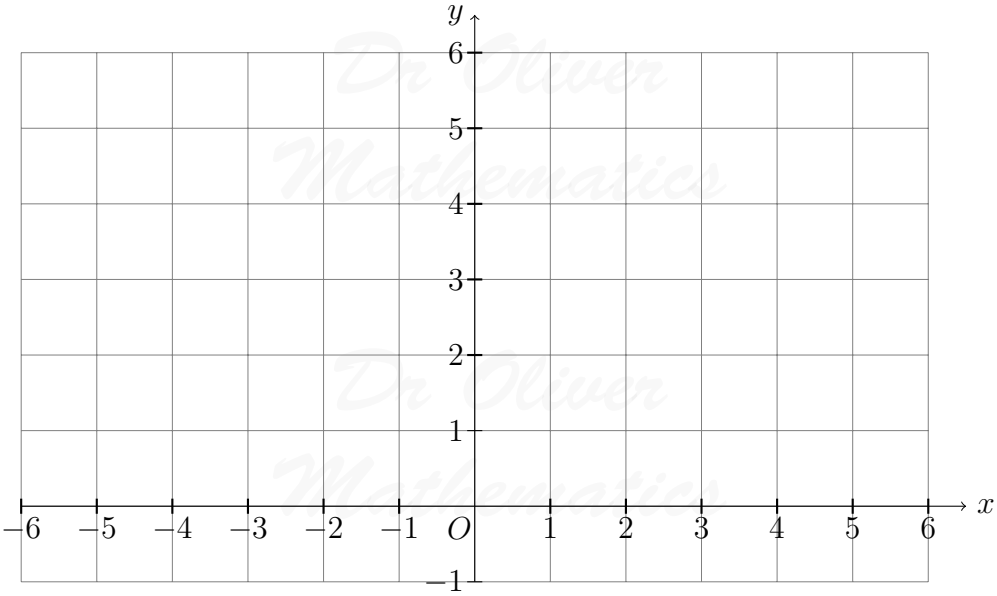
Dr Oliver Mathematics
AQA Further Maths Level 2
June 2016 Paper 2
2 hours

The total number of marks available is 105.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.

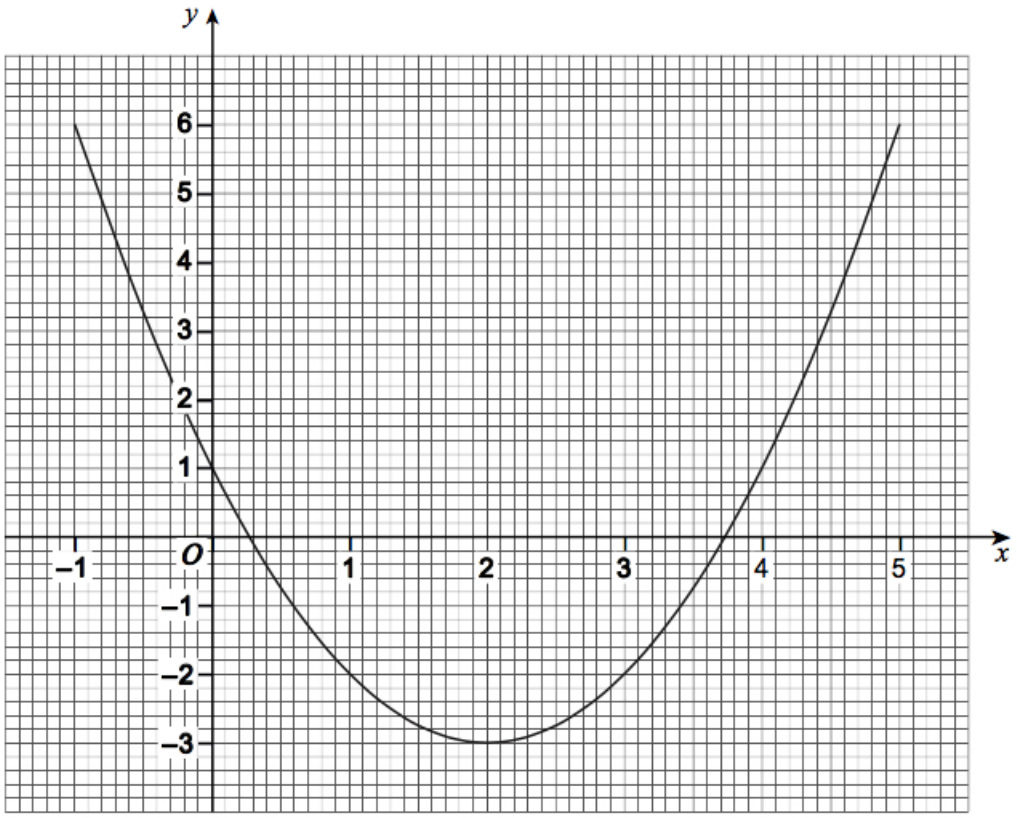
1. A triangle has vertices $A(2, 5)$, $B(2, 0)$, and $C(-4, 3)$. (3)
   Work out the area of triangle ABC.
   You may use the grid to help you.



