Dr Oliver Mathematics
Mathematics
Probability
Past Examination Questions

This booklet consists of 37 questions across a variety of examination topics.
The total number of marks available is 371.

1. A car dealer offers purchasers a three-year warranty on a new car. He sells two models, the Zippy and the Nifty. For the first 50 cars sold of each model the number of claims under the warranty is shown in the table below.

	Claim	No claim
Zippy	35	15
Nifty	40	10

One of the purchasers is chosen at random. Let A be the event that no claim is made by the purchaser under the warranty and B the event that the car purchased is a Nifty.

(a) Find $\mathrm{P}(A \cap B)$. (2)

Solution
$\mathrm{P}(A \cap B) = \underline{\underline{0.1}}$.

(b) Find $\mathrm{P}(A')$ (2)

Solution
$\mathrm{P}(A) = 0.25 \Rightarrow \underline{\underline{\mathrm{P}(A') = 0.75}}$.

Given that the purchaser chosen does not make a claim under the warranty,

(c) find the probability that the car purchased is a Zippy. (2)

Solution
$\mathrm{P}(B'|A) = \dfrac{\mathrm{P}(B' \cap A)}{\mathrm{P}(A)} = \dfrac{0.15}{0.25} = \underline{\underline{0.6}}$.

(d) Show that making a claim is not independent of the make of the car purchased. Comment on this result. (3)

Solution

$P(A \cap B) = 0.1$.
$P(A)\,P(B) = 0.25 \times 0.5 = 0.125$.
Since $P(A \cap B) \neq P(A)\,P(B)$ we conclude that the events are <u>not independent</u> and that one of the models is <u>less reliable</u>.

2. A fair die has six faces numbered 1, 2, 2, 3, 3 and 3. The die is rolled twice and the number showing on the uppermost face is recorded each time. (5)

Find the probability that the sum of the two numbers recorded is at least 5.

Solution

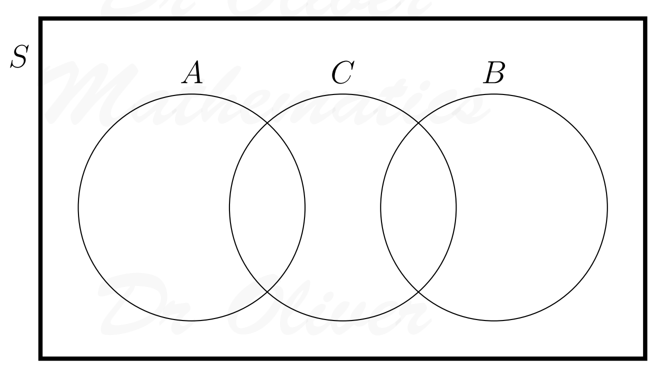
Of course, we can write out the possibility space or we can do

$$\begin{aligned} P(\text{at least } 5) &= P(5) + P(6) \\ &= 2 \times \tfrac{2}{6} \times \tfrac{3}{6} + \tfrac{3}{6} \times \tfrac{3}{6} \\ &= \tfrac{12}{36} + \tfrac{9}{36} \\ &= \tfrac{21}{36} \\ &= \underline{\underline{\tfrac{7}{12}}}. \end{aligned}$$

3. Three events A, B and C are defined in the sample space S. The events A and B are mutually exclusive and A and C are independent.

(a) Draw a Venn diagram to illustrate the relationships between the 3 events and the sample space. (3)

Solution



Given that $P(A) = 0.2$, $P(B) = 0.4$, and $P(A \cup C) = 0.7$, find

(b) $P(A|C)$, (2)

Solution

$P(A|C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{P(A)\,P(C)}{P(C)} = P(A) = \underline{\underline{0.2}}$.

(c) $P(A \cup B)$, (2)

Solution

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.2 + 0.4 - 0 \text{ (as they are mutually exclusive)} \\ &= \underline{\underline{0.6}}. \end{aligned}$$

(d) $P(C)$. (4)

Solution

$$\begin{aligned} & P(A \cap C) = P(A) + P(C) - P(A \cup C) \\ \Rightarrow\ & P(A)\,P(C) = P(A) + P(C) - P(A \cup C) \\ \Rightarrow\ & 0.2\,P(C) = 0.2 + P(C) - 0.7 \\ \Rightarrow\ & (1 - 0.2)\,P(C) = 0.5 \\ \Rightarrow\ & 0.8\,P(C) = 0.5 \\ \Rightarrow\ & \underline{\underline{P(C) = \tfrac{5}{8}}}. \end{aligned}$$

4. In a school there are 148 students in Years 12 and 13 studying Science, Humanities, or Arts subjects. Of these students, 89 wear glasses and the others do not. There are 30 Science students of whom 18 wear glasses. The corresponding figures for the Humanities students are 68 and 44 respectively.

A student is chosen at random.

Find the probability that this student

(a) s studying Arts subjects, (4)

Solution
Let P(S), P(H), and P(A) denote the probability that a particular does Science, Humanities, or Arts.

	Glasses	No glasses	Totals
Science	18		30
Humanities	44		68
Arts			
Totals	89		148

And this becomes

	Glasses	No glasses	Totals
Science	18	**12**	30
Humanities	44	**24**	68
Arts	**27**	**23**	**50**
Totals	89	**59**	148

and so

$$\mathrm{P}(A) = \tfrac{50}{148} = \underline{\underline{\tfrac{25}{74}}}.$$

(b) does not wear glasses, given that the student is studying Arts subjects. (2)

Solution
Let P(G) denote the probability that a particular does wear glasses.

$$\mathrm{P}(G'|A) = \frac{\frac{23}{148}}{\frac{50}{148}} = \underline{\underline{\tfrac{23}{50}}}.$$

Amongst the Science students, 80% are right-handed. Corresponding percentages for Humanities and Arts students are 75% and 70% respectively.

A student is again chosen at random.

(c) Find the probability that this student is right-handed. (3)

Solution

Let P(R) denote the probability that a particular is right-handed. Then

$$\begin{aligned}\mathrm{P}(R) &= (\tfrac{30}{148} \times 0.8) + (\tfrac{50}{148} \times 0.75) + (\tfrac{68}{148} \times 0.7) \\ &= \tfrac{24+51+35}{148} \\ &= \underline{\underline{\tfrac{55}{74}}}.\end{aligned}$$

(d) Given that this student is right-handed, find the probability that the student is studying Science subjects. (3)

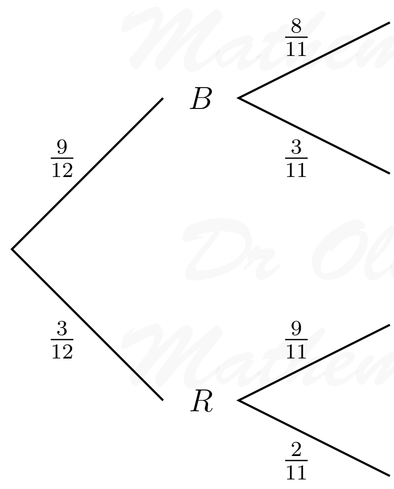
Solution

$$\begin{aligned}\mathrm{P}(S|R) &= \frac{\mathrm{P}(S \cap R)}{\mathrm{P}(R)} \\ &= \frac{\frac{12}{74}}{\frac{55}{74}} \\ &= \underline{\underline{\tfrac{12}{55}}}.\end{aligned}$$

5. A bag contains 9 blue balls and 3 red balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded.

(a) Draw a tree diagram to represent this information. (3)

Solution



$\mathrm{P}(BB) = \frac{9}{12} \times \frac{8}{11} = \frac{72}{132}$

$\mathrm{P}(BR) = \frac{3}{12} \times \frac{9}{11} = \frac{27}{132}$

$\mathrm{P}(RB) = \frac{3}{12} \times \frac{9}{11} = \frac{27}{132}$

$\mathrm{P}(RR) = \frac{3}{12} \times \frac{2}{11} = \frac{6}{132}$

Find the probability that

(b) the second ball selected is red, (2)

Solution

$\mathrm{P}(BR) + \mathrm{P}(RR) = \frac{27}{132} + \frac{6}{132} = \underline{\underline{\frac{1}{4}}}.$

(c) both balls selected are red, given that the second ball selected is red. (2)

Solution

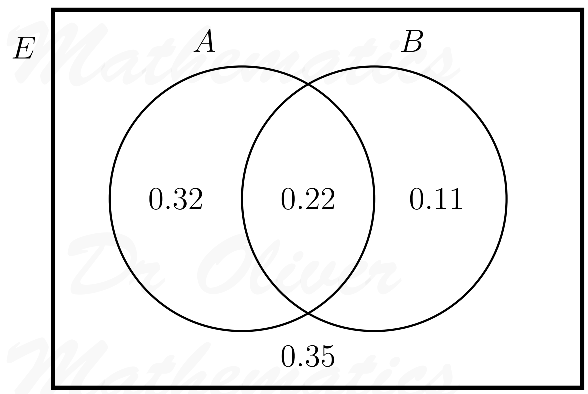
$\mathrm{P}(RR|\text{second one is } R) = \frac{\frac{6}{132}}{\frac{1}{4}} = \underline{\underline{\frac{2}{11}}}.$

6. For the events A and B,

$$\mathrm{P}(A \cap B') = 0.32,\ \mathrm{P}(A' \cap B) = 0.11, \text{ and } \mathrm{P}(A \cup B) = 0.65.$$

(a) Draw a Venn diagram to illustrate the complete sample space for the events A and B. (3)

Solution



(b) Write down the value of $\mathrm{P}(A)$ and the value of $\mathrm{P}(B)$. (3)

Solution

$\mathrm{P}(A) = \underline{\underline{0.54}}$ and $\mathrm{P}(B) = \underline{\underline{0.33}}$.

(c) Find $\mathrm{P}(A|B')$. (2)

Solution

$$\mathrm{P}(A|B') = \frac{\mathrm{P}(A \cap B')}{\mathrm{P}(B')} = \frac{0.32}{0.67} = \tfrac{32}{67}.$$

(d) Determine whether or not A and B are independent. (3)

Solution

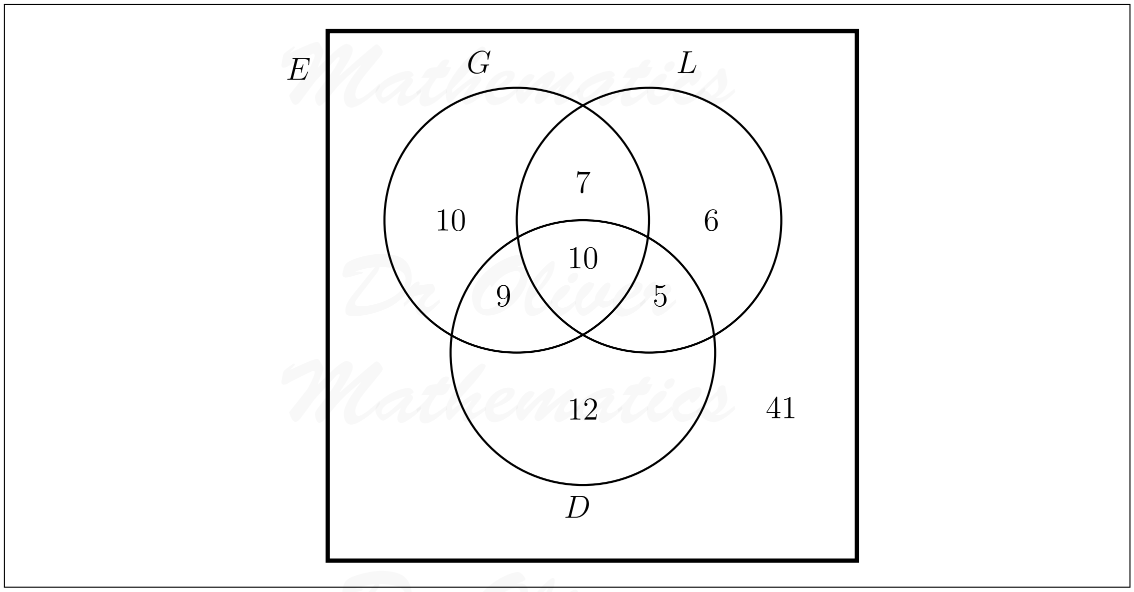
$\mathrm{P}(A \cap B) = 0.22$
$\mathrm{P}(A)\,\mathrm{P}(B) = 0.54 \times 0.33 = 0.1782$
So $\underline{\underline{\text{not independent}}}$.

