Dr Oliver Mathematics
Worked Examples
Probability 3

From: Edexcel 2004 June Paper 5H (Non-Calculator)

1. (a) (i) Factorise $2x^2 - 35x + 98$. (3)

Solution

$$\left.\begin{array}{rc} \text{add to:} & -35 \\ \text{multiply to:} & (+2) \times (+98) = 196 \end{array}\right\} -7,\ -28$$

Finally,

$$\begin{aligned} 2x^2 - 35x + 98 &= 2x^2 - 7x - 28x + 98 \\ &= x(2x - 7) - 14(2x - 7) \\ &= \underline{\underline{(x - 14)(2x - 7)}}. \end{aligned}$$

(ii) Solve the equation $2x^2 - 35x + 98 = 0$.

Solution

$$\begin{aligned} 2x^2 - 35x + 98 = 0 &\Rightarrow (x - 14)(2x - 7) = 0 \\ &\Rightarrow x - 14 = 0 \text{ or } 2x - 7 = 0 \\ &\Rightarrow \underline{\underline{x = 14 \text{ or } x = 3\tfrac{1}{2}}}. \end{aligned}$$

A bag contains $(n + 7)$ tennis balls.
n of the balls are yellow.
The other 7 balls are white.
John will take at random a ball from the bag.
He will look at its colour and then put it back in the bag.

(b) (i) Write down an expression, in terms of n, for the probability that John will take a white ball. (3)

Solution
The probability that John will take a white ball is $\underline{\underline{\frac{7}{n+7}}}$.

Bill states that the probability that John will take a white ball is $\frac{2}{5}$.

(ii) Prove that Bill's statement cannot be correct.

Solution

$$\begin{aligned}\tfrac{7}{n+7} = \tfrac{2}{5} &\Rightarrow 35 = 2(n+7)\\ &\Rightarrow 17\tfrac{1}{2} = n+7\\ &\Rightarrow n = 10\tfrac{1}{2};\end{aligned}$$

hence, it is not an integer and Bill's statement cannot be correct.

After John has put the ball back into the bag, Mary will then take at random a ball from the bag.
She will note its colour.

(c) Given that the probability that John and Mary will take balls with different colours is $\frac{4}{9}$, prove that (5)

$$2n^2 - 35n + 98 = 0.$$

Solution

$$\begin{aligned}2 \times \tfrac{7}{n+7} \times \tfrac{n}{n+7} = \tfrac{4}{9} &\Rightarrow 18 \times 7 \times n = 4(n+7)^2\\ &\Rightarrow 126n = 4(n^2 + 14n + 49)\\ &\Rightarrow 126n = 4n^2 + 56n + 196\\ &\Rightarrow 4n^2 - 70n + 196 = 0\\ &\Rightarrow 2n^2 - 35n + 98 = 0,\end{aligned}$$

as required.

(d) Using your answer to part (a) (ii) or otherwise, calculate the probability that John and Mary will both take white balls. (2)

Solution

There are $7 + 14 = 21$ tennis balls and

$$\tfrac{7}{21} \times \tfrac{7}{21} = \tfrac{1}{9}.$$