2. The function

$$\mathrm{f}(x) = x^2 - 4x + 1$$

   has domain $-1 \leqslant x \leqslant 5$.
   Here is the graph of $y = \mathrm{f}(x)$.



(a) Write down the equation of the line of symmetry of the graph. (1)

(b) Use the graph to work out the solutions of (2)

$$x^2 - 4x + 1 = 5.$$

Give your answers to 1 decimal place.

(c) Write down the range of $\mathrm{f}(x)$ for domain $-1 \leqslant x \leqslant 5$. (2)

3. L is a straight line with equation

$$ax + by = c,$$

where a, b, and c are non-zero integers.

(a) At which point does L intersect the x-axis? (1)
Circle your answer.

$$\left(\tfrac{a}{c}, 0\right) \qquad \left(\tfrac{c}{a}, 0\right) \qquad \left(\tfrac{b}{c}, 0\right) \qquad \left(\tfrac{c}{b}, 0\right)$$

(b) What is the gradient of a line parallel to L? (1)
Circle your answer.

$$-\tfrac{b}{a} \qquad \tfrac{b}{a} \qquad -\tfrac{a}{b} \qquad -\tfrac{a}{b}$$

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4. Work out the point of intersection of the lines (4)

$$2x + 3y = 11 \text{ and } 2y = 13 - 3x.$$

5. a, b, and c are numbers such that (4)

$$a < 0,\ b > 1,\ \text{and}\ -1 < c < 1.$$

Tick the correct box for each statement.

	Always true	Sometimes true	Never true
$a^3 < 0$	☐	☐	☐
$b < 10a^2$	☐	☐	☐
$ab > 0$	☐	☐	☐
$b - c > 1$	☐	☐	☐

6. For the curve $y = \mathrm{f}(x)$, (3)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{3}{2}x - kx^4 + k,$$

where k is a constant.

When $x = -2$ the gradient of the curve is 12.

Work out the value of k.