7. A group of 100 people produced the following information relating to three attributes. The attributes were wearing glasses, being left-handed and having dark hair. Glasses were worn by 36 people, 28 were left-handed and 36 had dark hair. There were 17 who wore glasses and were left-handed, 19 who wore glasses and had dark hair, and 15 who were left handed and had dark hair. Only 10 people wore glasses, were left-handed and had dark hair.

(a) Represent these data on a Venn diagram. (6)

Solution

Let G, L, and D represent glasses, left-handed, and dark hair respectively.



A person was selected at random from this group.

Find the probability that this person

(b) wore glasses but was not left-handed and did not have dark hair, (1)

Solution

$\frac{10}{100} = \underline{\underline{\frac{1}{10}}}$.

(c) did not wear glasses, was not left-handed and did not have dark hair, (1)

Solution

$\underline{\underline{\frac{41}{100}}}$.

(d) had only two of the attributes, (2)

Solution

P(only two of the attributes) $= \frac{9+7+5}{100} = \underline{\underline{\frac{21}{100}}}$.

(e) wore glasses given that they were left-handed and had dark hair. (3)

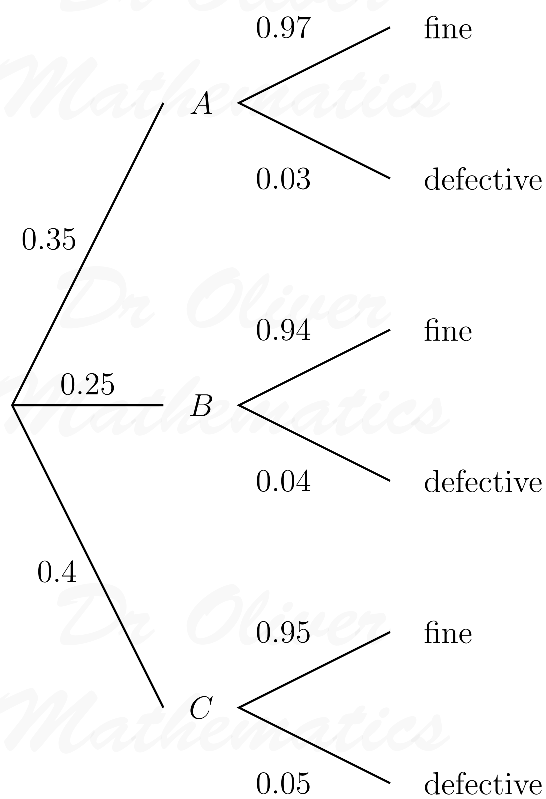
Solution

$$\text{P}(G|L \cap D) = \frac{\text{P}(G \cap L \cap D)}{\text{P}(L \cap D)} = \frac{\frac{10}{100}}{\frac{15}{100}} = \frac{2}{3}.$$

8. In a factory, machines A, B, and C are all producing metal rods of the same length. Machine A produces 35% of the rods, machine B produces 25%, and the rest are produced by machine C. Of their production of rods, machines A, B, and C produce 3%, 6%, and 5% defective rods respectively.

(a) Draw a tree diagram to represent this information. (3)

Solution



(b) Find the probability that a randomly selected rod is

(i) produced by machine A and is defective, (2)

Solution
$\text{P}(A \cap D) = 0.35 \times 0.03 = \underline{\underline{0.0105}}$

(ii) is defective. (3)

Solution

$$\begin{aligned}\mathrm{P}(\text{defective}) &= \mathrm{P}(A \cap D) + \mathrm{P}(B \cap D) + \mathrm{P}(C \cap D) \\ &= 0.35 \times 0.03 + 0.25 \times 0.06 + 0.4 \times 0.05 \\ &= 0.0105 + 0.015 + 0.02 \\ &= \underline{\underline{0.0455}}.\end{aligned}$$

(c) Given that a randomly selected rod is defective, find the probability that it was produced by machine C. (3)

Solution

$$\begin{aligned}\mathrm{P}(C|D) &= \frac{\mathrm{P}(C \cap D)}{\mathrm{P}(D)} \\ &= \frac{0.02}{0.0455} \\ &= \underline{\underline{\tfrac{40}{91}}}.\end{aligned}$$

9. A survey of the reading habits of some students revealed that, on a regular basis, 25% read quality newspapers, 45% read tabloid newspapers, and 40% do not read newspapers at all.

(a) Find the proportion of students who read both quality and tabloid newspapers. (3)

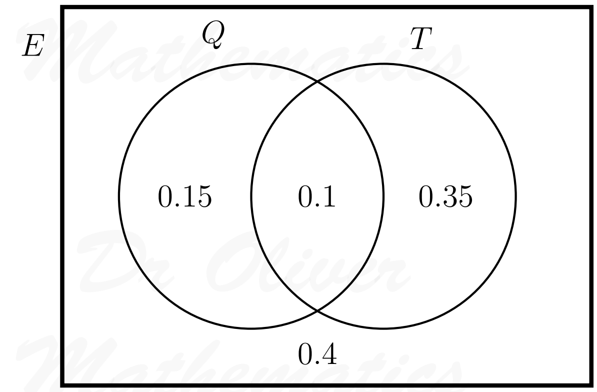
Solution

Let Q and T denote quality and tabloid newspapers respectively.

$$\begin{aligned}\mathrm{P}(Q \cap T) &= \mathrm{P}(Q) + \mathrm{P}(T) - \mathrm{P}(Q \cup T) \\ &= 0.25 + 0.45 - (1 - 0.4) \\ &= \underline{\underline{0.1}}.\end{aligned}$$

(b) Draw a Venn diagram to represent this information. (3)

Solution



A student is selected at random. Given that this student reads newspapers on a regular basis,

(c) find the probability that this student only reads quality newspapers. (3)

Solution

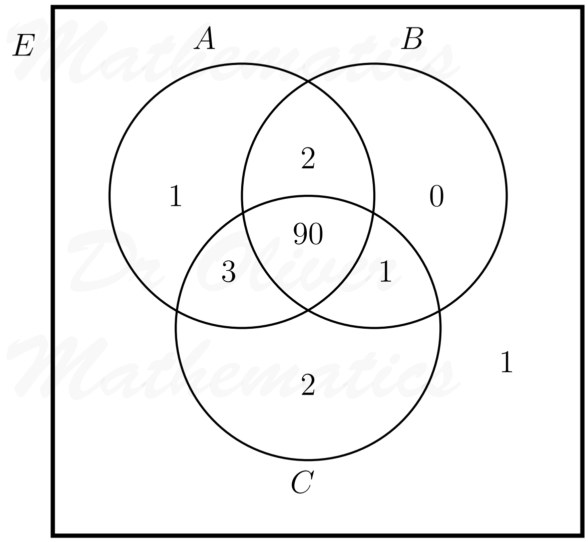
$$\begin{aligned} \mathrm{P}(Q|\text{reads a newspaper}) &= \frac{\mathrm{P}(Q \cup T')}{\mathrm{P}(\text{reads a newspaper})} \\ &= \frac{0.15}{0.6} \\ &= \underline{\underline{0.25}}. \end{aligned}$$

10. The following shows the results of a wine tasting survey of 100 people.
96 like wine A,
93 like wine B,
96 like wine C,
92 like A and B,
91 like B and C,
93 like A and C, and
90 like all three wines.

(a) Draw a Venn Diagram to represent these data. (6)

Solution

Let A, B, and C represent glasses, left-handed, and dark hair respectively.



Find the probability that a randomly selected person from the survey likes

(b) none of the three wines, (1)

Solution

$\underline{\underline{0.01}}$.

(c) wine A but not wine B, (2)

Solution

$\mathrm{P}(A \text{ but not } B) = \underline{\underline{0.04}}$.

(d) any wine in the survey except wine C, (2)

Solution

$\mathrm{P}(\text{except wine } C) = \underline{\underline{0.03}}$.

(e) exactly two of the three kinds of wine. (2)

Solution

$\mathrm{P}(\text{exactly two of the three kinds of wine}) = 0.03 + 0.02 + 0.01 = \underline{\underline{0.06}}$.

Given that a person from the survey likes wine A,

(f) find the probability that the person likes wine C. (3)

Solution

$$\begin{aligned} P(C|A) &= \frac{P(C \cap A)}{P(A)} \\ &= \frac{0.93}{0.96} \\ &= \underline{\underline{\tfrac{31}{32}}}. \end{aligned}$$

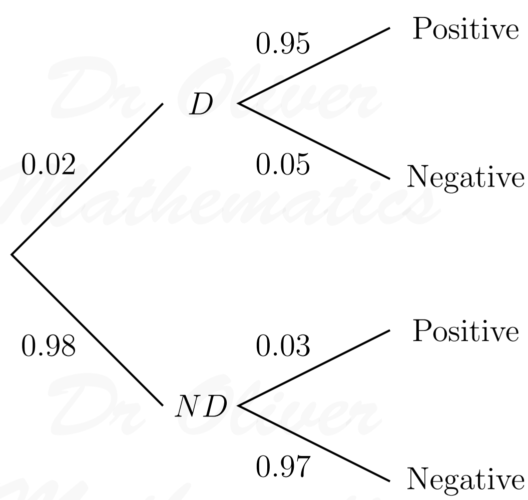
11. A disease is known to be present in 2% of a population. A test is developed to help determine whether or not someone has the disease.

Given that a person has the disease, the test is positive with probability 0.95.

Given that a person does not have the disease, the test is positive with probability 0.03.

(a) Draw a tree diagram to represent this information. (3)

Solution



A person is selected at random from the population and tested for this disease.

(b) Find the probability that the test is positive. (3)

Solution

$0.02 \times 0.95 + 0.98 \times 0.03 = 0.019 + 0.0294 = \underline{\underline{0.0484}}$.

A doctor randomly selects a person from the population and tests him for the disease. Given that the test is positive,

(c) find the probability that he does not have the disease. (2)

Solution

$$\begin{aligned} \mathrm{P}(ND|\text{positive}) &= \frac{0.0294}{0.0484} \\ &= \underline{\underline{\tfrac{147}{242}}}. \end{aligned}$$

(d) Comment on the usefulness of this test. (1)

Solution

There is a high probability of not having the disease for a person with a positive test so not useful at all.

