Dr Oliver Mathematics
Mathematics
Binomial Series
Past Examination Questions

This booklet consists of 23 questions across a variety of examination topics.
The total number of marks available is 184.

For $n \in \mathbb{R}$,

$$\begin{aligned}(a+bx)^n &= \left[a(1+\tfrac{b}{a}x)\right]^n \\ &= a^n\left[1+\tfrac{b}{a}x\right]^n \\ &= a^n\left[1+n(\tfrac{b}{a}x)+\frac{n(n-1)}{2!}(\tfrac{b}{a}x)^2+\frac{n(n-1)(n-2)}{3!}(\tfrac{b}{a}x)^3+\ldots\right]\end{aligned}$$

or

$$\boxed{(a+bx)^n = a^n + na^{n-1}bx + \frac{n(n-1)}{2!}a^{n-2}(bx)^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}(bx)^3 + \ldots}$$

provided $|x| < \dfrac{b}{a}$.

1. Use the binomial theorem to expand (5)

$$\sqrt{4-9x},\ |x| < \tfrac{4}{9},$$

in ascending powers of x, up to and including the term in x^3, simplifying each term.

2.

$$f(x) \equiv \frac{3x^2+16}{(1-3x)(2+x)^2} \equiv \frac{A}{1-3x} + \frac{B}{2+x} + \frac{C}{(2+x)^2},\ |x| < \tfrac{1}{3}.$$

(a) Find the values of A and C and show that $B = 0$. (4)

(b) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^3. Simplify each term. (7)

3.

$$f(x) = \frac{3x-1}{(1-2x)^2},\ |x| < \tfrac{1}{2}.$$

Given that, for $x \neq \frac{1}{2}$,

$$\frac{3x-1}{(1-2x)^2} \equiv \frac{A}{1-2x} + \frac{B}{(1-2x)^2},$$

where A and B are constants,

(a) find the values of A and B. (3)

(b) Hence, or otherwise, find the series expansion of $\mathrm{f}(x)$, in ascending powers of x, up to and including the term in x^3, simplifying each term. (6)

4. (5)

$$\mathrm{f}(x) = (2-5x)^{-2},\ |x| < \tfrac{2}{5}.$$

Find the binomial expansion of $\mathrm{f}(x)$, in ascending powers of x, up to and including the term in x^3, giving each coefficient as a simplified fraction.

5. (5)

$$\mathrm{f}(x) = (3+2x)^{-3},\ |x| < \tfrac{3}{2}.$$

Find the binomial expansion of $\mathrm{f}(x)$, in ascending powers of x, up to and including the term in x^3.

Give each coefficient as a simplified fraction.

6. (a) Use the binomial series to expand (5)

$$(8-3x)^{\frac{1}{3}},\ |x| < \tfrac{8}{3},$$

in ascending powers of x, up to and including the term in x^3, giving each terms as a simplified fraction.

(b) Use your expansion, with a suitable value of x to obtain an approximation to $\sqrt[3]{7.7}$. Give your answer to 7 decimal places. (2)

7. (a) Expand $\dfrac{1}{\sqrt{4-3x}}$, where $|x| < \frac{4}{3}$, in ascending powers of x, up to and including the term in x^2. Simplify each term. (5)

(b) Hence, or otherwise, find the first 3 term in the expansion of $\dfrac{x+8}{\sqrt{4-3x}}$ as a series in ascending powers of x. (4)

8.

$$\mathrm{f}(x) = \frac{27x^2+32x+16}{(3x+2)^2(1-x)},\ |x| < \tfrac{2}{3}.$$

Given that f(x can be expressed in the form

$$\mathrm{f}(x) = \frac{A}{3x+2} + \frac{B}{(3x+2)^2} + \frac{C}{1-x},$$

(a) find the values of B and C and show that $A = 0$. (4)

(b) Hence, or otherwise, find the series expansion of $\mathrm{f}(x)$, in ascending powers of x, up to and including the term in x^2. Simplify each term. (6)

(c) Find the percentage error made in using the series expansion in part (b) to estimate the value of $\mathrm{f}(0.2)$. Give your answer to 2 significant figures. (4)

9. (6)

$$\mathrm{f}(x) = \frac{1}{\sqrt{4+x}}.$$

Find the binomial expansion of $\mathrm{f}(x)$, in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

10. (a) Find the binomial expansion of (4)

$$(1-8x)^{\frac{1}{2}},\ |x| < \tfrac{1}{8},$$

in ascending powers of x, up to and including the term in x^3, simplifying each term.

