Dr Oliver Mathematics
GCSE Mathematics
2018 November Paper 3H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. (a) Write (1)

$$7\,357$$

correct to 3 significant figures.

Solution

$$7\,357 = \underline{\underline{7\,360 \quad (3 \text{ sf})}}.$$

(b) Work out (2)

$$\frac{\sqrt{17+4^2}}{7.3^2}.$$

Write down all the figures on your calculator display.

Solution

$$\begin{aligned}\frac{\sqrt{17+4^2}}{7.3^2} &= \frac{\sqrt{33}}{53.29} \\ &= \underline{\underline{0.107\,798\,135\,6 \text{ (FCD)}}}.\end{aligned}$$

2. Last year Jo paid £245 for her car insurance. (3)
This year she has to pay £883 for her car insurance.

Work out the percentage increase in the cost of her car insurance.

Solution

$$\begin{aligned}\text{Percentage increase} &= \left(\frac{883-245}{245}\right) \times 100\% \\ &= \underline{\underline{260\tfrac{20}{49}\%}}.\end{aligned}$$

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3. (a) Complete this table of values for (2)

$$y = x^2 + x - 4.$$

x	-3	-2	-1	0	1	2	3
y		-2	-4		-2		

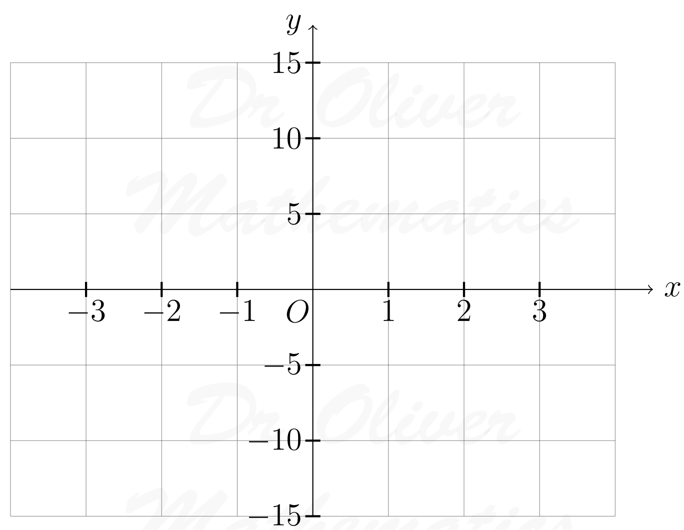
Solution

x	-3	-2	-1	0	1	2	3
y	$\underline{\underline{2}}$	-2	-4	$\underline{\underline{-4}}$	-2	$\underline{\underline{2}}$	$\underline{\underline{8}}$

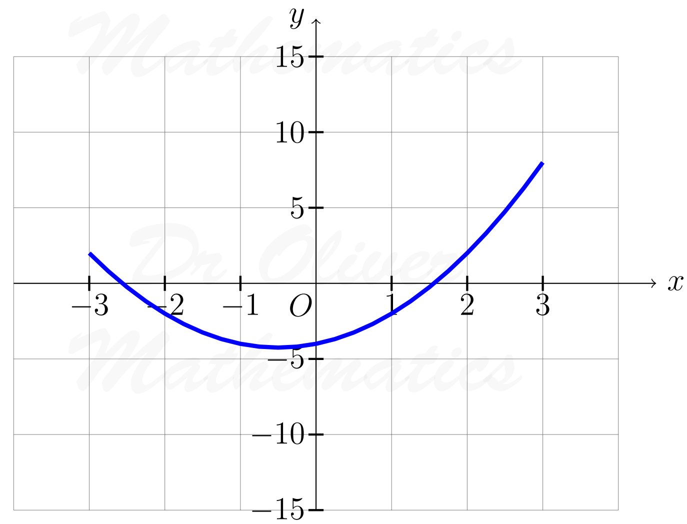
(b) On the grid, draw the graph of (2)

$$y = x^2 + x - 4$$

for values of x from -3 to 3.



Solution



(c) Use the graph to estimate a solution to (1)