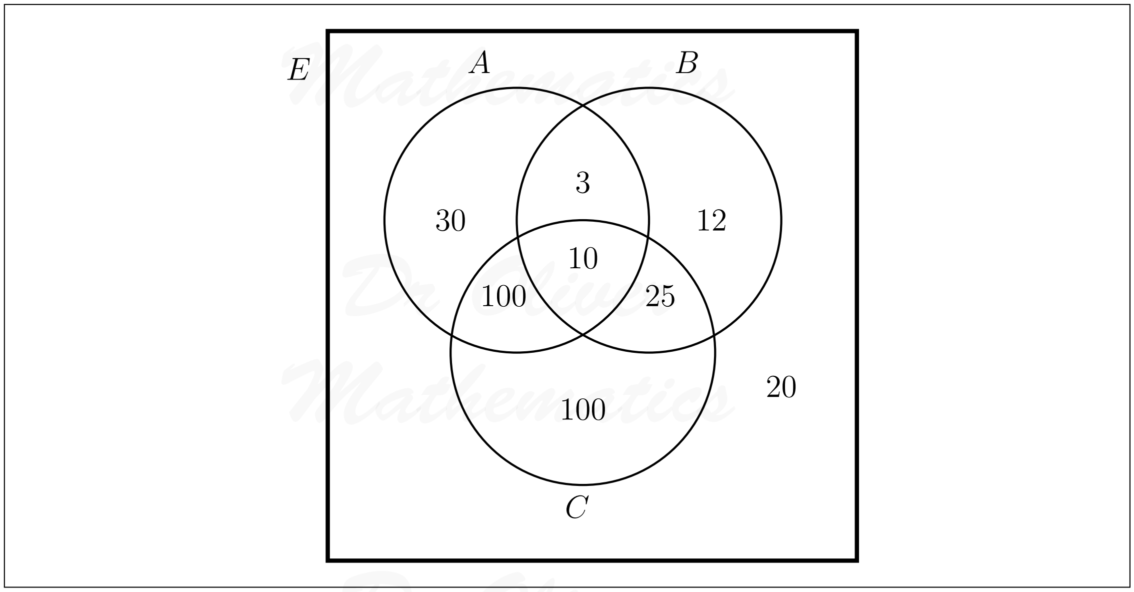
12. A person's blood group is determined by whether or not it contains any of 3 substances A, B, and C.

A doctor surveyed 300 patients' blood and produced the table below.

Blood contains	No. of patients
only C	100
A and C but not B	100
only A	30
B and C but not A	25
only B	12
A, B, and C	10
A and B but not C	3

(a) Draw a Venn diagram to represent this information. (4)

Solution



(b) Find the probability that a randomly chosen patient's blood contains substance C. (2)

Solution
$\mathrm{P}(C) = \frac{10+25+100+100}{300} = \frac{235}{300} = \underline{\underline{\frac{47}{60}}}$.

Harry is one of the patients. Given that his blood contains substance A,

(c) find the probability that his blood contains all 3 substances. (2)

Solution
$\mathrm{P}(\text{all three}|A) = \frac{\frac{10}{300}}{\frac{3+10+30+100}{300}} = \frac{\frac{10}{300}}{\frac{143}{300}} = \underline{\underline{\frac{10}{143}}}$.

Patients whose blood contains none of these substances are called universal blood donors.

(d) Find the probability that a randomly chosen patient is a universal blood donor. (2)

Solution
$\mathrm{P}(\text{universal blood donor}) = \frac{20}{300} = \underline{\underline{\frac{1}{15}}}$.

13. A group of office workers were questioned for a health magazine and $\frac{2}{5}$ were found to take regular exercise. When questioned about their eating habits $\frac{2}{3}$ said they always eat breakfast and, of those who always eat breakfast, $\frac{9}{25}$ also took regular exercise.

Find the probability that a randomly selected member of the group

(a) always eats breakfast and takes regular exercise, (2)

Solution

Let E and B be 'exercise' and 'breakfast' respectively. Then

$$\begin{aligned} \mathrm{P}(E \cap B) &= \mathrm{P}(E|B)\,\mathrm{P}(B) \\ &= \tfrac{9}{25} \times \tfrac{2}{3} \\ &= \underline{\underline{\tfrac{6}{25}}}. \end{aligned}$$

(b) does not always eat breakfast and does not take regular exercise. (4)

Solution

$$\begin{aligned} \mathrm{P}(E' \cap B') &= 1 - \mathrm{P}(E \cup B) \text{ (why?)} \\ &= 1 - [\mathrm{P}(E) + \mathrm{P}(B) - \mathrm{P}(E \cap B)] \\ &= 1 - \left[\tfrac{2}{5} + \tfrac{2}{3} - \tfrac{6}{25}\right] \\ &= \underline{\underline{\tfrac{13}{75}}}. \end{aligned}$$

(c) Determine, giving your reason, whether or not always eating breakfast and taking regular exercise are statistically independent. (2)

Solution

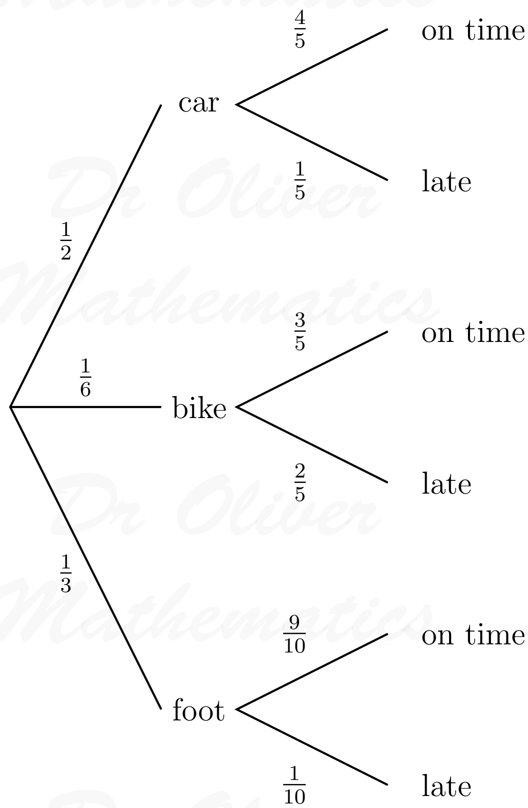
$$\begin{aligned} \mathrm{P}(E)\,\mathrm{P}(B) &= \tfrac{2}{5} \times \tfrac{2}{3} \\ &= \tfrac{4}{25} \\ &\neq \tfrac{6}{25} \\ &= \mathrm{P}(E \cap B); \end{aligned}$$

so, they are <u>not statistically independent</u>.

14. On a randomly chosen day the probability that Bill travels to school by car, by bicycle, or on foot is $\frac{1}{2}$, $\frac{1}{6}$, and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{1}{10}$ respectively.

(a) Draw a tree diagram to represent this information. (3)

Solution



(b) Find the probability that on a randomly chosen day

(i) Bill travels by foot and is late, (2)

Solution

$\frac{1}{3} \times \frac{1}{10} = \underline{\underline{\frac{1}{30}}}.$

(ii) Bill is not late. (2)

Solution

$$\begin{aligned} \text{P(not late)} &= \tfrac{1}{2} \times \tfrac{4}{5} + \tfrac{1}{6} \times \tfrac{3}{5} + \tfrac{1}{3} \times \tfrac{9}{10} \\ &= \tfrac{2}{5} + \tfrac{1}{10} + \tfrac{3}{10} \\ &= \underline{\underline{\tfrac{4}{5}}}. \end{aligned}$$

(c) Given that Bill is late, find the probability that he did not travel on foot. (4)

Solution

$$\begin{aligned}\mathrm{P}(F'|L) &= \frac{\mathrm{P}(F' \cap L)}{\mathrm{P}(L)} \\ &= \frac{\frac{1}{2} \times \frac{1}{5} + \frac{1}{6} \times \frac{2}{5}}{1 - \frac{4}{5}} \\ &= \frac{\frac{1}{6}}{\frac{1}{5}} \\ &= \underline{\underline{\tfrac{5}{6}}}.\end{aligned}$$

15. (a) Given that $\mathrm{P}(A) = a$ and $\mathrm{P}(B) = b$ express $\mathrm{P}(A \cup B)$ in terms of a and b when

(i) A and B are mutually exclusive, (1)

Solution
$\underline{\underline{a + b}}$.

(ii) A and B are independent. (1)

Solution
$\underline{\underline{a + b - ab}}$.

Two events R and Q are such that

$$\mathrm{P}(R \cap Q') = 0.15,\ \mathrm{P}(Q) = 0.35,\ \text{and}\ \mathrm{P}(R|Q) = 0.1.$$

Find the value of

(b) $\mathrm{P}(R \cup Q)$, (1)

Solution
$\mathrm{P}(R \cup Q) = \mathrm{P}(R \cap Q') + \mathrm{P}(Q) = 0.15 + 0.35 = \underline{\underline{0.5}}$.

(c) $\mathrm{P}(R \cap Q)$, (2)

Solution

$$
\begin{aligned}
P(R \cap Q) &= P(R|Q)\,P(Q) \\
&= 0.1 \times 0.35 \\
&= \underline{\underline{0.035}}.
\end{aligned}
$$

(d) P(R). (2)

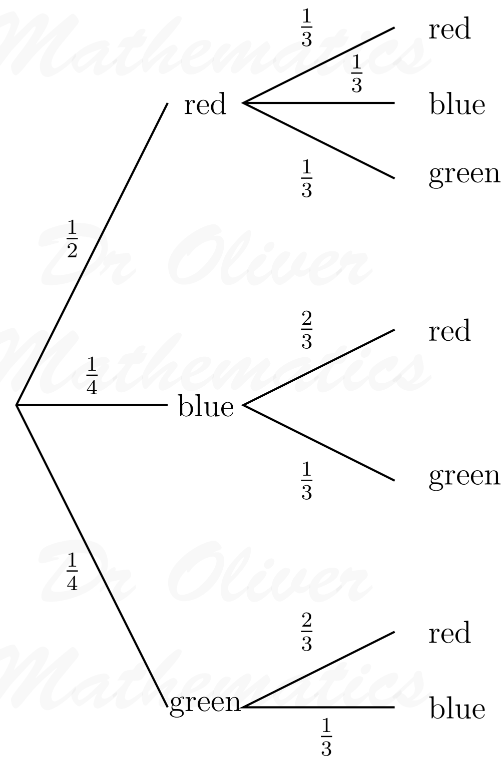
Solution

$$
\begin{aligned}
P(R) &= P(R \cap Q) + P(R \cap Q') \\
&= 0.035 + 0.15 \\
&= \underline{\underline{0.185}}.
\end{aligned}
$$

16. A jar contains 2 red, 1 blue, and 1 green bead. Two beads are drawn at random from the jar without replacement.

(a) Draw a tree diagram to illustrate all the possible outcomes and associated probabilities. State your probabilities clearly. (3)

Solution



(b) Find the probability that a blue bead and a green bead are drawn from the jar. (2)

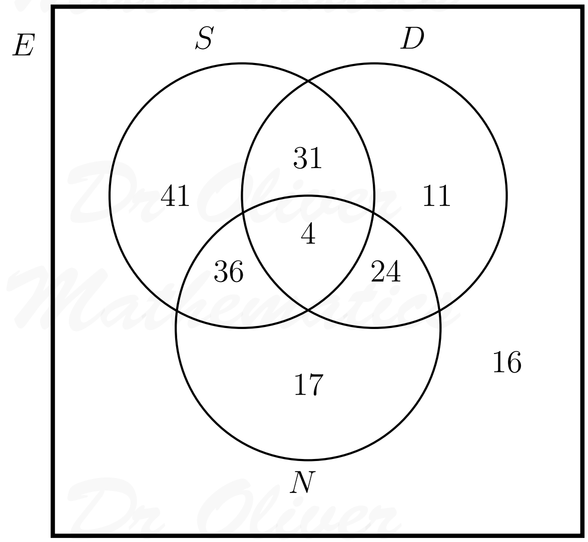
Solution

$$\begin{aligned} \text{P(blue and green)} &= 2 \times \tfrac{1}{4} \times \tfrac{1}{3} \\ &= \underline{\underline{\tfrac{1}{6}}}. \end{aligned}$$

17. There are 180 students at a college following a general course in computing. Students on this course can choose to take up to three extra options.
112 take systems support,
70 take developing software,
81 take networking,
35 take developing software and systems support,
28 take networking and developing software,
40 take systems support and networking,
4 take all three extra options.

