

Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2012 November Paper 1 Variant 2: Calculator
2 hours

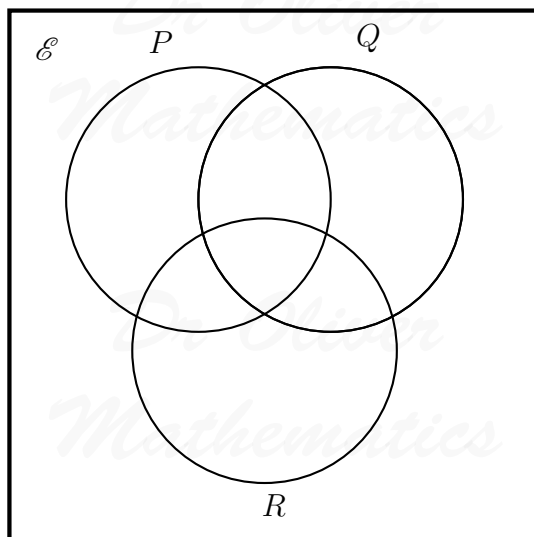
The total number of marks available is 80.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

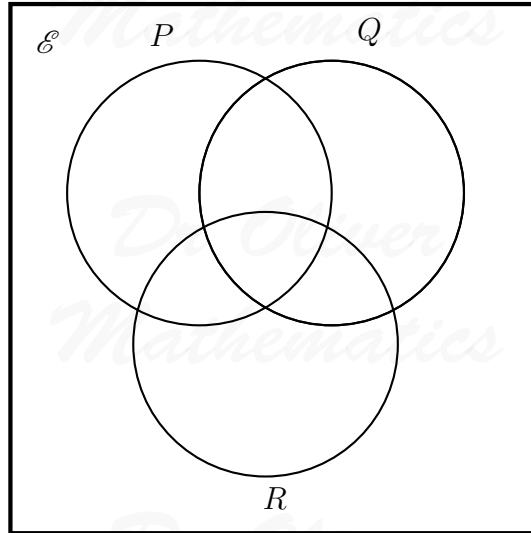
You must write down all the stages in your working.

1. (a) On the Venn diagrams below, shade the region corresponding to the set given below each Venn diagram. (2)

$$P \cup (Q \cap R)$$



$$P \cap (Q \cup R)$$



(b) It is given that sets $\mathcal{E}$, B , S , and F are such that

- $\mathcal{E} = \{\text{students in a school}\}$,
- $B = \{\text{students who are boys}\}$,
- $S = \{\text{students in the swimming team}\}$, and
- $F = \{\text{students in the football team}\}$.

Express each of the following statements in set notation.

- (i) All students in the football team are boys. (1)
- (ii) There are no students who are in both the swimming team and the football team. (1)

2. The rate of change of a variable x with respect to time t is

$$4 \cos^2 t.$$

- (a) Find the rate of change of x with respect to t when $t = \frac{1}{6}\pi$. (1)

The rate of change of a variable y with respect to time t is

$$3 \sin t.$$

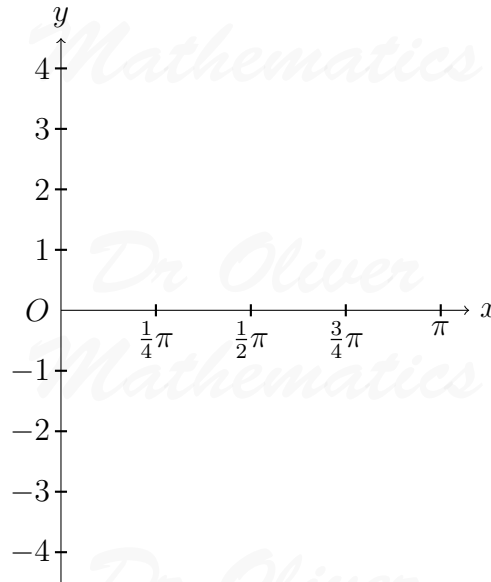
- (b) Using your result from part (a), find the rate of change of y with respect to x when $t = \frac{1}{6}\pi$. (3)

3. A committee of 7 members is to be selected from 6 women and 9 men.

Find the number of different committees that may be selected if

- (a) there are no restrictions, (1)
- (b) the committee must consist of 2 women and 5 men, (2)
- (c) the committee must contain at least 1 woman. (3)
4. (a) On the axes below sketch, for $0 \leq x \leq \pi$, the graphs of (3)

$$y = \tan x \text{ and } y = 1 + 3 \sin 2x.$$



Write down

- (b) the coordinates of the stationary points on the curve (2)

$$y = 1 + 3 \sin 2x \text{ for } 0 \leq x \leq \pi,$$

- (c) the number of solutions of the equation (1)

$$\tan x = 1 + 3 \sin 2x \text{ for } 0 \leq x \leq \pi.$$

5. A pilot flies his plane directly from a point A to a point B , a distance of 450 km.

The bearing of B from A is 030° .