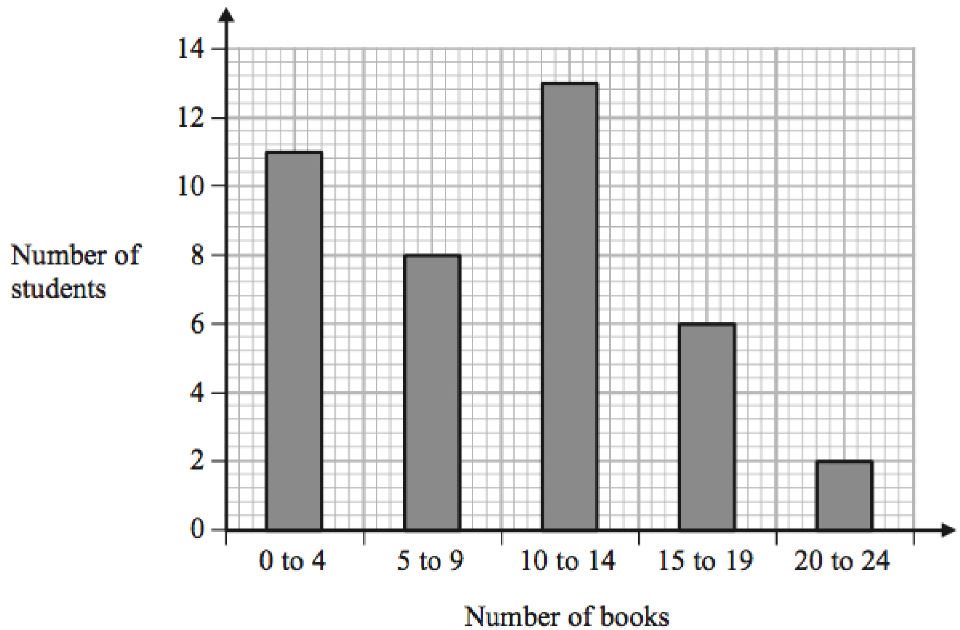
$$x^2 + x - 4 = 0.$$

Solution

Correct read-off: $\underline{\underline{x = -2.6 \text{ or } x = 1.6}}$.

4. Fran asks each of 40 students how many books they bought last year.

The chart below shows information about the number of books bought by each of the 40 students.



(a) Work out the percentage of these students who bought 20 or more books. (2)

Solution

$$\begin{aligned} \text{P(percentage)} &= \frac{2}{40} \times 100\% \\ &= \underline{\underline{5\%}}. \end{aligned}$$

(b) Show that an estimate for the mean number of books bought is 9.5. (4)
You must show all your working.

Solution

We need the midpoint of the interval: 2, 7, 12, 17, and 22.

$$\begin{aligned} \text{Mean} &\approx \frac{(2 \times 11) + (7 \times 8) + (12 \times 13) + (17 \times 6) + (22 \times 2)}{40} \\ &= \frac{22 + 56 + 156 + 102 + 44}{40} \\ &= \frac{380}{40} \\ &= \underline{\underline{9.5}}, \end{aligned}$$

as required.

5. Lara is a skier.

She completed a ski race in 1 minute 54 seconds.
The race was 475 m in length.

Lara assumes that her average speed is the same for each race.

(a) Using this assumption, work out how long Lara should take to complete a 700 m race. (3)
Give your answer in minutes and seconds.

Solution
x be the number of seconds to complete a 700 m race. Then

$$\frac{x}{114} = \frac{700}{475} \Rightarrow x = \frac{700 \times 114}{475}$$
$$\Rightarrow x = 168 \text{ s};$$

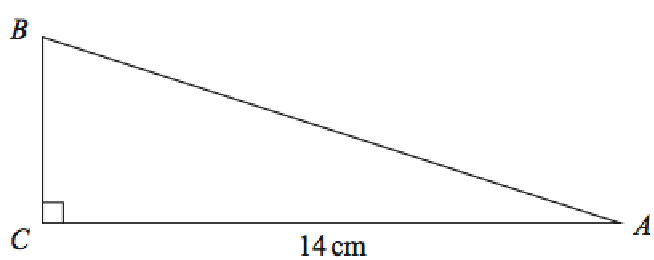
hence, it will take 2 minutes 48 seconds.

Lara's average speed actually increases the further she goes.

(b) How does this affect your answer to part (a)? (1)

Solution
E.g., Lara takes less time.

6. ABC is a right-angled triangle. (4)



$AC = 14$ cm.
Angle $C = 90°$.
Size of angle B : size of angle $A = 3 : 2$.

Work out the length of AB.
Give your answer correct to 3 significant figures.

