Dr Oliver Mathematics
Mathematics: Higher
2026 Paper 1: Non-Calculator
1 hour 15 minutes

The total number of marks available is 55.
You must write down all the stages in your working.

1. Express (3)
$$2x^2 + 20x + 3$$
in the form
$$p(x+q)^2 + r.$$

Solution

Well,
$$2x^2 + 20x + 3 = 2[x^2 + 10x] + 3$$
we complete the square:

$$\begin{array}{ll} \text{add to:} & +10 \\ \text{half it:} & +5 \\ \text{square it:} & (+5)^2 = +25 \end{array}$$

$$\begin{aligned} &= 2[(x^2 + 10x + 25) - 25] + 3 \\ &= 2[(x+5)^2 - 25] + 3 \\ &= 2(x+5)^2 - 50 + 3 \\ &= \underline{\underline{2(x+5)^2 - 47}}. \end{aligned}$$

2. Find (4)
$$\int \left(15x^{\frac{2}{3}} + \frac{7}{x^2}\right) \mathrm{d}x.$$

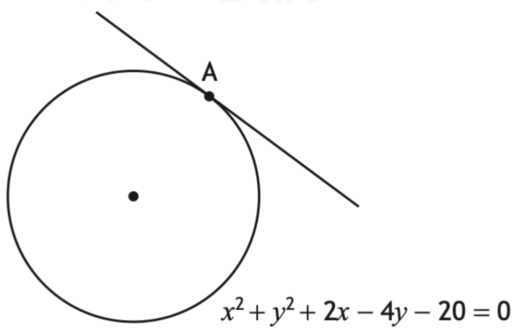
Solution

Now,

$$\int\left(15x^{\frac{2}{3}}+\frac{7}{x^2}\right)\,\mathrm{d}x=\int\left(15x^{\frac{2}{3}}+7x^{-2}\right)\,\mathrm{d}x$$
$$=15\times\tfrac{3}{5}x^{\frac{5}{3}}-7x^{-1}+c$$
$$=\underline{\underline{9x^{\frac{5}{3}}-7x^{-1}+c}}.$$

3. A circle has equation (4)

$$x^2+y^2+2x-4y-20=0.$$



Point $A(2,6)$ lies on the circle.

Find the equation of the tangent to the circle at A.

Solution

We complete the square twice:

add to:	$+2$
half it:	$+1$
square it:	$(+1)^2=+1$

and

add to:	-4
half it:	-2
square it:	$(-2)^2=+4$

Now,

$$\begin{aligned} x^2+y^2+2x-4y-20=0 &\Rightarrow x^2+2x+y^2-4y=20 \\ &\Rightarrow (x^2+2x+1)+(y^2-4y+4)=20+1+4 \\ &\Rightarrow (x+1)^2+(y-2)^2=25; \end{aligned}$$

so, the centre of $P(-1, 2)$ and the radius is $\sqrt{25}=5$.

Next,

$$\begin{aligned} m_{PA} &= \frac{6-2}{2-(-1)} \\ &= \tfrac{4}{3}, \end{aligned}$$

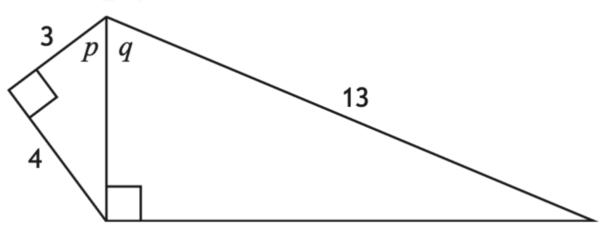
which means

$$m_{\text{tangent}} = -\tfrac{3}{4}.$$

Finally, the equation of the tangent to the circle at A is

$$\begin{aligned} y-6=-\tfrac{3}{4}(x-2) &\Rightarrow y-6=-\tfrac{3}{4}x+\tfrac{3}{2} \\ &\Rightarrow \underline{\underline{y=-\tfrac{3}{4}x+\tfrac{15}{2}}}. \end{aligned}$$

4. The diagram shows two right-angled triangles with angles p and q as marked.



(a) Determine the exact value of $\sin q$. (1)

Solution
Well, by the Pythagoras' theorem, the hypotenuse is 5. Now,

$$\begin{aligned} \cos = \frac{\text{adj}}{\text{hyp}} &\Rightarrow \cos q = \tfrac{5}{13} \\ &\Rightarrow \underline{\underline{\sin q = \tfrac{12}{13}}}, \end{aligned}$$

again, by the Pythagoras' theorem.

