2, +5$$

Now,

$$\begin{aligned} \frac{x^2 - 25}{x^2 + 7x + 10} &= \frac{(x + 5)(x - 5)}{(x + 2)(x + 5)} \\ &= \underline{\underline{\frac{x - 5}{x + 2}}}. \end{aligned}$$

For all values of x ,

$$x^2 + 6x - 2 \equiv (x + p)^2 + q.$$

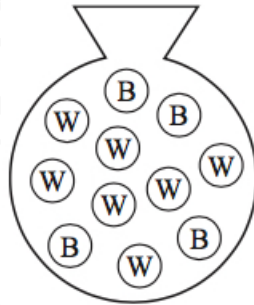
- (d) Find the value of p and the value of q . (2)

Solution

$$\begin{aligned} x^2 + 6x - 2 &\equiv (x^2 + 6x + 9) - 11 \\ &\equiv \underline{\underline{(x + 3)^2 - 11}}; \end{aligned}$$

hence, $p = 3$ and $q = -11$.

26. There are 11 buttons in a bag. (3)



7 buttons are white.

4 buttons are black.

Harley takes a button at random from the bag, and keeps it.

She now takes another button at random from the bag.

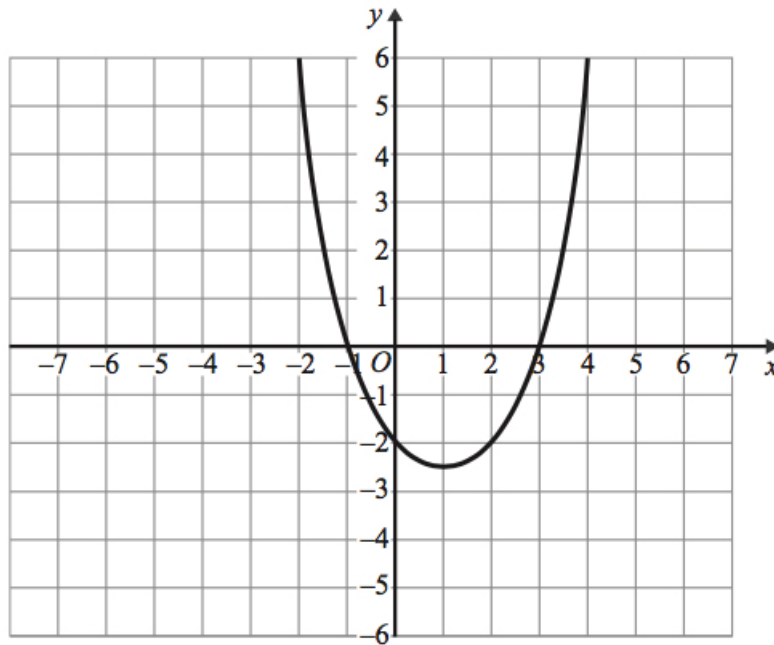
Work out the probability that Harley takes a button of each colour.

Solution

$$\begin{aligned} \text{P(button of each colour)} &= \text{P}(WB) + \text{P}(BW) \\ &= 2 \times \frac{7}{11} \times \frac{4}{10} \\ &= \underline{\underline{\frac{28}{55}}}. \end{aligned}$$

27. The graph of $y = f(x)$ is shown on the grids.

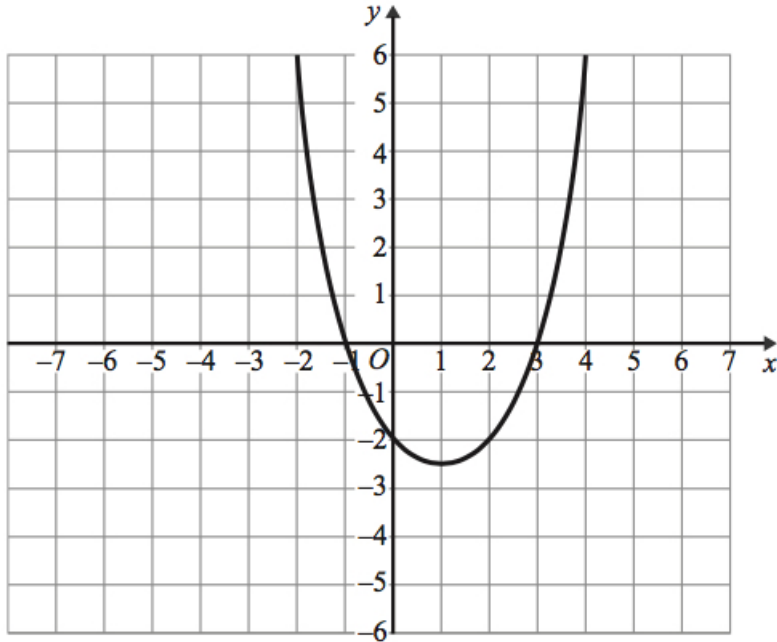
- (a) On this grid, sketch the graph of $y = f(x - 3)$. (2)



Solution

It goes through (1, 6), (2, 0), (3, -2), (5, -2), (6, 0), and (7, 6) with a minimum at (4, -2.5).

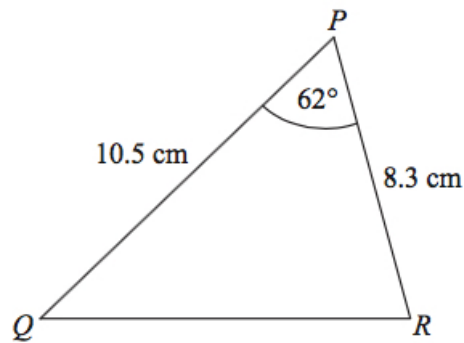
- (b) On this grid, sketch the graph of $y = -f(x)$. (2)



Solution

It goes through $(-2, -6)$, $(-1, 0)$, $(0, 2)$, $(2, 2)$, $(3, 0)$, and $(4, -6)$ with a maximum at $(1, 2.5)$.

28. In triangle PQR ,



$PQ = 10.5$ cm, $PR = 8.3$ cm, and angle $QPR = 62^\circ$.

(a) Calculate the area of triangle PQR .

Give your answer correct to 3 significant figures.

(2)

Solution

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 8.3 \times 10.5 \times \sin 62^\circ \\ &= 38.47444136 \text{ (FCD)} \\ &= \underline{\underline{38.5 \text{ cm}^2}} \text{ (3 sf)}.\end{aligned}$$

(b) Calculate the length of QR .

Give your answer correct to 3 significant figures.

(3)

Solution

$$\begin{aligned}QR &= \sqrt{8.3^2 + 10.5^2 - 2 \times 8.3 \times 10.5 \times \cos 62^\circ} \\ &= 9.864639203 \text{ (FCD)} \\ &= \underline{\underline{9.86 \text{ cm}}} \text{ (3 sf)}.\end{aligned}$$