Solution

$$3 + 2 = 5$$

and

$$\angle B = \tfrac{3}{5} \times 90 = 54^\circ.$$

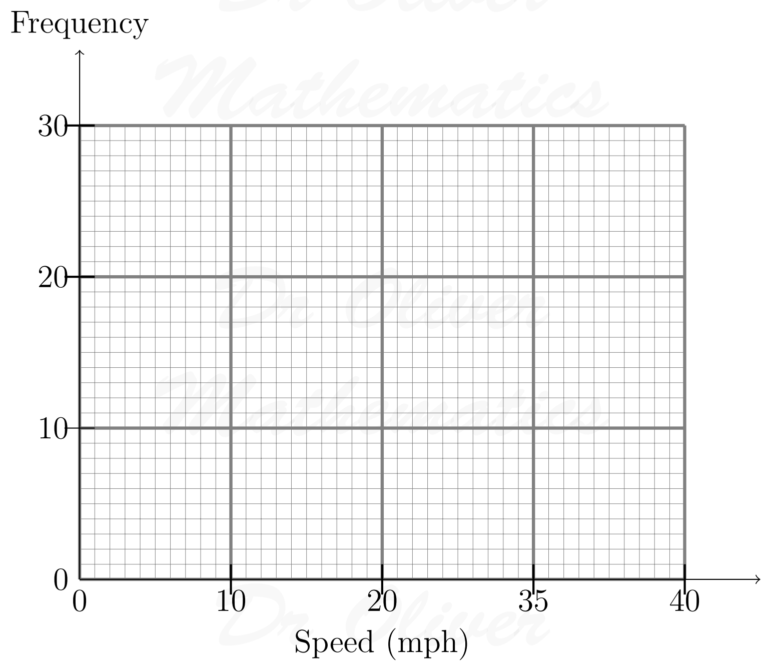
Hence,

$$\begin{aligned}\sin = \frac{\text{opp}}{\text{hyp}} &\Rightarrow \sin 54^\circ = \frac{14}{AB}\\ &\Rightarrow AB = \frac{14}{\sin 54^\circ}\\ &\Rightarrow AB = 17.304\,951\,68 \text{ (FCD)}\\ &\Rightarrow \underline{\underline{AB = 17.3 \text{ cm (3 sf)}}}.\end{aligned}$$

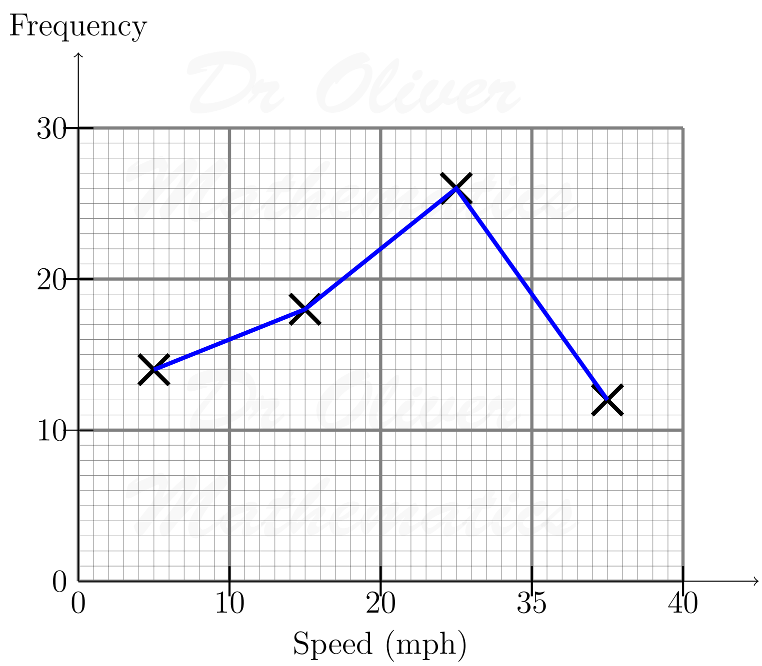
7. The table gives information about the speeds of 70 cars. (2)

Speed, s mph	Frequency
$0 < s \leqslant 10$	14
$10 < s \leqslant 20$	18
$20 < s \leqslant 30$	26
$30 < s \leqslant 40$	12

Draw a frequency polygon for this information.

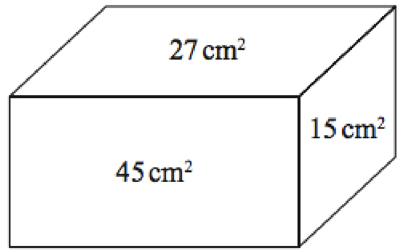


Solution



8. The diagram shows a solid metal cuboid. (5)

The areas of three of the faces are marked on the diagram.
The lengths, in cm, of the edges of the cuboid are whole numbers.



The metal cuboid is melted and made into cubes.
Each of the cubes has sides of length 2.5 cm.

Work out the greatest number of these cubes that can be made.

Solution

$$\begin{aligned} 15 &= 3 \times 5 \\ 27 &= 3 \times 9 \\ 45 &= 5 \times 9 \end{aligned}$$

and so the linear dimensions are 3 cm, 5 cm, and 9 cm.

$$\begin{aligned} \text{Volume of cuboid} &= 3 \times 5 \times 9 \\ &= 135 \text{ cm}^3. \end{aligned}$$

Now,

$$\text{volume of cube} = 2.5^3 = 15.625 \text{ cm}^3.$$

Next,

$$\frac{\text{volume of cuboid}}{\text{volume of cube}} = \frac{135}{15.625} = 8.64.$$

Hence, we can make 8 cubes.

9. (a) Expand and simplify (3)

$$(x-2)(2x+3)(x+1).$$

Solution

	$2x$	$+3$
x	$2x^2$	$+3x$
-2	$-4x$	-6

$$(x-2)(2x+3)(x+1) = (2x^2 - x - 6)(x+1)$$

	$2x^2$	$-x$	-6
x	$2x^3$	$-x^2$	$-6x$
$+1$	$+2x^2$	$-x$	-6

$$(x-2)(2x+3)(x+1) = \underline{\underline{2x^3 + x^2 - 7x - 6}}.$$

$$\frac{y^4 \times y^n}{y^2} = y^{-3}.$$

(b) Find the value of n. (2)

Solution

$$\begin{aligned}\frac{y^4 \times y^n}{y^2} = y^{-3} &\Rightarrow y^2 \times y^n = y^{-3}\\ &\Rightarrow y^n = y^{-5}\\ &\Rightarrow \underline{\underline{n = -5}}.\end{aligned}$$

(c) Solve (3)

$$5x^2 - 4x - 3 = 0.$$

Give your solutions correct to 3 significant figures.