(b) Find the exact value of:

(i) $\sin(p+q)$, (4)

Solution

Well,

$$\sin p = \tfrac{4}{5} \text{ and } \cos p = \tfrac{3}{5}.$$

Now,

$$\begin{aligned}\sin(p+q) &= \sin p \cos q + \cos p \sin q \\ &= (\tfrac{4}{5})(\tfrac{5}{13}) + (\tfrac{12}{13})(\tfrac{3}{5}) \\ &= \tfrac{20}{65} + \tfrac{36}{65} \\ &= \underline{\underline{\tfrac{56}{65}}}.\end{aligned}$$

(ii) $\cos(p+q)$. (2)

Solution

Next,

$$\begin{aligned}\cos(p+q) &= \cos p \cos q - \sin p \sin q \\ &= (\tfrac{3}{5})(\tfrac{5}{13}) - (\tfrac{4}{13})(\tfrac{12}{13}) \\ &= \tfrac{15}{65} - \tfrac{48}{65} \\ &= \underline{\underline{-\tfrac{33}{65}}}.\end{aligned}$$

(c) Hence find the exact value of $\tan(p+q)$. (1)

Solution

Finally,

$$\begin{aligned}\tan(p+q) &= \frac{\sin(p+q)}{\cos(p+q)} \\ &= \frac{\frac{56}{65}}{-\frac{33}{65}} \\ &= \underline{\underline{-\tfrac{56}{33}}}.\end{aligned}$$

5. Functions f and g are defined on $\mathbb{R}$, the set of real numbers, by

$$\mathrm{f}(x) = 2x^2 + 5 \text{ and } \mathrm{g}(x) = x + 3.$$

(a) Find an expression for:

(i) $\mathrm{f}(\mathrm{g}(x))$ and (2)

Solution
Well,

$$\begin{aligned} \mathrm{f}(\mathrm{g}(x)) &= \mathrm{f}(x+3) \\ &= 2(x+3)^2 + 5 \end{aligned}$$

$\times$	x	$+3$
x	x^2	$+3x$
$+3$	$+3x$	$+9$

$$\begin{aligned} &= 2(x^2 + 6x + 9) + 5 \\ &= 2x^2 + 12x + 18 + 5 \\ &= \underline{\underline{2x^2 + 12x + 23}}. \end{aligned}$$

(ii) $\mathrm{g}(\mathrm{f}(x))$. (1)

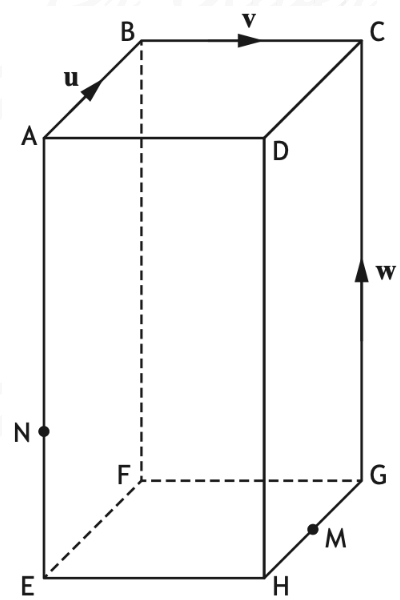
Solution
Now,

$$\begin{aligned} \mathrm{g}(\mathrm{f}(x)) &= \mathrm{g}(2x^2 + 5) \\ &= 2x^2 + 5 + 3 \\ &= \underline{\underline{2x^2 + 8}}. \end{aligned}$$

(b) State the range of $\mathrm{g}(\mathrm{f}(x))$. (1)

Solution
$\underline{\underline{\mathrm{g}(\mathrm{f}(x)) \geqslant 8}}$.

6. $ABCDEFGH$ is a cuboid. (2)



- $\overrightarrow{AB} = \mathbf{u}$.
- $\overrightarrow{BC} = \mathbf{v}$.
- $\overrightarrow{GC} = \mathbf{w}$.
- The point M is the mid-point of HG.
- The point N divides EA with the ratio $1:2$.

Express the vector $\overrightarrow{MN}$ in terms of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

Solution

Well,

$$\begin{aligned}\overrightarrow{MN} &= \overrightarrow{MH} + \overrightarrow{HE} + \overrightarrow{EN} \\ &= \tfrac{1}{2}\overrightarrow{GH} - \overrightarrow{EH} + \tfrac{1}{3}\overrightarrow{EA} \\ &= -\tfrac{1}{2}\overrightarrow{HG} - \overrightarrow{EH} + \tfrac{1}{3}\overrightarrow{EA} \\ &= \underline{\underline{-\tfrac{1}{2}\mathbf{u} - \mathbf{v} + \tfrac{1}{3}\mathbf{w}}}.\end{aligned}$$

7. Determine the gradient of the tangent to the curve with equation (4)

$$y = 2x + 4\sqrt{x},\ x > 0,$$

at the point where $x = 9$.