7. Simplify fully (2)

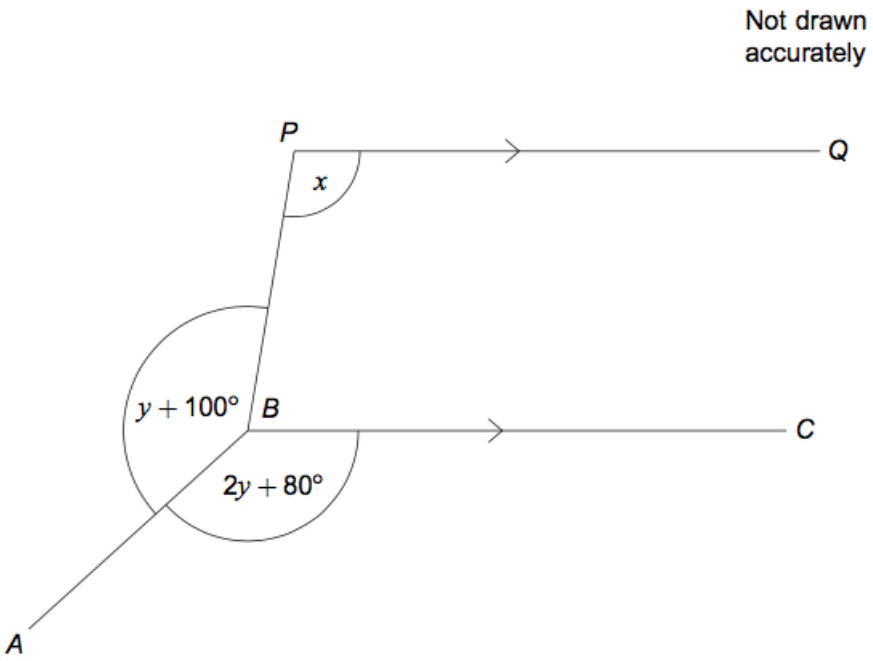
$$(\tfrac{2}{3}x^3y)^3.$$

8. $D(-6, 4)$ and $E(-2, 9)$ are joined by a straight line. (3)

- P is a point on DE.
- $DP : PE = 3 : 5$.

Work out the coordinates of P.

9. PQ is parallel to BC. (4)



Prove that

$$x = 3y.$$

10. (a) Simplify (2)

$$\frac{x^2 - 7x + 10}{x^2 - 2x - 15}.$$

(b) Factorise fully (2)

$$w^5x^3y^2 + w^2x^6y^3.$$

11. The x^2 term in the expansion of (3)

$$(3x + 4)(x^2 + px + 5)$$

is $-23x^2$.

Work out the value of p.

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12. Here are the first four terms of linear sequences X and Y and quadratic sequence Z.

Sequence X :	7	9	11	13	...
Sequence Y :	2	5	8	11	...
Sequence Z :	14	45	88	143	...

(a) Work out the nth term of sequence X. (2)

The nth term of sequence Y is $3n - 1$.

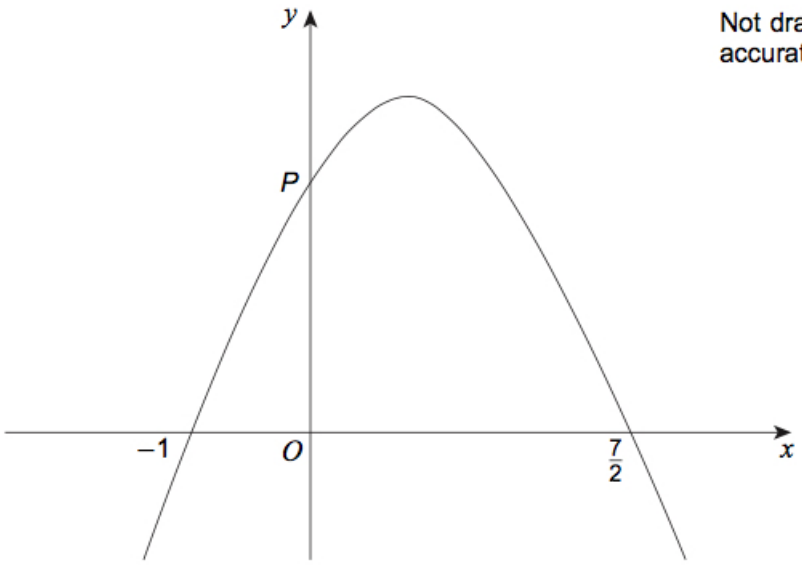
(b) Using your answer to part (a), or otherwise, work out the nth term of sequence Z. (3)
Give your answer in the form $an^2 + bn + c$, where a, b, and c are integers.

13. Here is a sketch of (4)

$$y = a + bx - 2x^2,$$

where a and b are constants.