Solution

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$a = 5$, $b = -4$, and $c = -3$:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{4 \pm \sqrt{(-4)^2 - 4 \times 5 \times (-3)}}{2 \times 5} \\ &= \frac{4 \pm \sqrt{76}}{10} \\ &= -0.471\,779\,788\,7,\ 1.271\,799\,789 \text{ (FCD)} \\ &= \underline{\underline{-0.472,\ 1.27 \text{ (3 sf)}}}. \end{aligned}$$

10.

$$f(x) = 4 \sin x^\circ.$$

(a) Find f(23). (1)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} f(23) &= 4 \sin 23^\circ \\ &= 1.562\,924\,514 \text{ (FCD)} \\ &= \underline{\underline{1.56 \text{ (3 sf)}}} \end{aligned}$$

$$g(x) = 2x - 3.$$

(b) Find f g(34). (2)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} f\,g(34) &= f(g(34)) \\ &= f(2 \times 34 - 3)) \\ &= f(65) \\ &= 3.625\,231\,114\,8 \text{ (FCD)} \\ &= \underline{\underline{3.63 \text{ (3 sf)}}}. \end{aligned}$$

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$$h(x) = (x + 4)^2.$$

Ivan needs to solve the following equation

$$h(x) = 25.$$

He writes

$$(x + 4)^2 = 25$$
$$x + 4 = 5$$
$$x = 1.$$

This is not fully correct.

(c) Explain why. (1)

Solution

The second line should read

$$\underline{\underline{x + 4 = \pm 5}}$$

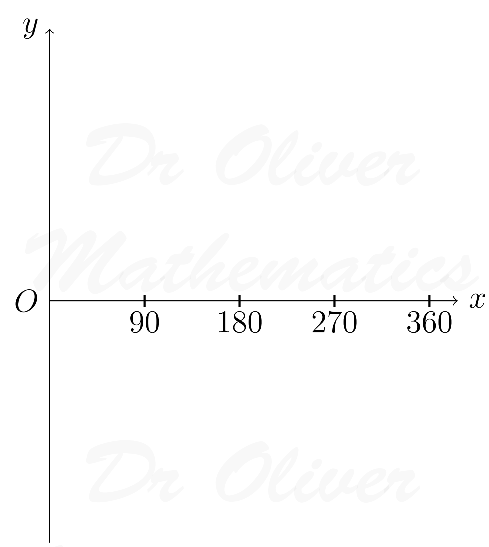
and the third line should read

$$\underline{\underline{x = -9 \text{ or } x = 1}}.$$

11. Sketch the graph of (2)

$$y = \tan x^\circ,$$

for $0 \leqslant x \leqslant 360$.



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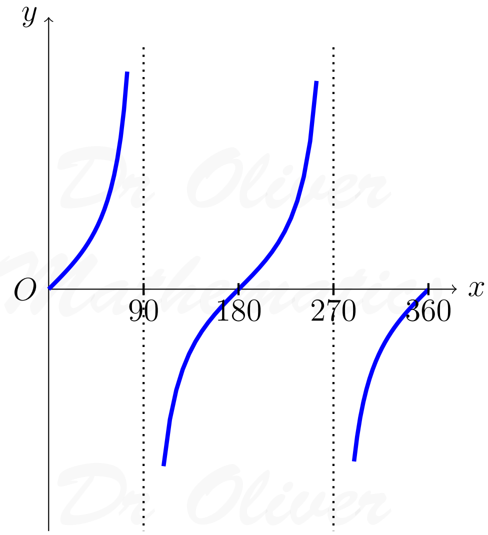
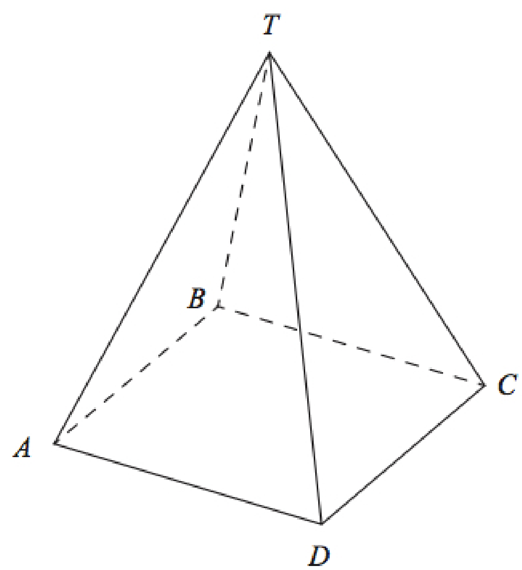
Solution



12. Here is a pyramid with a square base $ABCD$. (4)



$AB = 5$ m.