Solution

Well,

$$\begin{aligned} y = 2x + 4\sqrt{x} &\Rightarrow y = 2x + 4x^{\frac{1}{2}} \\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2 + 2x^{-\frac{1}{2}}. \end{aligned}$$

Now,

$$\begin{aligned} x = 9 &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2 + 2[9^{-\frac{1}{2}}] \\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2 + 2(\tfrac{1}{3}) \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{8}{3}}}. \end{aligned}$$

8. Solve (4)

$$\log_2 x + \log_2(x+2) = 3,$$

where $x > 0$.

Solution

Well,

$$\begin{aligned} \log_2 x + \log_2(x+2) = 3 &\Rightarrow \log_2[x(x+2)] = 3 \\ &\Rightarrow x(x+2) = 2^3 \\ &\Rightarrow x^2 + 2x = 8 \\ &\Rightarrow x^2 + 2x - 8 = 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & +2 \\ \text{multiply to:} & -8 \end{array}\right\} + 4,\ -2$$

$$\begin{aligned} &\Rightarrow (x+4)(x-2) = 0 \\ &\Rightarrow x + 4 = 0 \text{ or } x - 2 = 0 \\ &\Rightarrow x = -4 \text{ or } x = 2; \end{aligned}$$

but we know that $x > 0$ so the solution is $\underline{\underline{x = 2}}$.

9. (a) Show that the points $D(-1, 6, 1)$, $E(1, 2, 7)$, and $F(2, 0, 10)$ are collinear. (3)

Solution

Well,

$$\overrightarrow{DE} = \begin{pmatrix} 1-(-1) \\ 2-6 \\ 7-1 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$$

and

$$\overrightarrow{DF} = \begin{pmatrix} 2-(-1) \\ 0-6 \\ 10-1 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix} = \tfrac{3}{2}\begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} = \tfrac{3}{2}\overrightarrow{DE};$$

hence, the three points are collinear.

Point G is such that F divides DG in the ratio $3 : 4$.

(b) Find the coordinates of G. (2)

Solution

Well,

$$\begin{aligned}\overrightarrow{OG} &= \overrightarrow{OD} + \overrightarrow{DG} \\ &= \overrightarrow{OD} + \tfrac{7}{3}\overrightarrow{DF} \\ &= \begin{pmatrix} -1 \\ 6 \\ 1 \end{pmatrix} + \tfrac{7}{3}\begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix} \\ &= \begin{pmatrix} -1 \\ 6 \\ 1 \end{pmatrix} + \begin{pmatrix} 7 \\ -14 \\ 21 \end{pmatrix} \\ &= \begin{pmatrix} 6 \\ -8 \\ 22 \end{pmatrix};\end{aligned}$$

hence, $\underline{\underline{G(6, -8, 22)}}$.

10. Find (4)

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 6\sin(3x - \tfrac{1}{2}\pi)\,\mathrm{d}x.$$

Solution

Well,

$$\begin{aligned}\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 6\sin(3x - \tfrac{1}{2}\pi)\,\mathrm{d}x &= \left[-2\cos(3x - \tfrac{1}{2}\pi)\right]_{x=\frac{1}{6}\pi}^{\frac{1}{3}\pi} \\ &= 0 - (-2) \\ &= \underline{\underline{2}}.\end{aligned}$$

11. (a) (i) Show that $(x + 2)$ is a factor (2)

$$x^3 + 7x^2 + 18x + 16.$$

Solution
We use synthetic division:

$$\begin{array}{r|rrrr} -2 & 1 & 7 & 18 & 16 \\ & \downarrow & -2 & -10 & -16 \\ \hline & 1 & 5 & 8 & 0 \end{array}$$

As there is no remainder, $(x+2)$ is a factor of this cubic.

(ii) Explain why $(x+2)$ is the only linear factor of (2)

$$x^3 + 7x^2 + 18x + 16.$$

Give a reason for your answer.

Solution
Quadratic formula:

$$\begin{aligned} b^2 - 4ac &= 5^2 - 4 \times 1 \times 8 \\ &= 25 - 32 \\ &= -7 \\ &< 0; \end{aligned}$$

the curve $y = x^2 + 5x + 8$ does not cross the x-axis.

Hence, $(x+2)$ is the only linear factor of this cubic.

(b) Find the coordinates of the point(s) where the curves with equations (4)

$$y = 2x^3 + 20x^2 + 27x + 9 \text{ and } y = 6x^2 - 9x - 23$$

intersect.