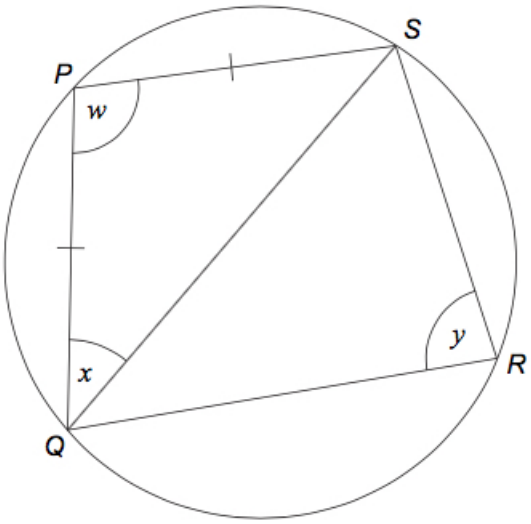
The graph intersects the x-axis at $(-1, 0)$ and $(\frac{7}{2}, 0)$ and the y-axis at point P.



Work out the coordinates of point P.
You **must** show your working.

14. P, Q, R, and S are points on the circumference of a circle. (4)

- $w : y = 7 : 5$.
- $PQ = PS$.



Not drawn accurately

Work out the size of angle x.

15. (a) Solve (2)

$$\tfrac{2}{5}\sqrt{x} = 1.$$

(b) Solve (2)

$$x^3 = 5x^2.$$

16. Rearrange (4)

$$y = \frac{8(w-x)}{x}$$

to make x the subject.

17. A cylinder has base radius x cm and height y cm. (3)
A hemisphere has radius $6y$ cm.
The cylinder and hemisphere have equal volumes.

Work out the value of

$$\frac{x}{y}.$$

You **must** show your working.

18. Angle y is acute.

$$\tan y = \frac{p+1}{p-1},$$

where p is a constant greater than 1.

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(a) Which of the statements below is correct? (1)
Circle your answer.

$y = 45^\circ$ $y < 45^\circ$ $y > 45^\circ$ y could be any acute angle

(b) Work out the expression for $\sin y$. (4)
Give your answer in the form

$$\frac{ap + b}{\sqrt{cp^2 + d}},$$

where a, b, c, and d are integers.
You may use a diagram to help you.

19. The continuous curve $y = g(x)$ has exactly two stationary points. (3)
The stationary points are

- a point of inflection at $P(1, 2)$ and
- a minimum point at $Q(a, b)$ where $a > 1$ and $b < 0$.

On the axes below, sketch the curve.
Label points P and Q on your sketch.