A wind of 80 kmh^{-1} is blowing from the east.

Given that the plane can travel at 320 kmh^{-1} in still air, find

- (a) the bearing on which the plane must be steered, (4)
- (b) the time taken to fly from A to B . (4)

6. In the expansion of

$$(p + x)^6,$$

where p is a positive integer, the coefficient of x^2 is equal to 1.5 times the coefficient of x^3 .

- (a) Find the value of p . (4)
- (b) Use your value of p to find the term independent of x in the expansion of (3)

$$(p + x)^6 \left(1 - \frac{1}{x}\right)^2.$$

7. A particle P moves along the x -axis such that its distance, x m, from the origin O at time t s is given by

$$x = \frac{t}{t^2 + 1}, \text{ for } t > 0.$$

- (a) Find the greatest distance of P from O . (4)
- (b) Find the acceleration of P at the instant when P is at its greatest distance from O . (3)
8. (a) Given that (5)

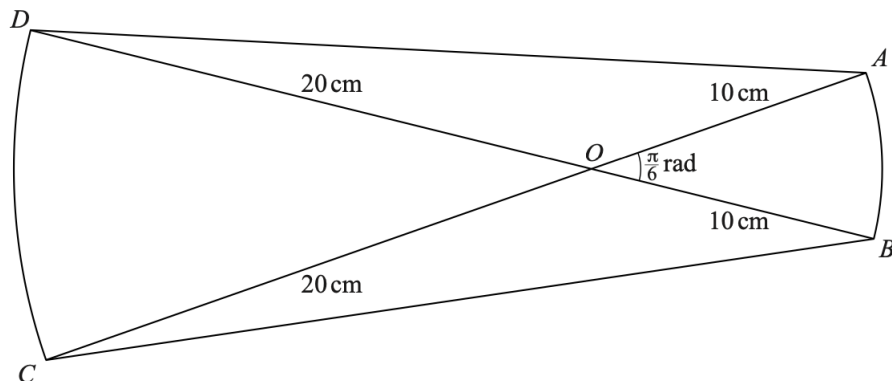
$$3x^3 + 5x^2 + px + 8 \equiv (x - 2)(ax^2 + bx + c),$$

find the value of each of the integers a , b , c , and p .

- (b) Using the values found in part (a), factorise completely (2)

$$3x^3 + 5x^2 + px + 8.$$

9. The diagram shows four straight lines: AD , BC , AC , and BD .



- Lines AC and BD intersect at O such that angle AOB is $\frac{1}{6}\pi$ radians.
- AB is an arc of the circle, centre O and radius 10 cm, and CD is an arc of the circle, centre O and radius 20 cm.

(a) Find the perimeter of $ABCD$. (4)

(b) Find the area of $ABCD$. (4)

10. (a) Solve (4)

$$\tan^2 x - 2 \sec x + 1 = 0 \text{ for } 0^\circ \leq x \leq 360^\circ.$$

(b) Solve (4)

$$\cos^2 3y = 5 \sin^2 3y \text{ for } 0 \leq y \leq 2 \text{ radians.}$$

(c) Solve (4)

$$2 \operatorname{cosec}(z + \frac{1}{4}\pi) = 5 \text{ for } 0 \leq z \leq 6 \text{ radians.}$$

EITHER

11. The tangent to the curve

$$y = 5e^x + 3e^{-x}$$

at the point where $x = \ln \frac{3}{5}$, meets the x -axis at the point P .

(a) Find the coordinates of P . (5)

The area of the region enclosed by the curve

$$y = 5e^x + 3e^{-x},$$

the y -axis, the positive x -axis, and the line $x = a$ is 12 square units.

(b) Show that (3)

$$5e^{2a} - 14e^a - 3 = 0.$$

(c) Hence find the value of a . (3)

OR

12. (a) Given that (4)

$$y = \frac{3e^{2x}}{1 + e^{2x}},$$

show that

$$\frac{dy}{dx} = \frac{Ae^{2x}}{(1 + e^{2x})^2},$$

where A is a constant to be found.

- (b) Find the equation of the tangent to the curve (3)

$$y = \frac{3e^{2x}}{1 + e^{2x}}$$

at the point where the curve crosses the y -axis.

- (c) Using your result from part (a), find (4)

$$\int \frac{e^{2x}}{(1 + e^{2x})^2} dx$$

and hence evaluate

$$\int_0^{\ln 3} \frac{e^{2x}}{(1 + e^{2x})^2} dx.$$