Dr Oliver Mathematics
Worked Examples
Probability 9

From: Edexcel GCSE Mathematics November 2024 Paper 2 (Calculator)

1. There are only red counters and yellow counters in a box. (5)

   $\frac{3}{5}$ of the counters are red.

   Sophie takes at random two counters from the box.

   The probability that the two counters are the same colour is $\frac{41}{80}$.

   Work out the number of yellow counters in the box.

Solution

We should take the number of counters to be 5 and the numbers of red counters to be 3, right?

No: that is not what the question says. The second line says, "$\frac{3}{5}$ of the counters are red."

Well,

- 5 counters: 3 red, 2 yellow,
- 10 counters: 6 red, 4 yellow,
- 15 counters: 9 red, 6 yellow, etc,

each follow $\frac{3}{5}$ of the counters are red.

Let the number of all of the counters be $5n$.

Then the number of red counters will be $3n$ and the number of yellow counters is

$$5n - 3n = 2n.$$

$\underline{\mathrm{P}(RR)}$:

- Pick any red counter — $3n$ — out of the total number — $5n$:

$$\frac{3n}{5n}.$$

- And then pick any red counter — $(3n-1)$ — out of the remaining counters — $(5n-1)$:

$$\frac{3n-1}{5n-1}.$$

- Multiply them:

$$\left(\frac{3n}{5n}\right) \times \left(\frac{3n-1}{5n-1}\right).$$

<u>P(YY)</u>:

- Pick any yellow counter — $2n$ — out of the total number — $5n$:

$$\frac{2n}{5n}.$$

- And then pick any yellow counter — $(2n-1)$ — out of the remaining counters — $(5n-1)$:

$$\frac{2n-1}{5n-1}.$$

- Multiply them:

$$\left(\frac{2n}{5n}\right) \times \left(\frac{2n-1}{5n-1}\right).$$

Now,

$$\begin{aligned} & \text{P(same colour)} = \tfrac{41}{80} \\ \Rightarrow \quad & \text{P}(RR) + \text{P}(YY) = \tfrac{41}{80} \\ \Rightarrow \quad & \left(\frac{3n}{5n}\right) \times \left(\frac{3n-1}{5n-1}\right) + \left(\frac{2n}{5n}\right) \times \left(\frac{2n-1}{5n-1}\right) = \tfrac{41}{80} \end{aligned}$$

cross-multiply:

$$
\begin{aligned}
&\Rightarrow \quad 80[3n(3n-1)+2n(2n-1)] = 41(5n)(5n-1) \\
&\Rightarrow \quad 80[9n^2-3n+4n^2-2n] = 205n(5n-1) \\
&\Rightarrow \quad 80(13n^2-5n) = 205n(5n-1) \\
&\Rightarrow \quad 1\,040n^2-400n = 1\,025n^2-205n \\
&\Rightarrow \quad 15n^2-195n = 0 \\
&\Rightarrow \quad 15n(n-13) = 0 \\
&\Rightarrow \quad n = 0 \text{ (cannot happen - why?) or } n-13 = 0 \\
&\Rightarrow \quad n = 13 \\
&\Rightarrow \quad 2n = 26;
\end{aligned}
$$

hence, the number of yellow counters in the box is $\underline{\underline{26}}$.

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