(a) Draw a Venn diagram to represent this information. (5)

Solution



A student from the course is chosen at random.

Find the probability that this student takes

(b) none of the three extra options, (1)

Solution

$\frac{16}{180} = \underline{\underline{\frac{4}{45}}}$.

(c) networking only. (1)

Solution

$\underline{\underline{\frac{17}{180}}}$.

Students who want to become technicians take systems support and networking. Given that a randomly chosen student wants to become a technician,

(d) find the probability that this student takes all three extra options. (2)

Solution

$$\begin{aligned} P(S \cap D \cap N | S \cap N) &= \frac{P(S \cap D \cap N)}{P(S \cap N)} \\ &= \tfrac{4}{40} \\ &= \tfrac{1}{10} \end{aligned}$$

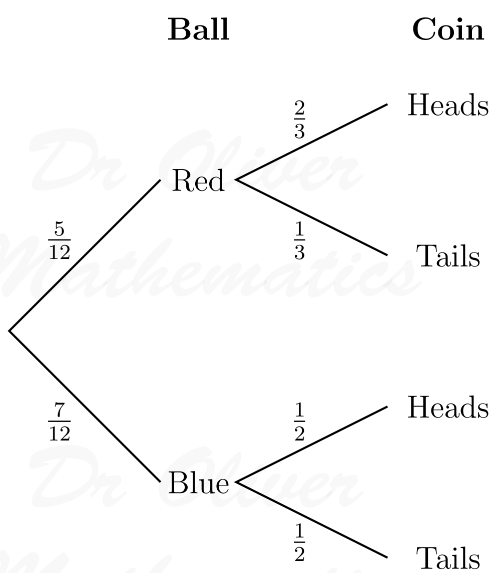
18. An experiment consists of selecting a ball from a bag and spinning a coin. The bag contains 5 red balls and 7 blue balls. A ball is selected at random from the bag, its colour is noted, and then the ball is returned to the bag.

When a red ball is selected, a biased coin with probability $\frac{2}{3}$ of landing heads is spun.

When a blue ball is selected a fair coin is spun.

(a) Draw a tree diagram below to show the possible outcomes and associated probabilities. (2)

Solution



Shivani selects a ball and spins the appropriate coin.

(b) Find the probability that she obtains a head. (2)

Solution

$\mathrm{P}(H) = \frac{5}{12} \times \frac{2}{3} + \frac{7}{12} \times \frac{1}{2} = \frac{10}{36} + \frac{7}{24} = \underline{\underline{\frac{41}{72}}}.$

Given that Tom selected a ball at random and obtained a head when he spun the appropriate coin,

(c) find the probability that Tom selected a red ball. (3)

Solution

$$\begin{aligned}\mathrm{P}(R|H) &= \frac{\mathrm{P}(R \cap H)}{\mathrm{P}(H)} \\ &= \frac{\frac{10}{36}}{\frac{41}{72}} \\ &= \underline{\underline{\frac{21}{41}}}.\end{aligned}$$

Shivani and Tom each repeat this experiment.

(d) Find the probability that the colour of the ball Shivani selects is the same as the colour of the ball Tom selects. (3)

Solution

$$\begin{aligned}\mathrm{P}(\text{both the same colour}) &= \mathrm{P}(RR) + \mathrm{P}(BB) \\ &= \frac{5}{12} \times \frac{5}{12} + \frac{7}{12} \times \frac{7}{12} \\ &= \frac{25}{144} \times \frac{49}{144} \\ &= \frac{74}{144} \\ &= \underline{\underline{\frac{37}{72}}}.\end{aligned}$$

19. The Venn diagram in Figure 1 shows the number of students in a class who read any of three popular magazines A, B, and C.

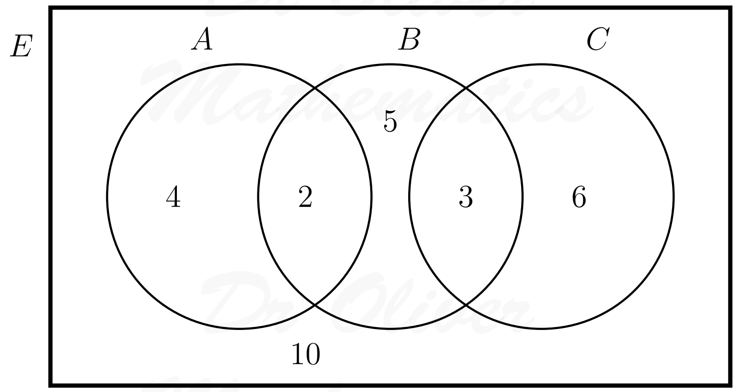


Figure 1: three popular magazines

One of these students is selected at random.

(a) Show that the probability that the student reads more than one magazine is $\frac{1}{6}$. (2)

Solution

P(more than one magazine) $= \frac{2+3}{4+2+5+3+6+10} = \frac{5}{30} = \underline{\underline{\frac{1}{6}}}$.

(b) Find the probability that the student reads A or B (or both). (2)

Solution

P(A or B or both) $= \frac{4+2+5+3}{30} = \frac{14}{30} = \underline{\underline{\frac{7}{15}}}$.

(c) Write down the probability that the student reads both A and C. (1)

Solution

$\underline{\underline{0}}$.

Given that the student reads at least one of the magazines,

(d) find the probability that the student reads C. (2)

Solution

$$\begin{aligned} \text{P}(C|\text{at least one magazine}) &= \frac{\text{P}(C)}{\text{P}(\text{at least one magazine})} \\ &= \frac{\frac{9}{30}}{\frac{20}{30}} \\ &= \underline{\underline{\tfrac{9}{20}}}. \end{aligned}$$

(e) Determine whether or not reading magazine B and reading magazine C are statistically independent. (3)

Solution

$\text{P}(B)\,\text{P}(C) = \frac{1}{3} \times \frac{3}{10} = \frac{1}{10}$

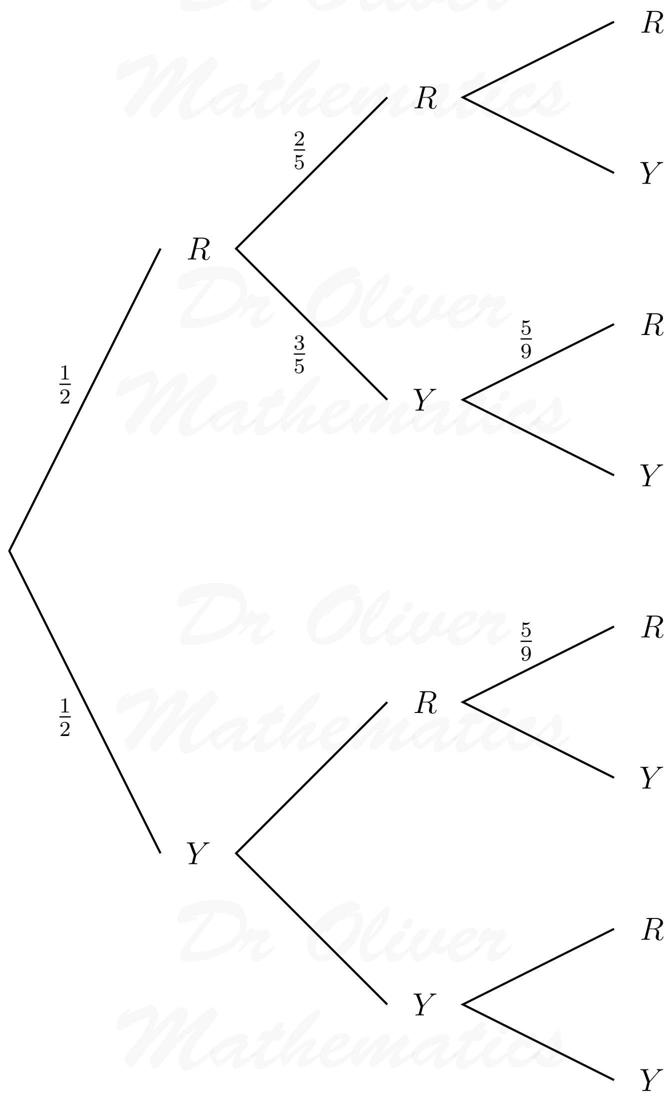
$\text{P}(B \cap C) = \frac{3}{30} = \frac{1}{10}$

Yes, they are <u>statistically independent</u>.

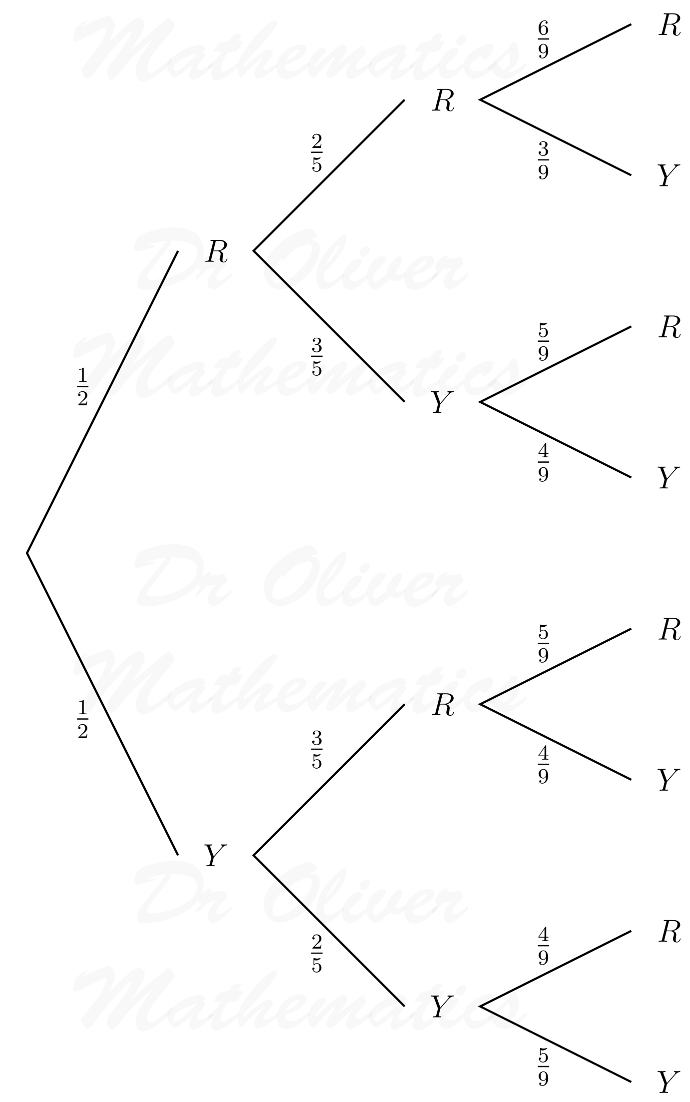
20. The bag P contains 6 balls of which 3 are red and 3 are yellow.
The bag Q contains 7 balls of which 4 are red and 3 are yellow.
A ball is drawn at random from bag P and placed in bag Q. A second ball is drawn at random from bag P and placed in bag Q.
A third ball is then drawn at random from the 9 balls in bag Q.