The vertex T is 12 m vertically above the midpoint of AC.

Calculate the size of angle TAC.

Solution

Let E be the midpoint of AB. Then $AE = 2.5$ m. Let F be right in the centre of $ABCD$. Then $FT = 12$ m. Now,

$$\begin{aligned} AT^2 = AE^2 + FT^2 &\Rightarrow AT^2 = 2.5^2 + 12^2 \\ &\Rightarrow AT^2 = 150.25 \\ &\Rightarrow AT = \sqrt{150.25} \end{aligned}$$

and

$$\begin{aligned} AF^2 = AE^2 + AE^2 &\Rightarrow AF^2 = 2 \times 2.5^2 \\ &\Rightarrow AF^2 = 12.5 \\ &\Rightarrow AF = \sqrt{12.5}. \end{aligned}$$

Finally,

$$\begin{aligned} \tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan TAC = \frac{\sqrt{150.25}}{\sqrt{12.5}} \\ &\Rightarrow \angle TAC = 73.910\,594\,15 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\angle TAC = 73.9^\circ \text{ (3 sf)}}}. \end{aligned}$$

13. The number of animals in a population at the start of year t is P_t. (2)
The number of animals at the start of year 1 is 400.

Given that

$$P_{t+1} = 1.01 P_t,$$

work out the number of animals at the start of year 3.

Solution

$$\begin{aligned} P_2 &= 1.01 \times P_1 = 1.01 \times 400 = 404 \\ P_3 &= 1.01 \times P_2 = 1.01 \times 404 = 408.04; \end{aligned}$$

hence, there are $\underline{\underline{408}}$ animals.

14. y is inversely proportional to x^3. (3)

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$y = 44$ when $x = a$.

Show that $y = 5.5$ when $x = 2a$.

Solution

$$y \propto \frac{1}{x^3} \Rightarrow y = \frac{k}{x^3},$$

for some constant k. Now,

$$44 = \frac{k}{a^3} \Rightarrow k = 44a^3,$$

and

$$y = \frac{44a^3}{x^3}.$$

Next,

$$\begin{aligned} x = 2a \Rightarrow y &= \frac{44a^3}{(2a)^3} \\ \Rightarrow y &= \frac{44a^3}{8a^3} \\ \Rightarrow y &= \frac{44\cancel{a^3}}{8\cancel{a^3}} \\ \Rightarrow \underline{\underline{y}} &\underline{\underline{= 5.5}}, \end{aligned}$$

as required.

15. Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8. (3)

Solution

Let $(2n - 1)$ and $(2n + 1)$ be two consecutive odd numbers.

	$2n$	$+1$
$2n$	$4n^2$	$+2n$
$+1$	$+2n$	$+1$

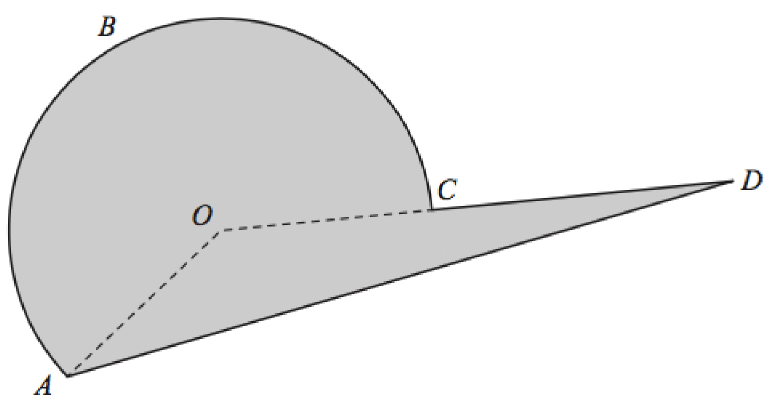
	$2n$	-1
$2n$	$4n^2$	$-2n$
-1	$-2n$	$+1$

Then

$$\begin{aligned}(2n+1)^2-(2n-1)^2 &= (4n^2+4n+1)-(4n^2-4n+1)\\ &= 8n^2+8n\\ &= 8\times n(n+1);\end{aligned}$$

hence, we have proved that the difference between the squares of any two consecutive odd numbers is always a multiple of 8.

16. Here is a shaded shape $ABCD$. (5)



The shape is made from a triangle and a sector of a circle, centre O, and radius 6 cm.
OCD is a straight line.
$AD = 14$ cm.
Angle $AOD = 140°$.
Angle $OAD = 24°$.

Calculate the perimeter of the shape.
Give your answer correct to 3 significant figures.

