

Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2010 June Paper 1 Variant 1: Calculator
2 hours

The total number of marks available is 80.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You must write down all the stages in your working.

1. Differentiate with respect to x :

(a) $\sqrt{1+x^3}$, (2)

(b) $x^2 \cos 2x$. (3)

2. (a) Find the first 3 terms of the expansion, in ascending powers of x , of (2)

$$(1+3x)^6.$$

(b) Hence find the coefficient of x^2 in the expansion of (3)

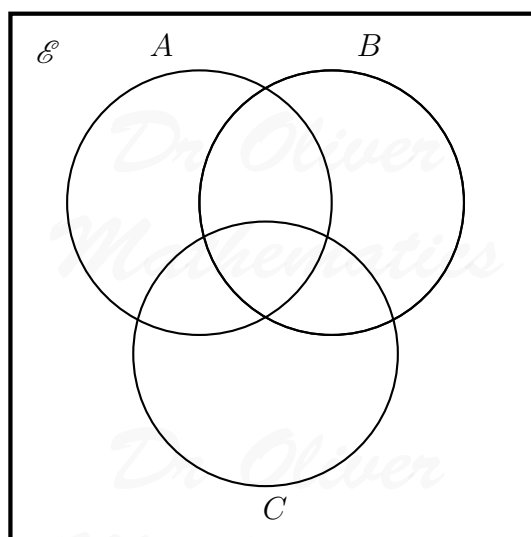
$$(1+3x)^6(1-3x-5x^2).$$

3. Find the set of values of k for which the equation (5)

$$x^2 + (k-2)x + (2k-4) = 0$$

has real roots.

4. We have a Venn diagram.



- (i) Copy the Venn diagram above and shade the region that represents $(A \cap B) \cup C$. (1)
 (ii) Copy the Venn diagram above and shade the region that represents $A' \cap B'$. (1)
 (iii) Copy the Venn diagram above and shade the region that represents $(A \cup B) \cap C$. (1)

(b) It is given that the

- universal set $\mathcal{E} = \{x : 2 \leq x \leq 20, x \text{ is an integer}\}$,
- $X = \{x : 4 < x < 15, x \text{ is an integer}\}$,
- $Y = \{x : x \geq 9, x \text{ is an integer}\}$, and
- $Z = \{x : x \text{ is a multiple of } 5\}$.

- (i) List the elements of $X \cap Y$. (1)
 (ii) List the elements of $X \cup Y$. (1)
 (iii) Find $(X \cup Y)' \cap Z$. (1)

5. Solve the equation

$$3x(x^2 + 6) = 8 - 17x^2. \quad (6)$$

6. Given that

$$\log_8 p = x \text{ and } \log_8 q = y,$$

express in terms of x and/or y :

- (a) $\log_8 \sqrt{p} + \log_8 q^2$, (2)
 (b) $\log_8 \left(\frac{q}{8}\right)$, (2)
 (c) $\log_2(64p)$. (3)

7. The function f is defined by

$$f(x) = (2x + 1)^2 - 3, \text{ for } x \geq -\frac{1}{2}.$$

Find

- (a) the range of f . (1)
 (b) an expression for $f^{-1}(x)$. (3)

The function g is defined by

$$g(x) = \frac{3}{1+x}, \text{ for } x > -1.$$

- (c) Find the value of x for which $fg(x) = 13$. (4)

8. (a) Solve the equation

$$(2^{3-4x})(4^{x+4}) = 2. \quad (3)$$

- (b) (i) Simplify (2)

$$\sqrt{108} - \frac{12}{\sqrt{3}},$$

giving your answer in the form $k\sqrt{3}$, where k is an integer.

- (ii) Simplify (3)

$$\frac{\sqrt{5} + 3}{\sqrt{5} - 2},$$

giving your answer in the form

$$a\sqrt{5} + b,$$

where a and b are integers.

9. (a) Variables x and y are related by the equation

$$y = 5x + 2 - 4e^{-x}.$$

- (i) Find $\frac{dy}{dx}$. (2)

- (ii) Hence find the approximate change in y when x increases from 0 to p , where p is small. (2)

- (b) A square of area A cm² has a side of length x cm. (4)

Given that the area is increasing at a constant rate of 0.5 cm²s⁻¹, find the rate of increase of x when $A = 9$.

10. Solve

(a) $4 \sin x = \cos x$ for $0^\circ < x < 360^\circ$, (3)

(b) $3 + \sin y = 3 \cos^2 y$ for $0^\circ < y < 360^\circ$, (5)

(c) $\sec\left(\frac{z}{3}\right) = 4$ for $0 < z < 5$ radians. (3)

EITHER

11. A curve has equation

$$y = \frac{\ln x}{x^2}, \text{ where } x > 0.$$

- (a) Find the exact coordinates of the stationary point of the curve. (6)

- (b) Show that $\frac{d^2y}{dx^2}$ can be written in the form (3)

$$\frac{a \ln x + b}{x^4},$$

where a and b are integers.

(c) Hence, or otherwise, determine the nature of the stationary point of the curve. (2)

OR

12. A curve is such that

$$\frac{dy}{dx} = 6 \cos\left(2x + \frac{1}{2}\pi\right) \text{ for } -\frac{1}{4}\pi \leq x \leq \frac{5}{4}\pi.$$

The curve passes through the point $\left(\frac{1}{4}\pi, 5\right)$.

Find

(a) the equation of the curve, (4)

(b) the x -coordinates of the stationary points of the curve, (3)

(c) the equation of the normal to the curve at the point on the curve where $x = \frac{3}{4}\pi$. (4)