The event A occurs when the 2 balls drawn from bag P are of the same colour.
The event B occurs when the ball drawn from bag Q is red.

(a) Complete the tree diagram shown below. (4)


R
Y
R
Y
R
Y
R
Y
R
Y
R
Y
R
Y
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{2}{5}$
$\frac{3}{5}$
$\frac{5}{9}$
$\frac{5}{9}$

Solution



(b) P(A). (3)

Solution

$$\begin{aligned}\mathrm{P}(A) &= \mathrm{P}(RR) + \mathrm{P}(YY) \\ &= \tfrac{1}{2} \times \tfrac{2}{5} + \tfrac{1}{2} \times \tfrac{2}{5} \\ &= \tfrac{1}{5} + \tfrac{1}{5} \\ &= \underline{\underline{\tfrac{2}{5}}}.\end{aligned}$$

(c) Show that $\mathrm{P}(B) = \frac{5}{9}$. (3)

Solution

$$\begin{aligned}
\mathrm{P}(B) &= \mathrm{P}(RRR) + \mathrm{P}(RYR) + \mathrm{P}(YRR) + \mathrm{P}(YYR) \\
&= \tfrac{1}{2} \times \tfrac{2}{5} \times \tfrac{6}{9} + \tfrac{1}{2} \times \tfrac{3}{5} \times \tfrac{5}{9} + \tfrac{1}{2} \times \tfrac{3}{5} \times \tfrac{5}{9} + \tfrac{1}{2} \times \tfrac{2}{5} \times \tfrac{4}{9} \\
&= \tfrac{12}{90} + \tfrac{15}{90} + \tfrac{15}{90} + \tfrac{8}{90} \\
&= \tfrac{50}{90} \\
&= \underline{\underline{\tfrac{5}{9}}}.
\end{aligned}$$

(d) Show that $\mathrm{P}(A \cap B) = \frac{2}{9}$. (2)

Solution

$$\begin{aligned}
\mathrm{P}(A \cap B) &= \mathrm{P}(RRR) + \mathrm{P}(YYR) \\
&= \tfrac{12}{90} + \tfrac{8}{90} \\
&= \tfrac{20}{90} \\
&= \underline{\underline{\tfrac{2}{9}}}.
\end{aligned}$$

(e) Hence find $\mathrm{P}(A \cup B)$. (2)

Solution

$$\begin{aligned}
\mathrm{P}(A \cup B) &= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B) \\
&= \tfrac{2}{5} + \tfrac{5}{9} - \tfrac{2}{9} \\
&= \underline{\underline{\tfrac{11}{15}}}.
\end{aligned}$$

(f) Given that all three balls drawn are the same colour, find the probability that they are all red. (3)

Solution

Now,

$$\mathrm{P}(YYY) = \tfrac{1}{2} \times \tfrac{2}{5} \times \tfrac{5}{9} = \tfrac{1}{9}$$

and

$$\begin{aligned}\mathrm{P}(RRR|\text{all the same colour}) &= \frac{\mathrm{P}(RRR)}{\mathrm{P}(\text{all the same colour})}\\ &= \frac{\frac{12}{90}}{\frac{12}{90}+\frac{1}{9}}\\ &= \underline{\underline{\tfrac{6}{11}}}.\end{aligned}$$

21. Jake and Kamil are sometimes late for school.
The events J and K are defined as follows:

$$J = \text{the event that Jake is late for school}$$
$$K = \text{the event that Kamil is late for school.}$$

$\mathrm{P}(J) = 0.25$, $\mathrm{P}(J \cap K) = 0.15$, and $\mathrm{P}(J' \cap K') = 0.7$.

On a randomly selected day, find the probability that

(a) at least one of Jake or Kamil are late for school, (1)

Solution
$\mathrm{P}(J \cup K) = 1 - \mathrm{P}(J' \cap K') = 1 - 0.7 = \underline{\underline{0.3}}$.

(b) Kamil is late for school. (2)

Solution

$$\begin{aligned}& \mathrm{P}(J \cup K) = \mathrm{P}(J) + \mathrm{P}(K) - \mathrm{P}(J \cap K)\\ \Rightarrow\quad & 0.3 = 0.25 + \mathrm{P}(K) - 0.15\\ \Rightarrow\quad & \underline{\underline{\mathrm{P}(K) = 0.2}}.\end{aligned}$$

Given that Jake is late for school,

(c) find the probability that Kamil is late. (3)

Solution

$$\begin{aligned} P(K|J) &= \frac{P(K \cap J)}{P(J)} \\ &= \frac{0.15}{0.25} \\ &= \underline{\underline{\tfrac{3}{5}}}. \end{aligned}$$

The teacher suspects that Jake being late for school and Kamil being late for school are linked in some way.

(d) Determine whether or not J and K are statistically independent. (2)

Solution
$P(J \cap K) = 0.15$.
$P(J)\,P(K) = 0.25 \times 0.2 = 0.05$.
No, they are not statistically independent.

(e) Comment on the teacher's suspicion in the light of your calculation in (d). (1)

Solution
They are not statistically independent so the teacher's suspicion is confirmed.

22. A spinner is designed so that the score S is given by the following probability distribution.

s	0	1	2	4	5
$P(S = s)$	0.1	0.25	0.25	0.2	0.2

Tom and Jess play a game with this spinner. The spinner is spun repeatedly and S counters are awarded on the outcome of each spin. If S is even then Tom receives the counters and if S is odd then Jess receives them. The first player to collect 10 or more counters is the winner.

(a) Find the probability that Jess wins after 2 spins. (2)

Solution
$P(5, 5) = 0.2 \times 0.2 = \underline{\underline{0.04}}$.

(b) Find the probability that Tom wins after exactly 3 spins. (4)

Solution

$$\begin{aligned}\mathrm{P}(T \text{ wins after 3 spins}) &= \mathrm{P}(4,4,4) + \mathrm{P}(4,4,2) + \mathrm{P}(4,2,4) + \mathrm{P}(2,4,4) \\ &= (0.2)^3 + 3 \times (0.2)^2 \times (0.25) \\ &= 0.008 + 0.03 \\ &= \underline{\underline{0.038}}.\end{aligned}$$

(c) Find the probability that Jess wins after exactly 3 spins. (3)

Solution

$$\begin{aligned}\mathrm{P}(J \text{ wins after 3 spins}) &= \mathrm{P}(5,\bar{5},5) + \mathrm{P}(\bar{5},5,5) \\ &= 2 \times (0.2)^2 \times (0.8) \\ &= \underline{\underline{0.064}}.\end{aligned}$$

23. (a) State in words the relationship between two events R and S when $\mathrm{P}(R \cap S) = 0$. (1)

Solution

It means that R and S are mutually exclusive.

The events A and B are independent with $\mathrm{P}(A) = \frac{1}{4}$ and $\mathrm{P}(A \cup B) = \frac{2}{3}$.
Find

(b) $\mathrm{P}(B)$, (4)

Solution

$$\begin{aligned} & \mathrm{P}(A \cap B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cup B) \\ \Rightarrow\quad & \mathrm{P}(A)\,\mathrm{P}(B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cup B) \\ \Rightarrow\quad & \tfrac{1}{4}\,\mathrm{P}(B) = \tfrac{1}{4} + \mathrm{P}(B) - \tfrac{2}{3} \\ \Rightarrow\quad & \mathrm{P}(B)(1 - \tfrac{1}{4}) = \tfrac{5}{12} \\ \Rightarrow\quad & \tfrac{3}{4}\,\mathrm{P}(B) = \tfrac{5}{12} \\ \Rightarrow\quad & \mathrm{P}(B) = \underline{\underline{\tfrac{5}{9}}}.\end{aligned}$$

(c) $\mathrm{P}(A' \cap B)$, (2)

Solution

$\mathrm{P}(A' \cap B) = \mathrm{P}(A')\,\mathrm{P}(B) = \frac{3}{4} \times \frac{5}{9} = \underline{\underline{\frac{5}{12}}}$.

(d) $\mathrm{P}(B'|A)$. (2)

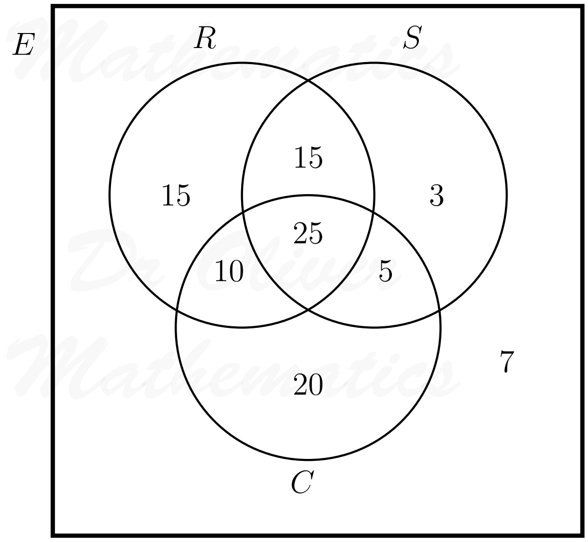
Solution

$$\begin{aligned}\mathrm{P}(B'|A) &= \frac{\mathrm{P}(B' \cap A)}{\mathrm{P}(A)} \\ &= \frac{\mathrm{P}(B')\,\mathrm{P}(A)}{\mathrm{P}(A)} \\ &= \mathrm{P}(B') \\ &= \underline{\underline{\tfrac{4}{9}}}.\end{aligned}$$

24. The following shows the results of a survey on the types of exercise taken by a group of 100 people:
65 run,
48 swim,
60 cycle,
40 run and swim,
30 swim and cycle,
35 run and cycle, and
25 do all three.

(a) Draw a Venn Diagram to represent these data. (4)

Solution



Find the probability that a randomly selected person from the survey

(b) takes none of these types of exercise, (2)

Solution

$\frac{7}{100}$.

(c) swims but does not run, (2)

Solution

$\text{P}(S \cap R') = \frac{3+5}{100} = \frac{8}{100}$.

(d) takes at least two of these types of exercise. (2)

Solution

$\text{P(at least two)} = \frac{15+25+5+10}{100} = \frac{55}{100} = \frac{11}{20}$.