Solution

Well,

$$\angle ODA = 180 - (140 + 24) = 16^\circ$$

and we apply the sine rule:

$$\begin{aligned}\frac{OD}{\sin 24^\circ} = \frac{14}{\sin 140^\circ} &\Rightarrow OD = \frac{14 \sin 24^\circ}{\sin 140^\circ}\\ &\Rightarrow CD = \frac{14 \sin 24^\circ}{\sin 140^\circ} - 6.\end{aligned}$$

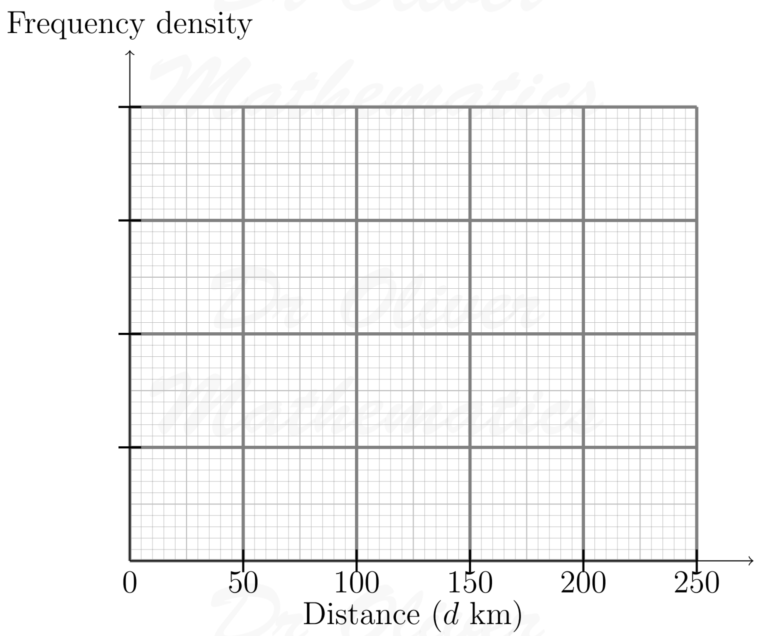
Finally,

$$\begin{aligned}\text{perimeter} &= \text{circular part} + CD + AD\\ &= \left(\tfrac{(360-140)}{360} \times 2 \times \pi \times 6\right) + \left(\frac{14 \sin 24^\circ}{\sin 140^\circ} - 6\right) + 14\\ &= 23.008\,346\,13 + 2.858\,778\,416 + 14 \text{ (FCD)}\\ &= 39.897\,124\,54 \text{ (FCD)}\\ &= \underline{\underline{39.9 \text{ cm (3 sf)}}}.\end{aligned}$$

17. The table shows information about the distances 570 students travelled to a university open day.

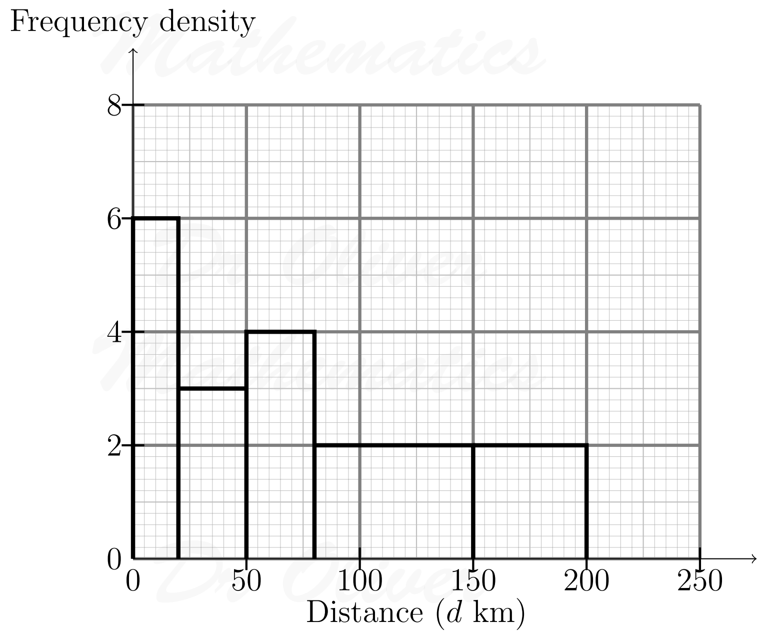
Distance (d km)	Frequency
$0 < d \leqslant 20$	120
$20 < d \leqslant 50$	90
$50 < d \leqslant 80$	120
$80 < d \leqslant 150$	140
$150 < d \leqslant 200$	100

(a) Draw a histogram for the information in the table. (3)



Solution

Distance (d km)	Frequency	Width	Frequency Density
$0 < d \leqslant 20$	120	20	$\frac{120}{20} = 6$
$20 < d \leqslant 50$	90	30	$\frac{90}{30} = 3$
$50 < d \leqslant 80$	120	30	$\frac{120}{30} = 4$
$80 < d \leqslant 150$	140	70	$\frac{140}{70} = 2$
$150 < d \leqslant 200$	100	50	$\frac{100}{50} = 2$



(b) Estimate the median distance. (2)

Solution

Distance (d km)	Frequency	Cumulative Frequency
$0 < d \leqslant 20$	120	120
$20 < d \leqslant 50$	90	210
$50 < d \leqslant 80$	120	330
$80 < d \leqslant 150$	140	470
$150 < d \leqslant 200$	100	570

So, since the median is in

$$\tfrac{1}{2}(570 + 1) = 285.5\text{th}$$

place, it is the 75.5th member of the $50 < d \leqslant 80$ group. Hence,

$$\begin{aligned} \text{median} &= 50 + \frac{75.5}{120} \times 30 \\ &= \underline{\underline{68.875}}. \end{aligned}$$

18. A high speed train travels a distance of 487 km in 3 hours. (5)

The distance is measured correct to the nearest kilometre.
The time is measured correct to the nearest minute.

