

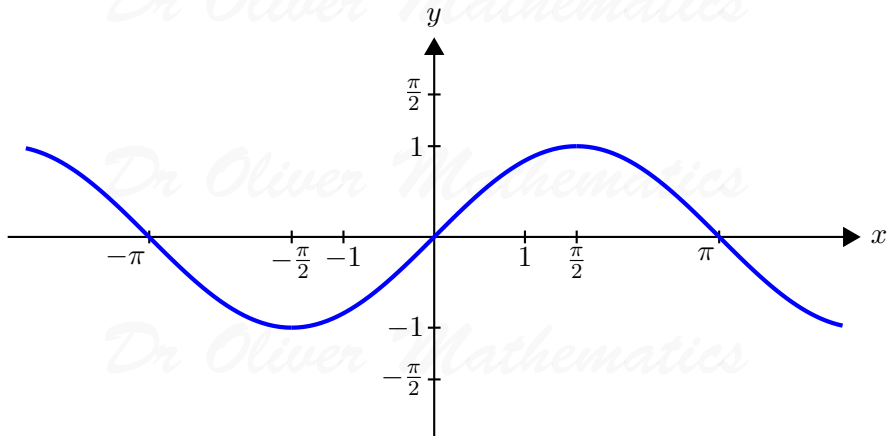
# Dr Oliver Mathematics

Inverse Trigonometric Functions

Core Mathematics 3, Chapter 6

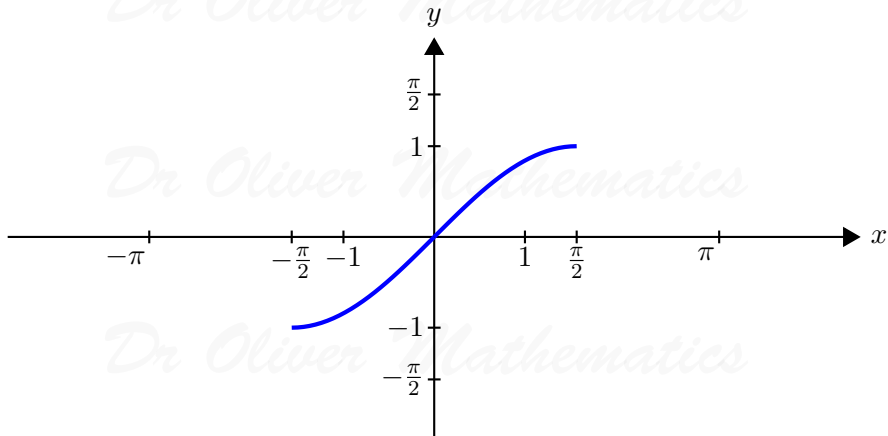
# Inverse Sine Function

Step 1: Draw the sine curve



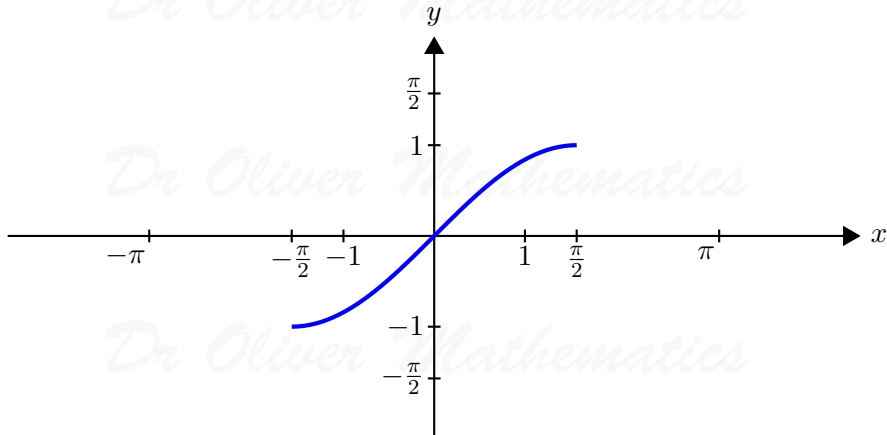
# Inverse Sine Function

Step 2: Restrict the domain to get a one-to-one function



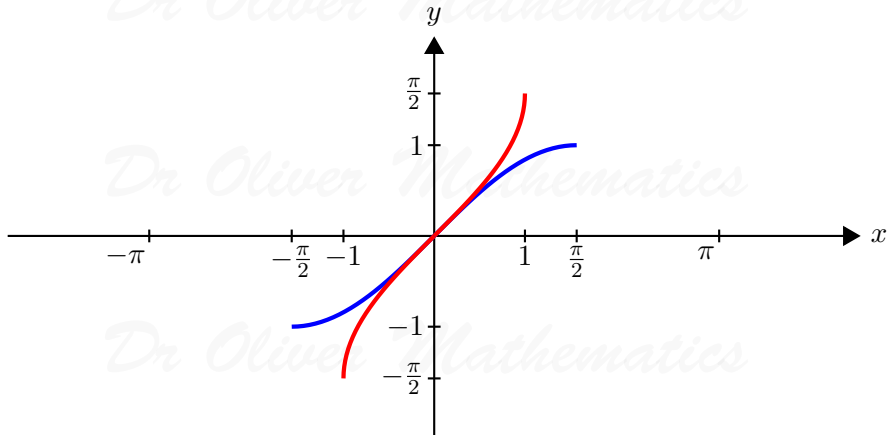
# Inverse Sine Function

Step 2: In this case, an appropriate domain is  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$