(b) Find the exact value of $(1-8x)^{\frac{1}{2}}$ when $x = 0.01$. (2)

(c) Hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places. (3)

11.

$$\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}.$$

(a) Find the values of the constants A, B, and C. (4)

(b) Hence, or otherwise, expand $\dfrac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of x, up to and including the term in x^2. Give each coefficient as a simplified fraction. (7)

12. (a) Use the binomial theorem to expand (5)

$$(3-2x)^{-2},\ |x| < \tfrac{2}{3},$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

$$\mathrm{f}(x) = \frac{a+bx}{(3-2x)^2},\ |x| < \tfrac{2}{3},\ \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of $\mathrm{f}(x)$, in ascending powers of x, the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$. Find

(b) the value of a and the value of b, (5)

(c) the coefficient of x^3, giving your answer as a simplified fraction. (3)

13. (6)

$$\mathrm{f}(x) = \frac{1}{\sqrt{9+4x^2}},\ |x| < \tfrac{3}{2}.$$

Find the first three non-zero terms of the binomial expansion of $\mathrm{f}(x)$, in ascending powers of x. Give each coefficient as a simplified fraction.

14. (a) Expand (5)

$$\frac{1}{(2-5x)^2},\ |x| < \tfrac{2}{5}$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

Given that the binomial expansion of $\dfrac{2+kx}{(2-5x)^2}$, $|x| < \tfrac{2}{5}$ is

$$\tfrac{1}{2} + \tfrac{7}{4}x + Ax^2 + \ldots$$

(b) find the value of the constant k, (2)

(c) find the value of the constant A. (2)

15.

$$\mathrm{f}(x) = \frac{6}{\sqrt{9-4x}},\ |x| < \tfrac{9}{4}.$$

(a) Find the binomial expansion of $\mathrm{f}(x)$, in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction. (6)

Use your answer to part (a) to find the binomial expansion in ascending powers of x, up to and including the term in x^3, of

(b) $\mathrm{g}(x) = \dfrac{6}{\sqrt{9+4x}}$, $|x| < \tfrac{9}{4}$, (1)

(c) $\mathrm{h}(x) = \dfrac{6}{\sqrt{9-8x}}$, $|x| < \tfrac{9}{8}$. (2)

16. Given (5)

$$\mathrm{f}(x) = (2+3x)^{-3},\ |x| < \tfrac{2}{3},$$

find the binomial expansion of $\mathrm{f}(x)$, in ascending powers of x, up to and including the term in x^3.

Give each coefficient as a simplified fraction.

17. (a) Use the binomial expansion to show that (6)

$$\sqrt{\frac{1+x}{1-x}} \approx 1 + x + \tfrac{1}{2}x^2,\ |x| < 1.$$

(b) Substitute $x = \frac{1}{26}$ into (3)

$$\sqrt{\frac{1+x}{1-x}} = 1 + x + \tfrac{1}{2}x^2$$

to obtain an approximation to $\sqrt{3}$.
Give your answer in the form $\frac{a}{b}$ where a and b are integers.

18. (a) Find the binomial expansion of (6)

$$\sqrt[3]{8-9x}, \ |x| < \tfrac{8}{9},$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(b) Use your expansion to estimate an approximate value for $\sqrt[3]{7100}$, giving your answer to 4 decimal places. State the value of x, which you use in your expansion, and show all your working. (3)

19. Given that the binomial expansion of $(1+kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \dots,$$

(a) find the value of the constant k, (2)

(b) find the value of the constant A, giving your answer in its simplest form. (3)

20. (a) Find the binomial expansion of (5)

$$\frac{1}{\sqrt{9-10x}}, \ |x| < \tfrac{9}{10}$$

in ascending powers of x, up to and including the term in x^2. Give each coefficient as a simplified fraction.

(b) Hence, or otherwise, find the expansion of (3)

$$\frac{3+x}{\sqrt{9-10x}}, \ |x| < \tfrac{9}{10}$$

in ascending powers of x, up to and including the term in x^2. Give each coefficient as a simplified fraction.

21. (a) Find the binomial series expansion of (5)

$$(4+5x)^{\frac{1}{2}}, |x| < \tfrac{4}{5},$$

in ascending powers of x, up to and including the term in x^2. Give each coefficient in its simplest form.

(b) Find the exact value of $(4+5x)^{\frac{1}{2}}$ when $x = \frac{1}{10}$. Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined. (1)

(c) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (a) and hence find an approximate value for $\sqrt{2}$. Give your answer in the form $\frac{p}{q}$ where p and q are integers. (2)

22. Use the binomial series to find the expansion of (6)

$$\frac{1}{(2+5x)^3},\ |x| < \tfrac{2}{5}$$

in ascending powers of x, up to and including the term in x^3.

Give each coefficient in its simplest form.

23.

$$f(x) = (2+kx)^{-3},\ |kx| < 2,\ \text{where } k \text{ is a positive constant.}$$

The binomial expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2 is

$$A + Bx + \tfrac{243}{16}x^2.$$

(a) Write down the value of A. (1)

(b) Find the value of k. (3)

(c) Write down the value of B. (2)