By considering bounds, work out the average speed, in km/minute, of the train to a suitable degree of accuracy.
You must show all your working and give a reason for your answer.

Solution

Well,

$$486.5 \leqslant \text{distance} < 487.5$$

and we convert hours to minutes:

$$179.5 \leqslant \text{time} < 180.5.$$

Now,

$$\begin{aligned} & \frac{\text{LB distance}}{\text{UB time}} < \text{speed} < \frac{\text{UB distance}}{\text{LB time}} \\ \Rightarrow\ & \frac{486.5}{180.5} < \text{speed} < \frac{487.5}{179.5} \\ \Rightarrow\ & 2.695\,290\,859 < \text{speed} < 2.715\,877\,437 \text{ (FCD)}. \end{aligned}$$

Number	Lower Bound	Upper Bound	Agree?
Nearest Natural Number	3	3	✓
1 dp	2.7	2.7	✓
2 dp	2.70	2.72	✗

Hence, as the lower and upper bounds agree to 1, but not 2, decimal places,

$$\text{average speed} = \underline{\underline{2.7 \text{ km/minute (1 dp)}}}.$$

19. Solve algebraically the simultaneous equations: (5)

$$\begin{aligned} 2x^2 - y^2 &= 17 \\ x + 2y &= 1. \end{aligned}$$

Solution

$$x + 2y = 1 \Rightarrow x = 1 - 2y.$$

Now,

$$2x^2 - y^2 = 17 \Rightarrow 2[(1-2y)^2] - y^2 = 17$$

	1	$-2y$
1	1	$-2y$
$-2y$	$-2y$	$+4y^2$

$$\begin{aligned} &\Rightarrow 2(1 - 4y + 4y^2) - y^2 = 17 \\ &\Rightarrow 2 - 8y + 7y^2 = 17 \\ &\Rightarrow 7y^2 - 8y - 15 = 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & -8 \\ \text{multiply to:} & +(7) \times (-15) = -105 \end{array}\right\} -15,\ +7$$

$$\begin{aligned} &\Rightarrow 7y^2 - 15y + 7y - 15 = 0 \\ &\Rightarrow y(7y - 15) + 1(7y - 15) = 0 \\ &\Rightarrow (y+1)(7y-15) = 0 \\ &\Rightarrow y + 1 = 0 \text{ or } 7y - 15 = 0 \\ &\Rightarrow y = -1 \text{ or } y = \tfrac{15}{7} \\ &\Rightarrow x = 3 \text{ or } x = -\tfrac{23}{7}; \end{aligned}$$

hence,

$$\underline{\underline{x = 3, y = -1 \text{ or } x = -\tfrac{23}{7}, y = \tfrac{15}{7}}}.$$

20. Triangle **A** is transformed by the combined transformation of a rotation of 180° about the point $(-2, 0)$ followed by a translation with vector (2)