Jason is one of the above group. Given that Jason runs,

(e) find the probability that he swims but does not cycle. (3)

Solution

$$\mathrm{P}(S \cap C' | R) = \frac{\mathrm{P}(S \cap C' \cap R)}{\mathrm{P}(R)} = \frac{\frac{15}{100}}{\frac{65}{100}} = \frac{15}{65} = \underline{\underline{\frac{3}{13}}}.$$

25. Figure 2 shows how 25 people travelled to work.

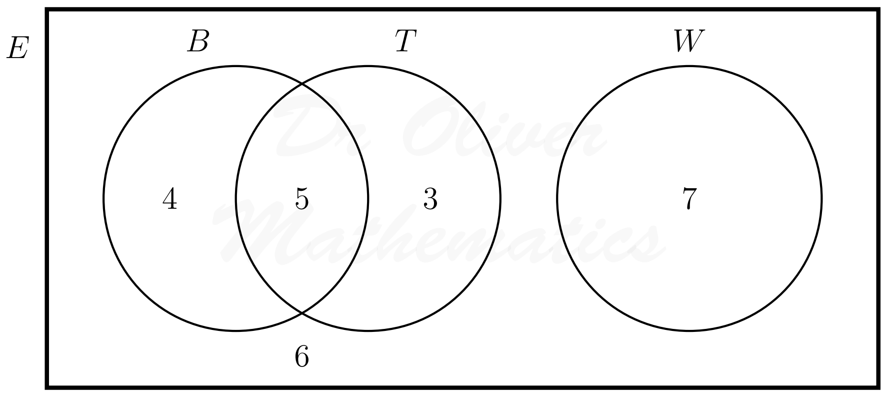


Figure 2: by bicycle, train, or walk

Their travel to work is represented by the events:

B	bicycle
T	train
W	walk.

(a) Write down 2 of these events that are mutually exclusive. Give a reason for your answer. (2)

Solution
Either B and W **or** T and W: there is no overlap between them.

(b) Determine whether or not B and T are independent events. (3)

Solution

$\mathrm{P}(B \cap T) = \frac{5}{25} = \frac{1}{5}$.
$\mathrm{P}(B)\,\mathrm{P}(T) = \frac{9}{25} \times \frac{8}{25} = \frac{72}{625}$.
No, they are $\underline{\underline{\text{not independent}}}$.

One person is chosen at random.

Find the probability that this person

(c) walks to work, (1)

Solution
$\mathrm{P}(W) = \underline{\underline{\frac{7}{25}}}$.

(d) travels to work by bicycle and train. (1)

Solution
$\mathrm{P}(B \cap T) = \underline{\underline{\frac{1}{5}}}$.

(e) Given that this person travels to work by bicycle, find the probability that they will also take the train. (2)

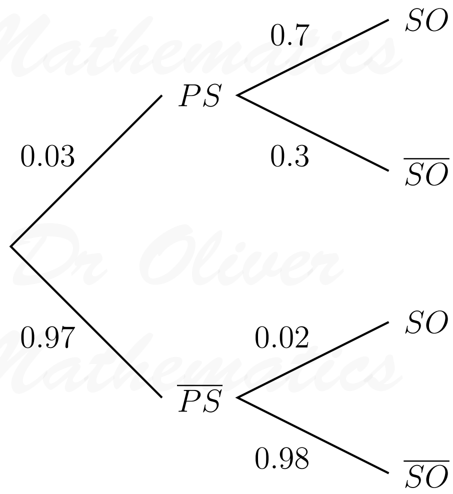
Solution

$$\begin{aligned} \mathrm{P}(T|B) &= \frac{\mathrm{P}(T \cap B)}{\mathrm{P}(B)} \\ &= \frac{\frac{5}{25}}{\frac{9}{25}} \\ &= \underline{\underline{\frac{5}{9}}}. \end{aligned}$$

26. A manufacturer carried out a survey of the defects in their soft toys. It is found that the probability of a toy having poor stitching is 0.03 and that a toy with poor stitching has a probability of 0.7 of splitting open. A toy without poor stitching has a probability of 0.02 of splitting open.

(a) Draw a tree diagram to represent this information. (3)

Solution



(b) Find the probability that a randomly chosen soft toy has exactly one of the two defects, poor stitching or splitting open. (3)

Solution

$0.03 \times 0.3 + 0.97 \times 0.02 = 0.009 + 0.0194 = \underline{\underline{0.0284}}.$

The manufacturer also finds that soft toys can become faded with probability 0.05 and that this defect is independent of poor stitching or splitting open. A soft toy is chosen at random.

(c) Find the probability that the soft toy has none of these 3 defects. (2)

Solution

$\text{P(none)} = 0.97 \times 0.98 \times 0.95 = \underline{\underline{0.903\,07}}.$

(d) Find the probability that the soft toy has exactly one of these 3 defects. (4)

Solution

$$\begin{aligned}\text{P(exactly one)} &= \text{P(poor stiching)} + \text{P(splitting open)} + \text{P(faded)}\\ &= 0.03 \times 0.3 \times 0.95 + 0.97 \times 0.02 \times 0.95 + 0.97 \times 0.98 \times 0.05\\ &= 0.008\,55 + 0.018\,43 + 0.047\,53\\ &= \underline{\underline{0.074\,51}}.\end{aligned}$$

27. The random variable B represents the score when the blue die is rolled (4)

b	1	3	5
$\mathrm{P}(B=b)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

The random variable R represents the score when the blue die is rolled

r	2	4	6
$\mathrm{P}(R=r)$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

Tom invites Avisha to play a game with these dice.

Tom spins a fair coin with one side labelled 2 and the other side labelled 5. When Avisha sees the number showing on the coin she then chooses one of the dice and rolls it. If the number showing on the die is *greater* than the number showing on the coin, Avisha wins, otherwise Tom wins.

Avisha chooses the die which gives her the best chance of winning each time Tom spins the coin.

Find the probability that Avisha wins the game, stating clearly which die she should use in each case.

Solution

If the coin lands on **2**, then Avisha should choose the **blue** die.
If the coin lands on **5**, then Avisha should choose the **red** die. (Why?)

$$\begin{aligned}\mathrm{P}(\text{Avisha wins}) &= \tfrac{1}{2} \times \tfrac{2}{3} + \tfrac{1}{2} \times \tfrac{1}{6} \\ &= \tfrac{1}{3} + \tfrac{1}{12} \\ &= \underline{\underline{\tfrac{5}{12}}}.\end{aligned}$$

28. Given that find

$$\mathrm{P}(A) = 0.35,\ \mathrm{P}(B) = 0.45,\ \text{and}\ \mathrm{P}(A \cap B) = 0.13,$$

find

(a) $\mathrm{P}(A \cup B)$, (2)

Solution

$$\begin{aligned}P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.35 + 0.45 - 0.13 \\ &= \underline{\underline{0.67}}.\end{aligned}$$

(b) $P(A'|B')$. (2)

Solution

Now,

$$P(A' \cap B') = 1 - 0.67 = 0.33$$

and

$$\begin{aligned}P(A'|B') &= \frac{P(A' \cap B')}{P(B')} \\ &= \frac{P(A' \cap B')}{1 - P(B)} \\ &= \frac{0.33}{1 - 0.45} \\ &= \frac{0.33}{0.55} \\ &= \underline{\underline{\tfrac{3}{5}}}.\end{aligned}$$

The event C has $P(C) = 0.20$.

The events A and C are mutually exclusive and the events B and C are independent.

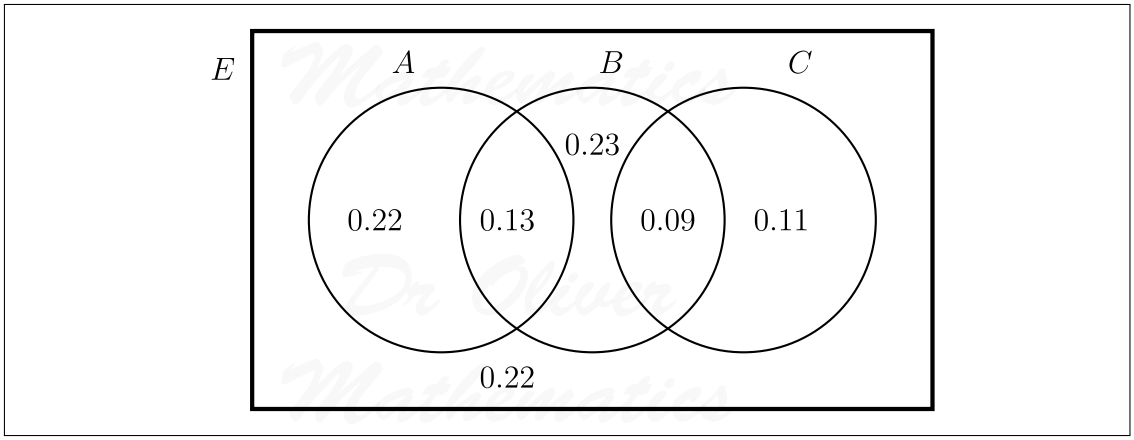
(c) Find $P(B \cap C)$. (2)

Solution

$P(B \cap C) = P(B)\,P(C) = 0.45 \times 0.20 = \underline{\underline{0.09}}$.

(d) Draw a Venn diagram to illustrate the events A, B, and C and the probabilities for each region. (4)

Solution



(e) Find $\mathrm{P}\left([B \cup C]'\right)$. (2)

Solution

$$\begin{aligned}\mathrm{P}\left([B \cup C]'\right) &= 1 - \mathrm{P}(B \cup C) \\ &= 1 - (0.45 + 0.11) \\ &= 1 - 0.56 \\ &= \underline{\underline{0.44}}.\end{aligned}$$

29. In a company the 200 employees are classified as full-time workers, part-time workers or contractors. The table below shows the number of employees in each category and whether they walk to work or use some form of transport.

	Walk	Transport
Full-time worker	2	8
Part-time worker	35	75
Contractor	30	50

The events F, H, and C are that an employee is a full-time worker, part-time worker or contractor respectively. Let W be the event that an employee walks to work.

An employee is selected at random.

Find

(a) $\mathrm{P}(H)$, (2)

Solution

$\mathrm{P}(H) = \frac{35+75}{200} = \frac{110}{200} = \underline{\underline{\frac{11}{20}}}$.