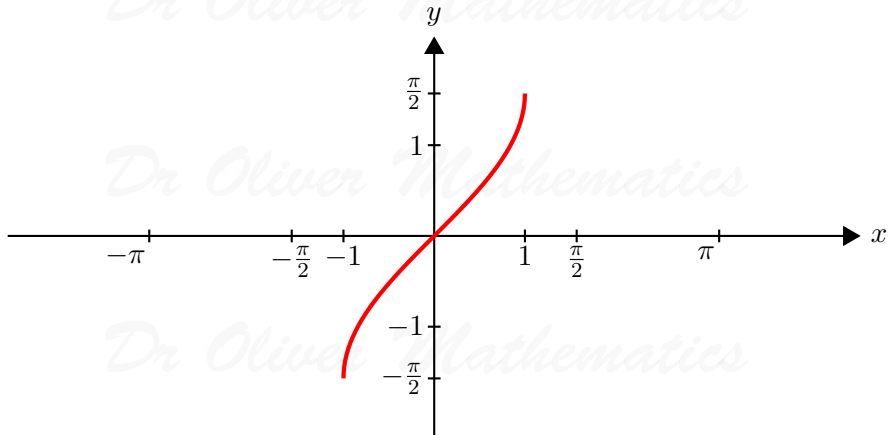
# Inverse Sine Function

Step 3: Reflect in the line  $y = x$



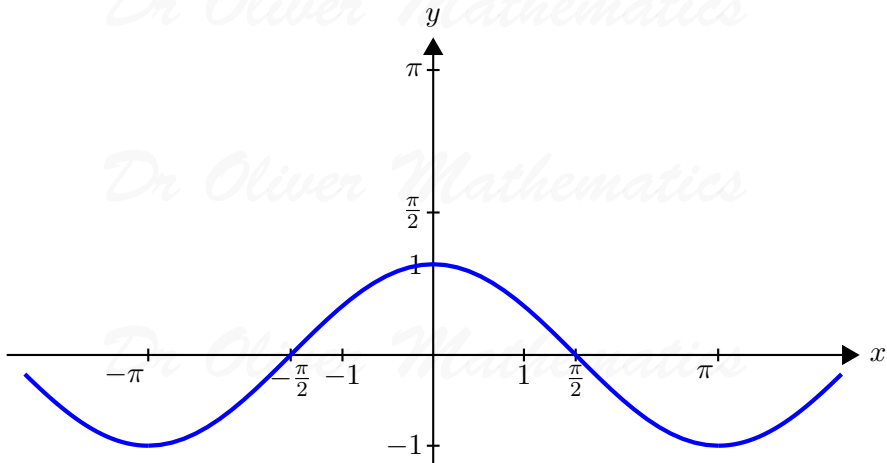
# Inverse Sine Function

Step 4: You now have the graph of  $y = \arcsin x$



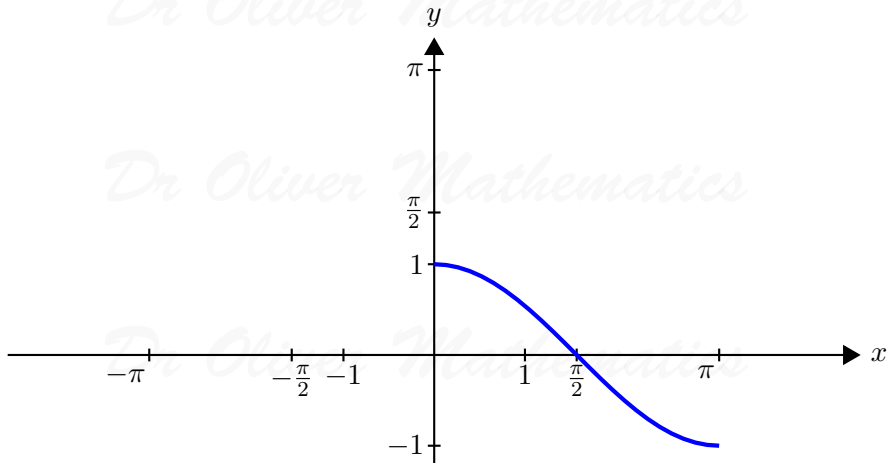
# Inverse Cosine Function

Step 1: Draw the cosine curve



# Inverse Cosine Function

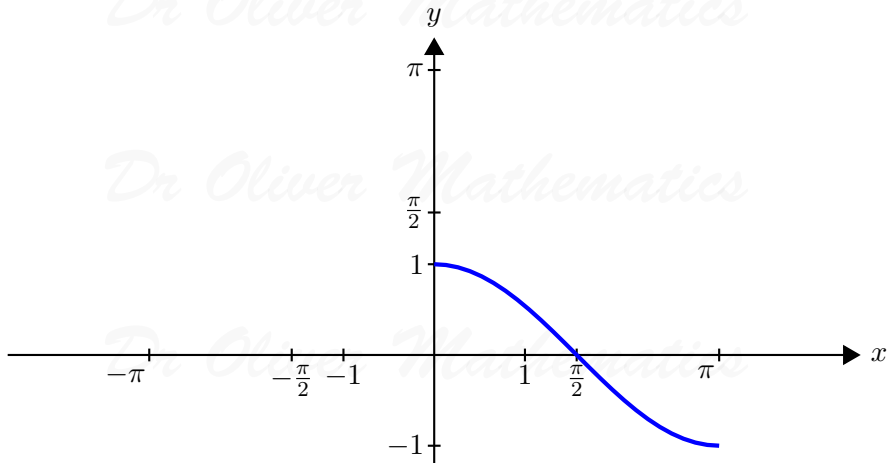
Step 2: Restrict the domain to get a one-to-one function