$$\begin{pmatrix} -3 \\ 2 \end{pmatrix}.$$

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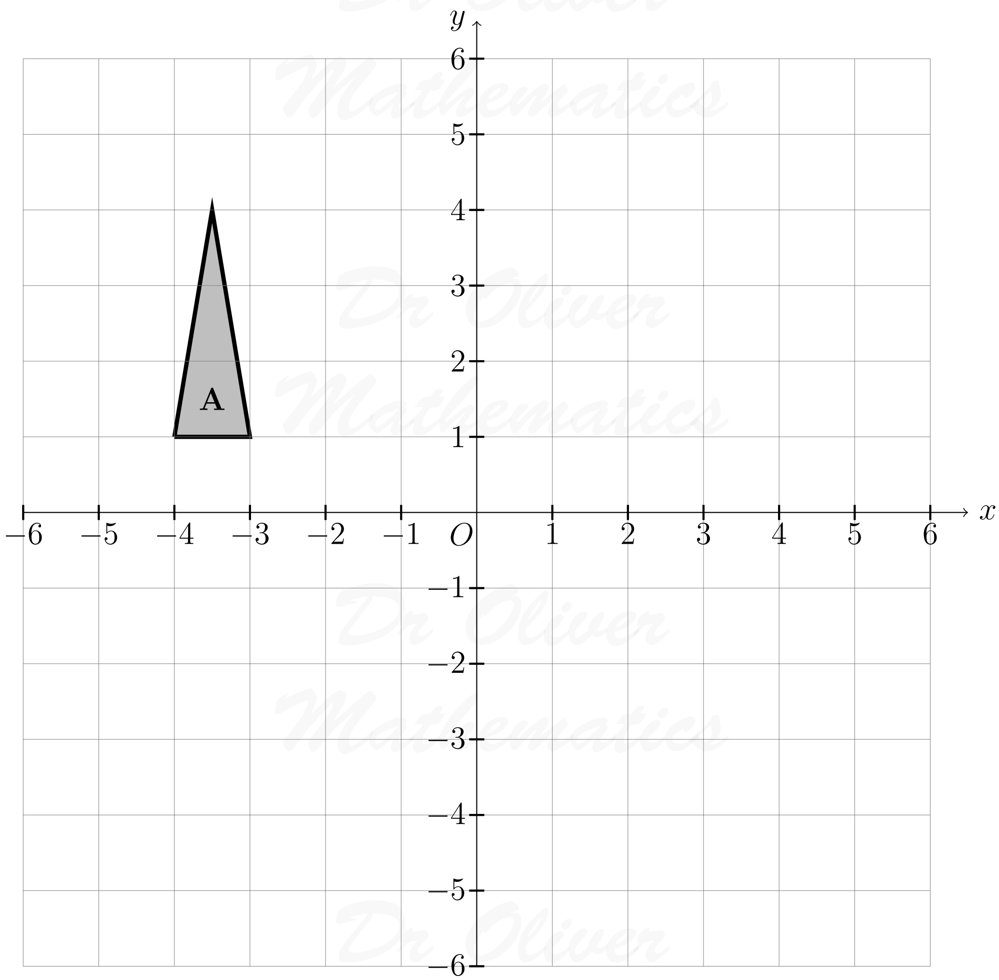
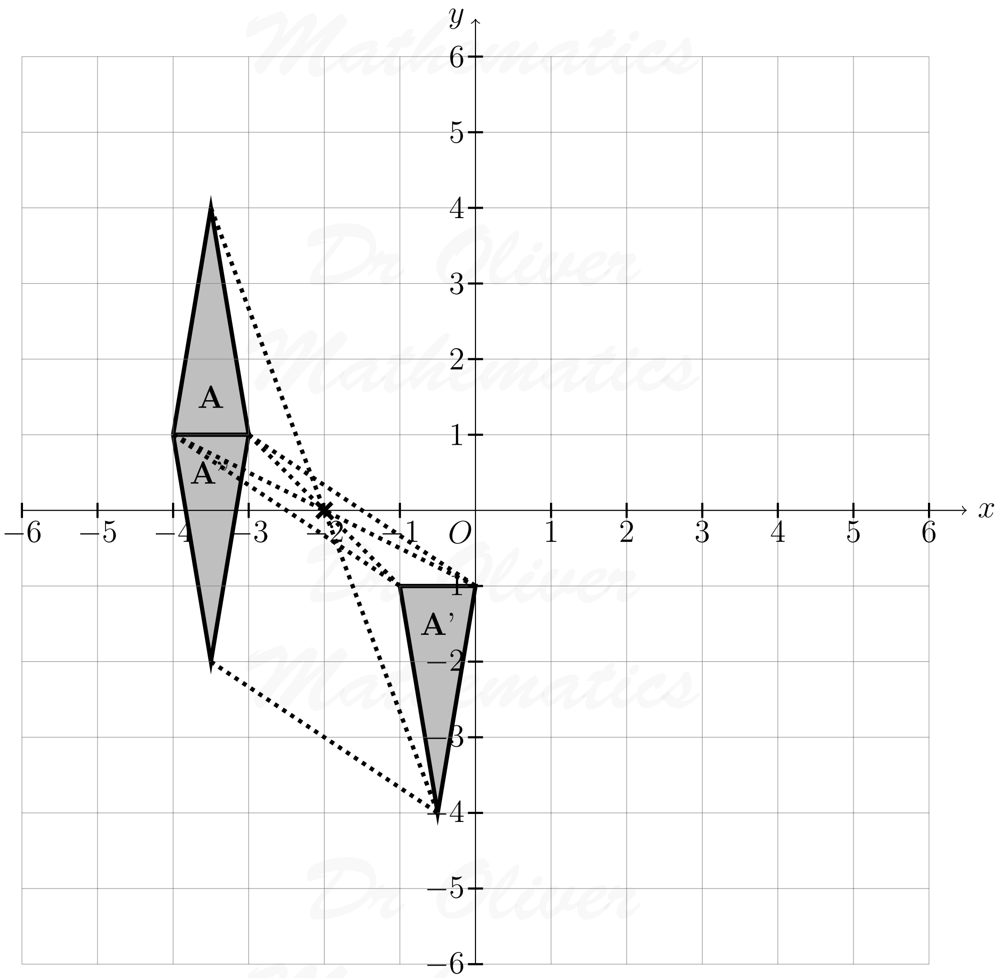


One point on triangle **A** is invariant under the combined transformation.
Find the coordinates of this point.

Solution



It is the midpoint of the base: $\underline{\underline{(-3\frac{1}{2}, 1)}}$.

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