(b) $\mathrm{P}\left([F \cap W]'\right)$, (2)

Solution

$$\begin{aligned}\mathrm{P}\left([F \cap W]'\right) &= 1 - \mathrm{P}\left(F \cap W\right) \\ &= 1 - 0.01 \\ &= \underline{\underline{0.99}}.\end{aligned}$$

(c) $\mathrm{P}(W|C)$. (2)

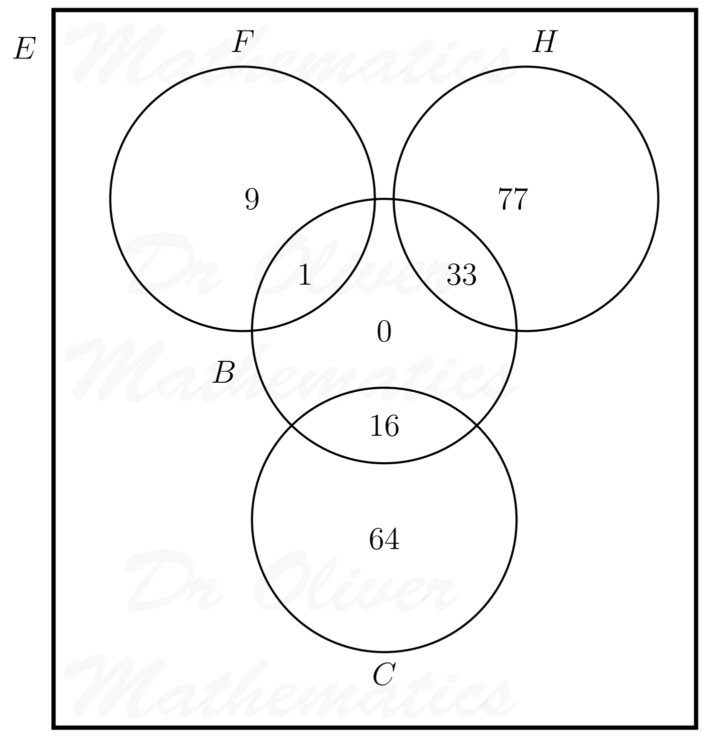
Solution

$$\begin{aligned}\mathrm{P}(W|C) &= \frac{\mathrm{P}(W \cap C)}{\mathrm{P}(C)} \\ &= \frac{\frac{30}{200}}{\frac{80}{200}} \\ &= \underline{\underline{\frac{3}{8}}}.\end{aligned}$$

Let B be the event that an employee uses the bus.
Given that 10% of full-time workers use the bus, 30% of part-time workers use the bus, and 20% of contractors use the bus,

(d) draw a Venn diagram to represent the events F, H, C, and B, (4)

Solution



(e) find the probability that a randomly selected employee uses the bus to travel to work. (2)

Solution
P(bus) $= \frac{1+16+33}{200} = \frac{50}{200} = \underline{\underline{\frac{1}{4}}}$.

30. The Venn diagram in Figure 3 shows three events A, B, and C and the probabilities associated with each region of B.

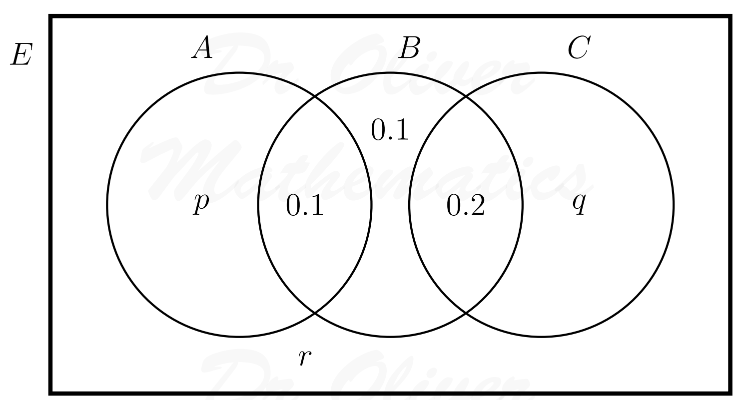


Figure 3: A, B, and C

The constants p, q, and r each represent probabilities associated with the three separate regions outside B.

The events A and B are independent.

(a) Find the value of p. (3)

Solution

$$\begin{aligned} \mathrm{P}(A)\,\mathrm{P}(B) = \mathrm{P}(A \cap B) &\Rightarrow 0.4(p + 0.1) = 0.1 \\ &\Rightarrow 0.4p + 0.04 = 0.1 \\ &\Rightarrow 0.4p = 0.06 \\ &\Rightarrow \underline{\underline{p = 0.15}}. \end{aligned}$$

Given that $\mathrm{P}(B|C) = \frac{5}{11}$,

(b) find the value of q and the value of r. (4)

Solution

$$\begin{aligned} \mathrm{P}(B|C) = \frac{5}{11} &\Rightarrow \frac{\mathrm{P}(B \cap C)}{\mathrm{P}(C)} = \frac{5}{11} \\ &\Rightarrow \frac{0.2}{q + 0.2} = \frac{5}{11} \\ &\Rightarrow 5(q + 0.2) = 2.2 \\ &\Rightarrow 5q + 1 = 2.2 \\ &\Rightarrow 5q = 1.2 \\ &\Rightarrow \underline{\underline{q = 0.24}} \end{aligned}$$

and

$$r = 1 - (0.15 + 0.1 + 0.1 + 0.2 + 0.24) = 1 - 0.79 = \underline{\underline{0.21}}.$$

(c) Find $\mathrm{P}(A \cup C|B)$. (2)

Solution

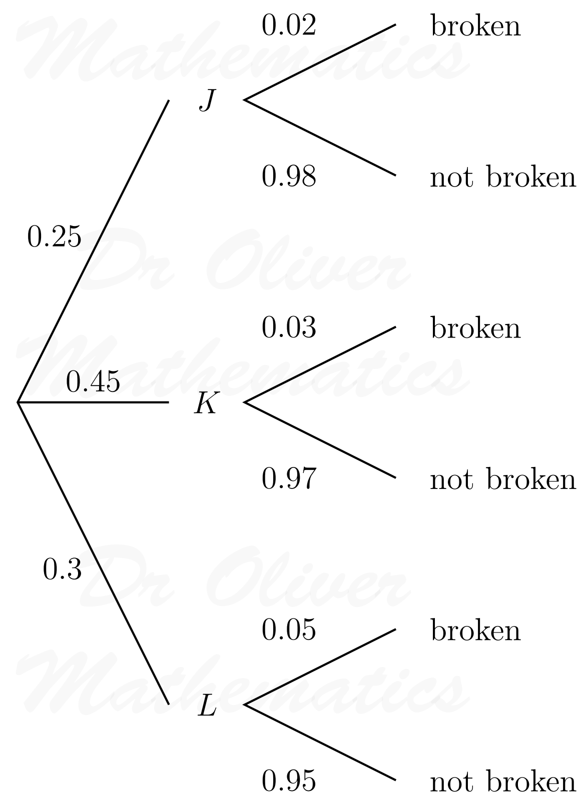
$$\begin{aligned} P(A \cup C|B) &= \frac{P(A \cup C \cap B)}{P(B)} \\ &= \frac{0.1 + 0.2}{0.4} \\ &= \frac{0.3}{0.4} \\ &= \underline{\underline{0.75}}. \end{aligned}$$

31. In a factory, three machines, J, K, and L, are used to make biscuits. Machine J makes 25% of the biscuits. Machine K makes 45% of the biscuits. The rest of the biscuits are made by machine L.

It is known that 2% of the biscuits made by machine J are broken, 3% of the biscuits made by machine K are broken, and 5% of the biscuits made by machine L are broken.

(a) Draw a tree diagram to illustrate all the possible outcomes and associated probabilities. (2)

Solution



A biscuit is selected at random.

(b) Calculate the probability that the biscuit is made by machine J and is not broken. (2)

Solution

$0.25 \times 0.98 = \underline{\underline{0.245}}$.

(c) Calculate the probability that the biscuit is broken. (2)

Solution

$$\begin{aligned} \mathrm{P}(\text{broken}) &= \mathrm{P}(J \cap B) + \mathrm{P}(K \cap B) + \mathrm{P}(L \cap B) \\ &= 0.25 \times 0.02 + 0.45 \times 0.03 + 0.3 \times 0.05 \\ &= 0.005 + 0.0135 + 0.015 \\ &= \underline{\underline{0.0335}}. \end{aligned}$$

(d) Given that the biscuit is broken, find the probability that it was not made by machine K. (3)

Solution

$$\begin{aligned} \mathrm{P}(K'|B) &= \frac{\mathrm{P}(K' \cap B)}{\mathrm{P}(B)} \\ &= \frac{0.005 + 0.015}{0.0355} \\ &= \underline{\underline{\tfrac{40}{67}}}. \end{aligned}$$

32. For the events A and B,

$$\mathrm{P}(A' \cap B) = 0.22 \text{ and } \mathrm{P}(A' \cap B') = 0.18.$$

(a) Find $\mathrm{P}(A)$. (1)

Solution

$\mathrm{P}(A) = 1 - \mathrm{P}(A' \cap B) - \mathrm{P}(A' \cap B') = 1 - 0.22 - 0.18 = \underline{\underline{0.6}}$.

(b) Find $\mathrm{P}(A \cup B)$. (1)

Solution

$\mathrm{P}(A \cup B) = 1 - \mathrm{P}(A' \cap B') = 1 - 0.18 = \underline{\underline{0.82}}$.

Given that $\mathrm{P}(A|B) = 0.6$,

(c) find $\mathrm{P}(A \cap B)$. (3)

Solution

$$\begin{aligned} & \mathrm{P}(A \cap B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cup B) \\ \Rightarrow\quad & \mathrm{P}(A \cap B) = \mathrm{P}(A) + \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(A|B)} - \mathrm{P}(A \cup B) \\ \Rightarrow\quad & \mathrm{P}(A \cap B) = 0.6 + \frac{\mathrm{P}(A \cap B)}{0.6} - 0.82 \\ \Rightarrow\quad & \mathrm{P}(A \cap B) = \tfrac{5}{3}\,\mathrm{P}(A \cap B) - 0.22 \\ \Rightarrow\quad & \tfrac{2}{3}\,\mathrm{P}(A \cap B) = 0.22 \\ \Rightarrow\quad & \mathrm{P}(A \cap B) = \underline{\underline{0.33}}. \end{aligned}$$

(d) Determine whether or not A and B are independent. (2)

Solution
Now,
$$P(B) = P(A \cap B) + P(A' \cap B) = 0.33 + 0.22 = 0.55$$
and
$$\begin{aligned} P(A)\,P(B) &= 0.6 \times 0.55 \\ &= 0.33 \\ &= P(A \cap B); \end{aligned}$$
thus, they are statistically independent.

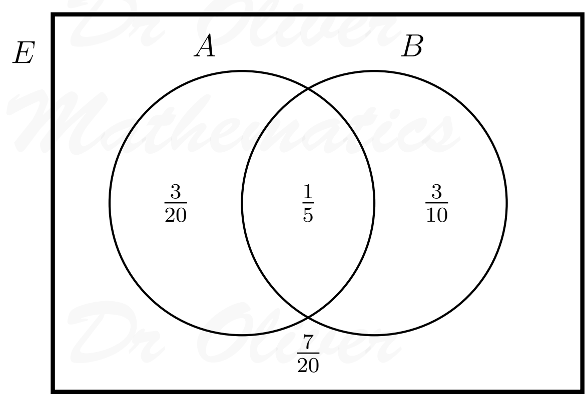
33. A and B are two events such that
$$P(B) = \tfrac{1}{2},\ P(A|B) = \tfrac{2}{5},\ \text{and}\ P(A \cup B) = \tfrac{13}{20}.$$

(a) Find $P(A \cap B)$. (2)

Solution
$$\begin{aligned} P(A \cap B) &= P(A|B)\,P(B) \\ &= \tfrac{2}{5} \times \tfrac{1}{2} \\ &= \underline{\underline{\tfrac{1}{5}}}. \end{aligned}$$

(b) Draw a Venn diagram to show the events A, B, and and all the associated probabilities. (3)

Solution



Find

(c) $\mathrm{P}(A)$, (1)

Solution
$\mathrm{P}(A) = \frac{3}{20} + \frac{1}{5} = \underline{\underline{\frac{7}{20}}}$.