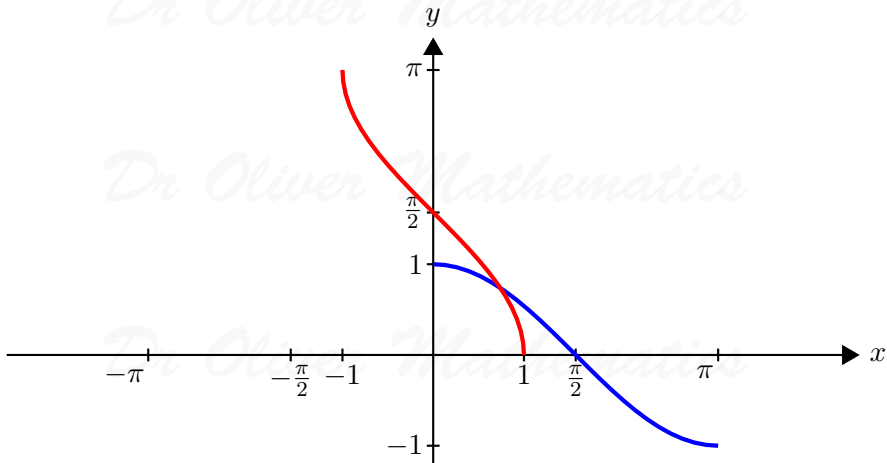
# Inverse Cosine Function

Step 2: In this case, an appropriate domain is  $0 \leq x \leq \pi$



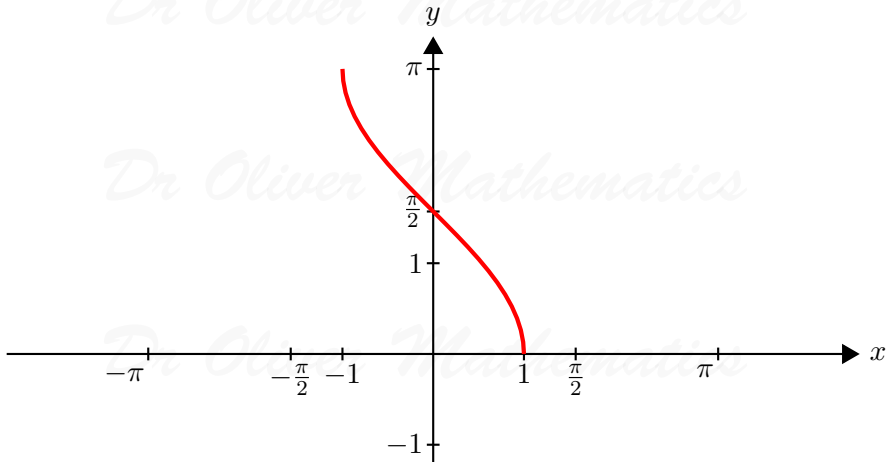
# Inverse Cosine Function

Step 3: Reflect in the line  $y = x$



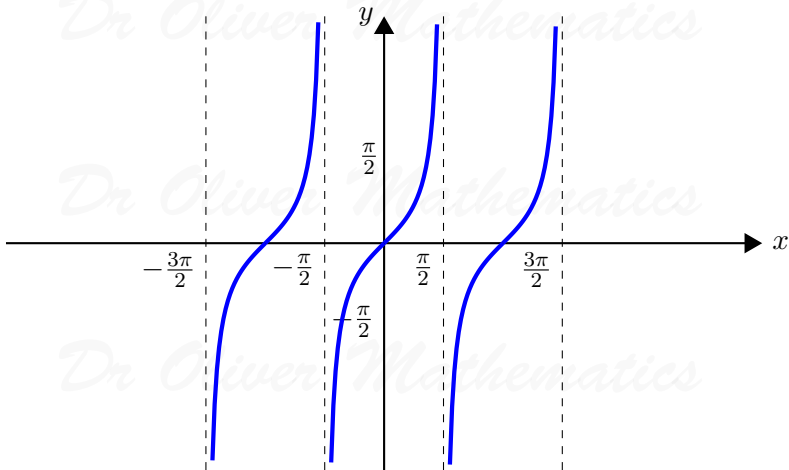
# Inverse Cosine Function

Step 4: You now have the graph of  $y = \arccos x$



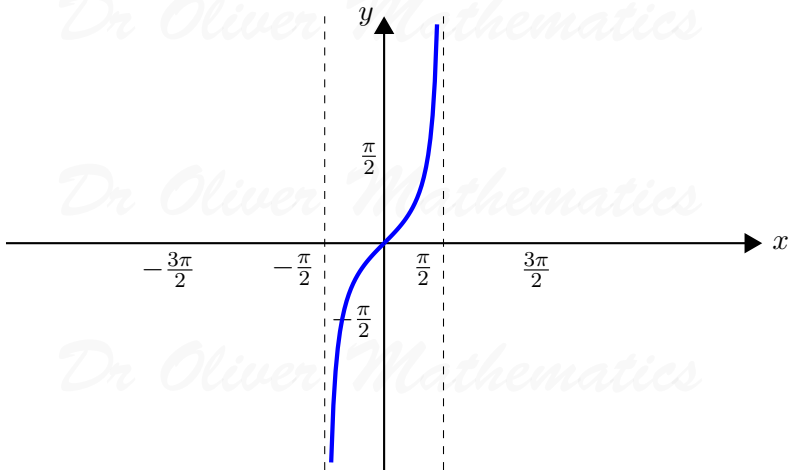
# Inverse Tangent Function

Step 1: Draw the tangent curve



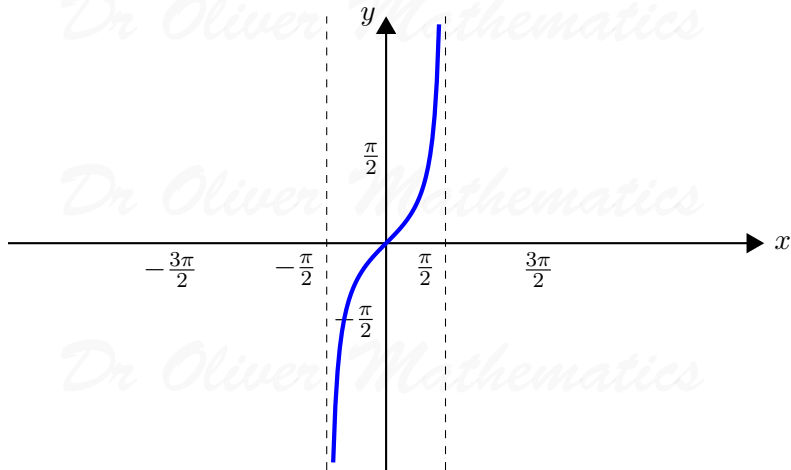
# Inverse Tangent Function

Step 2: Restrict the domain to get a one-to-one function



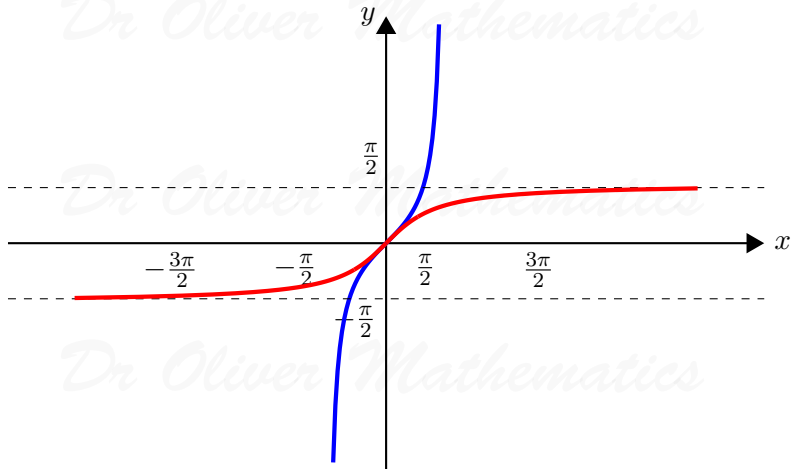
# Inverse Tangent Function

Step 2: In this case, an appropriate domain is  $-\frac{\pi}{2} < x < \frac{\pi}{2}$



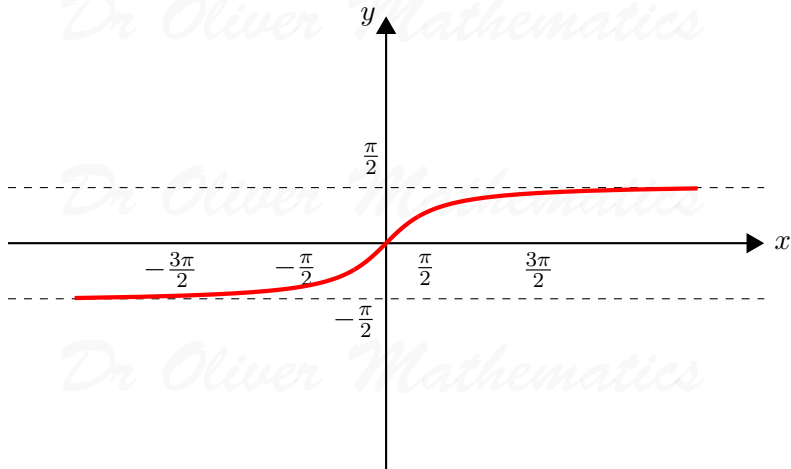
# Inverse Tangent Function

Step 3: Reflect in the line  $y = x$