Solution

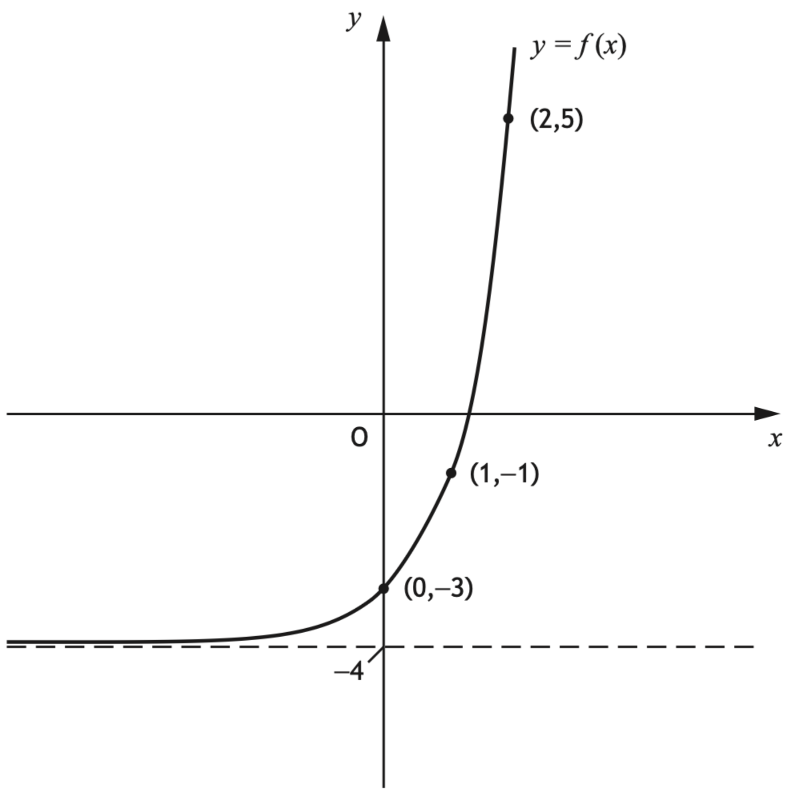
Well,

$$\begin{aligned}2x^3+20x^2+27x+9=6x^2-9x-23 &\Rightarrow 2x^3+14x^2+36x+32=0\\ &\Rightarrow 2(x^3+7x^2+18x+14)=0\\ &\Rightarrow 2(x+2)(x^2+5x+8)=0\\ &\Rightarrow x+2=0\\ &\Rightarrow x=-2\\ &\Rightarrow y=6[(-2)^2]-9(-2)-23\\ &\Rightarrow y=24+18-23\\ &\Rightarrow y=19;\end{aligned}$$

hence, the coordinates of the point are $\underline{\underline{(-2, 19)}}$.

12. An exponential function, f, is defined for $x \in \mathbb{R}$.

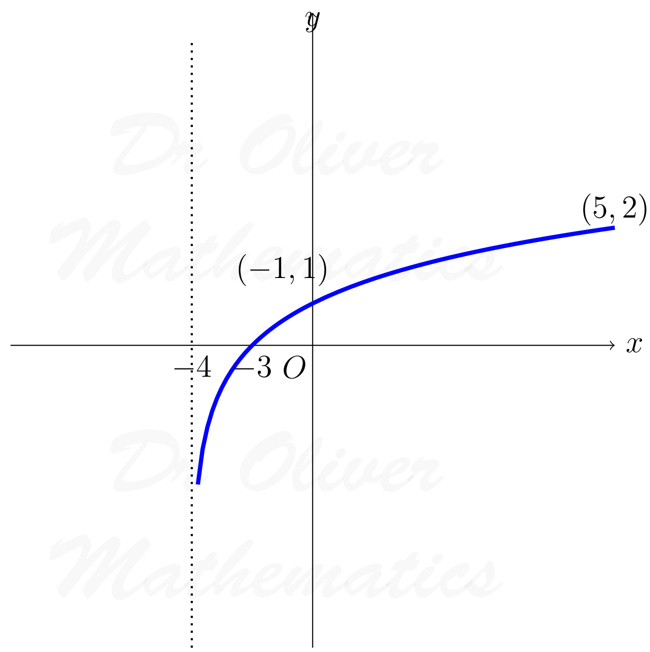
The diagram shows the graph of $y = \mathrm{f}(x)$.



The inverse function, $\mathrm{f}^{-1}(x)$, exists.

(a) On the diagram in your answer booklet, sketch the graph of the inverse function. (2)

Solution



The inverse function is of the form

$$f^{-1}(x) = \log_a(x + b).$$

(b) (i) Determine the values of a and b. (2)

Solution
Well,

$$\underline{\underline{b = 4}}$$

and

$$\begin{aligned} x = -1, f^{-1}(-1) = 1 &\Rightarrow 1 = \log_a(-1 + 4) \\ &\Rightarrow 1 = \log_a 3 \\ &\Rightarrow \underline{\underline{a = 3}}. \end{aligned}$$

(ii) State the domain of the inverse function, $f^{-1}(x)$. (1)

Solution
The domain is
$$\underline{\underline{x > -4}}.$$