(d) $\mathrm{P}(B|A)$, (2)

Solution

$$\begin{aligned}\mathrm{P}(B|A) &= \frac{\mathrm{P}(B \cap A)}{\mathrm{P}(A)} \\ &= \frac{\frac{1}{5}}{\frac{7}{20}} \\ &= \underline{\underline{\frac{4}{7}}}.\end{aligned}$$

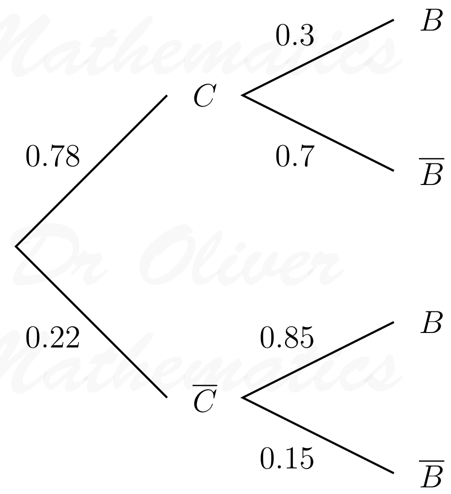
(e) $\mathrm{P}(A' \cap B)$. (1)

Solution
$\mathrm{P}(A' \cap B) = \underline{\underline{\frac{3}{10}}}$.

34. In a large company, 78% of employees are car owners, 30% of these car owners are also bike owners, and 85% of those who are not car owners are bike owners.

(a) Draw a tree diagram to represent this information. (3)

Solution



An employee is selected at random.

(b) Find the probability that the employee is a car owner or a bike owner but not both. (2)

Solution

$$\begin{aligned}\text{P(but not both)} &= 0.78 \times 0.7 + 0.22 \times 0.85 \\ &= 0.546 + 0.187 \\ &= \underline{\underline{0.733}}.\end{aligned}$$

Another employee is selected at random. Given that this employee is a bike owner,

(c) find the probability that the employee is a car owner. (3)

Solution

$$\begin{aligned}\text{P}(C|B) &= \frac{\text{P}(C \cap B)}{\text{P}(B)} \\ &= \frac{0.78 \times 0.3}{0.78 \times 0.3 + 0.22 \times 0.85} \\ &= \frac{0.234}{0.234 + 0.187} \\ &= \frac{0.234}{0.421} \\ &= \underline{\underline{\tfrac{234}{421}}}.\end{aligned}$$

Two employees are selected at random.

(d) Find the probability that only one of them is a bike owner. (3)

Solution

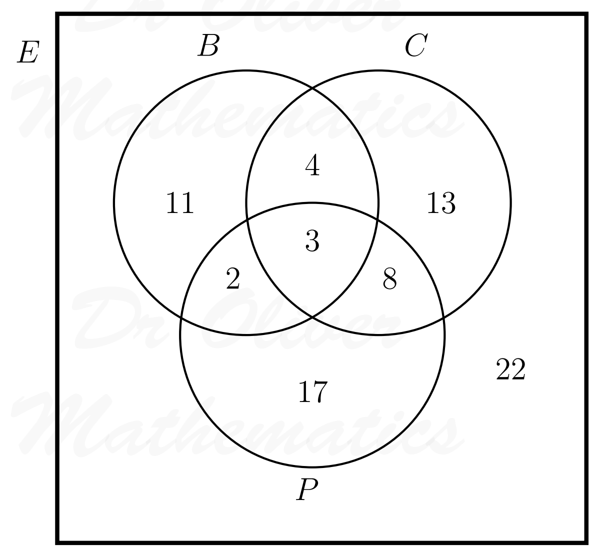
Recall $\mathrm{P}(B) = 0.421$:

$$\begin{aligned}\mathrm{P}(\text{only one of them is a bike owner}) &= 2 \times 0.421 \times (1 - 0.421)\\ &= 2 \times 0.421 \times 0.579\\ &= \underline{\underline{0.487\,518}}.\end{aligned}$$

35. A college has 80 students in Year 12:
20 students study Biology,
28 students study Chemistry,
30 students study Physics,
7 students study both Biology and Chemistry,
11 students study both Chemistry and Physics,
5 students study both Physics and Biology, and
3 students study all 3 of these subjects.

(a) Draw a Venn diagram to represent this information. (5)

Solution



A Year 12 student at the college is selected at random.

(b) Find the probability that the student studies Chemistry but not Biology or Physics. (1)

Solution

$\underline{\underline{\frac{13}{80}}}$.

(c) Find the probability that the student studies Chemistry or Physics or both. (2)

Solution

$\frac{17+2+3+8+4+13}{80} = \underline{\underline{\frac{47}{80}}}$.

Given that the student studies Chemistry or Physics or both,

(d) find the probability that the student does not study Biology. (2)

Solution

$$\begin{aligned} \mathrm{P}(\overline{B}|C,\ P,\ \text{or both}) &= \frac{\mathrm{P}(\overline{B} \cap C,\ P,\ \text{or both})}{\mathrm{P}(C,\ P,\ \text{or both})} \\ &= \frac{\frac{33}{80}}{\frac{47}{80}} \\ &= \underline{\underline{\frac{33}{47}}}. \end{aligned}$$

(e) Determine whether studying Biology and studying Chemistry are statistically independent. (3)

Solution

$\mathrm{P}(B \cap C) = \frac{7}{80}$.
$\mathrm{P}(B)\,\mathrm{P}(C) = \frac{20}{80} \times \frac{28}{80} = \frac{7}{80}$.
Hence, they are <u>statistically independent</u>.

36. Figure 4 shows the Venn diagram of the probabilities of customer bookings at Harry's hotel.

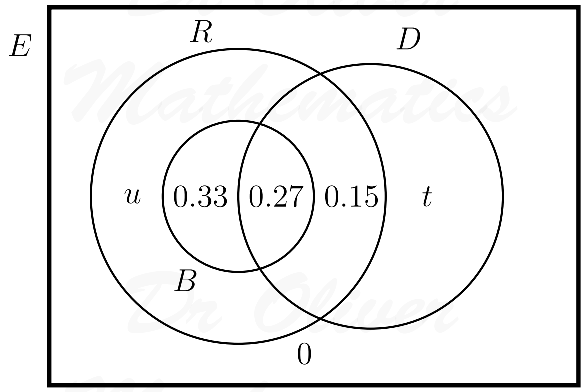


Figure 4: customer bookings at Harry's hotel

R is the event that a customer books a room, B is the event that a customer books breakfast, D is the event that a customer books dinner, and u and t are probabilities.

(a) Write down the probability that a customer books breakfast but does not book a room. (1)

Solution

$\underline{\underline{0}}$.

Given that the events B and D are independent,

(b) find the value of t, (4)

Solution

$$\begin{aligned} \mathrm{P}(B \cap D) = \mathrm{P}(B)\,\mathrm{P}(D) &\Rightarrow 0.27 = 0.6\,\mathrm{P}(D) \\ &\Rightarrow \mathrm{P}(D) = \frac{0.27}{0.6} \\ &\Rightarrow \mathrm{P}(D) = 0.45 \\ &\Rightarrow 0.27 + 0.15 + t = 0.45 \\ &\Rightarrow \underline{\underline{t = 0.03}}. \end{aligned}$$

(c) hence find the value of u. (2)

Solution

$u + 0.33 + 0.27 + 0.15 + 0.03 = 1 \Rightarrow \underline{\underline{u = 0.22}}$.

(d) Find

(i) $\mathrm{P}(D|R \cap B)$, (2)

Solution

$$\begin{aligned}\mathrm{P}(D|R \cap B) &= \frac{\mathrm{P}(D \cap R \cap B)}{\mathrm{P}(R \cap B)} \\ &= \frac{0.27}{0.6} \\ &= \underline{\underline{0.45}}.\end{aligned}$$

(ii) $\mathrm{P}(D|R \cap B')$. (2)

Solution

$$\begin{aligned}\mathrm{P}(D|R \cap B') &= \frac{\mathrm{P}(D \cap R \cap B')}{\mathrm{P}(R \cap B')} \\ &= \frac{0.15}{0.22 + 0.15} \\ &= \frac{0.15}{0.37} \\ &= \underline{\underline{\tfrac{15}{37}}}.\end{aligned}$$

A coach load of 77 customers arrive at Harry's hotel. Of these 77 customers, 40 have booked a room and breakfast and 37 have booked a room without breakfast.

(e) Estimate how many of these 77 customers will book dinner. (2)

Solution

$40 \times 0.45 + 37 \times \frac{15}{37} = \underline{\underline{33}}$.

37. The Venn diagram in Figure 5 shows three events A, B, and C, where p, q, r, s, and t are probabilities.

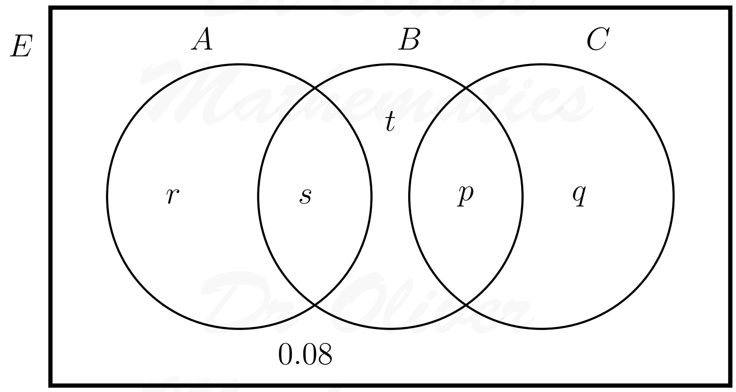


Figure 5: three events A, B, and C

$\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.6$, and $\mathrm{P}(C) = 0.25$, and the events B and C are independent.

(a) Find the value of p and the value of q. (2)

Solution

$p = \mathrm{P}(B \cap C) = \mathrm{P}(B)\,\mathrm{P}(C) = 0.6 \times 0.25 = \underline{\underline{0.15}}$ and $q = 0.25 - 0.15 = \underline{\underline{0.1}}$.

(b) Find the value of r. (2)

Solution

$r = 1 - 0.6 - 0.1 - 0.08 = \underline{\underline{0.22}}$.

(c) Hence write down the value of s and the value of t. (2)

Solution

$s = 0.5 - 0.22 = \underline{\underline{0.28}}$ and $t = 1 - 0.6 - 0.25 - 0.08 = \underline{\underline{0.07}}$.

(d) State, giving a reason, whether or not the events A and B are independent. (2)

Solution

$\mathrm{P}(A \cap B) = 0.28$.
$\mathrm{P}(A)\,\mathrm{P}(B) = 0.5 \times 0.6 = 0.3$.
Hence, they are not statistically independent.

(e) Find $\mathrm{P}(B|A \cup C)$. (3)

Solution

$$\begin{aligned}\mathrm{P}(B|A\cup C) &= \frac{\mathrm{P}(B\cap(A\cup C))}{\mathrm{P}(B)}\\ &= \frac{0.28+0.15}{0.75}\\ &= \underline{\underline{\tfrac{43}{75}}}.\end{aligned}$$