# Inverse Tangent Function

Step 4: You now have the graph of  $y = \arctan x$





# Summary: the six functions in radians

Function	Domain	Range
$\sin x$	$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$	$-1 \leq \sin x \leq 1$
$\arcsin x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2}$
$\cos x$	$0 \leq x \leq \pi$	$-1 \leq \cos x \leq 1$
$\arccos x$	$-1 \leq x \leq 1$	$0 \leq \arccos x \leq \pi$
$\tan x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	$-\infty < \tan x < \infty$
$\arctan x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$

Recall from Core Mathematics 3, chapter 2 that the domain of a function is precisely the same as the range of the inverse function and vice versa.

# Summary: the six functions in degrees

Function	Domain	Range
$\sin x^\circ$	$-90 \leq x \leq 90$	$-1 \leq \sin x \leq 1$
$\arcsin x$	$-1 \leq x \leq 1$	$-90^\circ \leq \sin^{-1} x \leq 90^\circ$
$\cos x^\circ$	$0 \leq x \leq 180$	$-1 \leq \cos x \leq 1$
$\arccos x$	$-1 \leq x \leq 1$	$0^\circ \leq \cos^{-1} x \leq 180^\circ$
$\tan x^\circ$	$-90 < x < 90$	$-\infty < \tan x < \infty$
$\arctan x$	$-\infty < x < \infty$	$-90^\circ < \tan^{-1} x < 90^\circ$

The functions  $\arcsin x$  and  $\sin^{-1} x$  are interchangeable and you should be familiar with both of them.

# Summary: the six functions in radians

$$y = \arcsin x \Leftrightarrow x = \sin y$$
$$-1 \leq x \leq 1$$
$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$y = \arccos x \Leftrightarrow x = \cos y$$
$$-1 \leq x \leq 1$$
$$0 \leq y \leq \pi$$

$$y = \arctan x \Leftrightarrow x = \tan y$$
$$-\infty < x < \infty$$
$$-\frac{\pi}{2} < y < \frac{\pi}{2}$$