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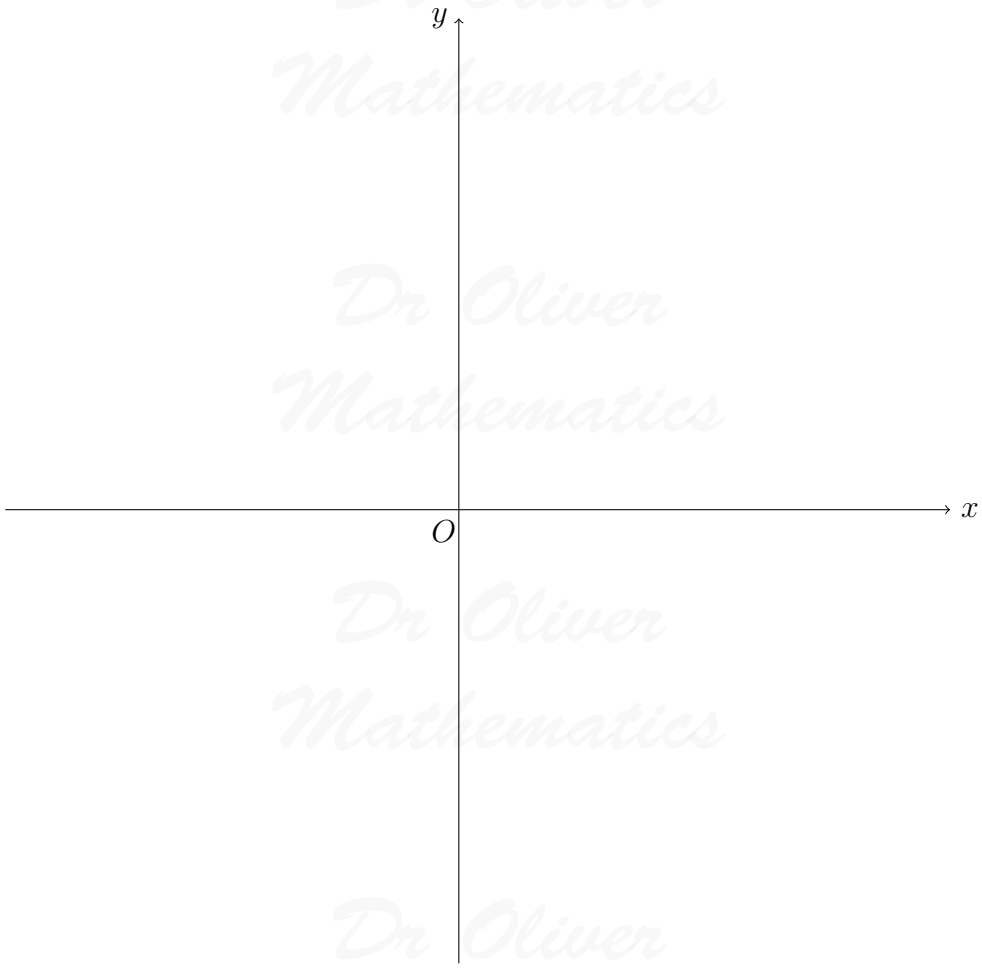


20. Under the transformation represented by (4)

$$\begin{pmatrix} -1 & -3 \\ 2 & 4 \end{pmatrix},$$

the image of point $P(a, 2)$ is point Q.

Can point Q be the same as point P?
You **must** show your working.

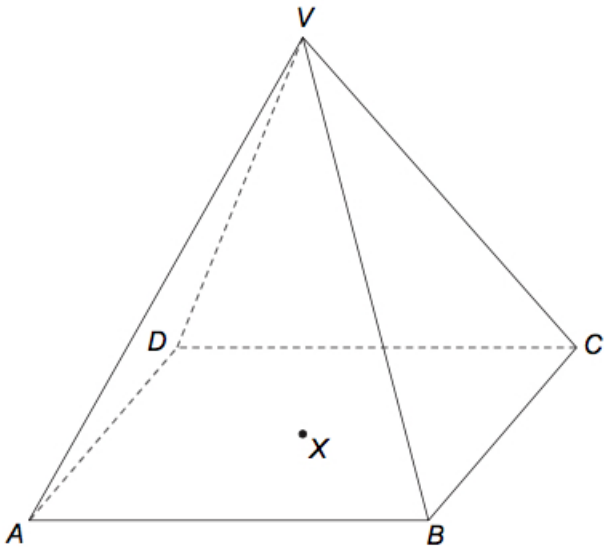
21. Solve (6)

$$\frac{3}{x-2} + \frac{2}{x-1} = 5.$$

Do **not** use trial and improvement.
Write your solutions to 3 significant figures.

22. Pyramid $VABCD$ has a horizontal rectangular base. (4)

- X is the centre of the base.
- V is vertically above X.
- $VB = VC = 17$ cm.
- $AB = 22$ cm.
- $BC = 16$ cm



Work out the angle between the planes VBC and $ABCD$.

23. Shape A maps to shape B by an enlargement, scale factor 3, centre the origin. (5)
Shape B maps to shape C by a rotation through 180°, centre the origin.

Shape A can be mapped to shape C by a single transformation.

Use matrices to show that the single transformation is an enlargement, centre the origin.
State the scale factor of the enlargement.

24. (5)

$$\mathrm{f}(x) = \frac{x}{2x+1},$$

for positive values of x.

Work out

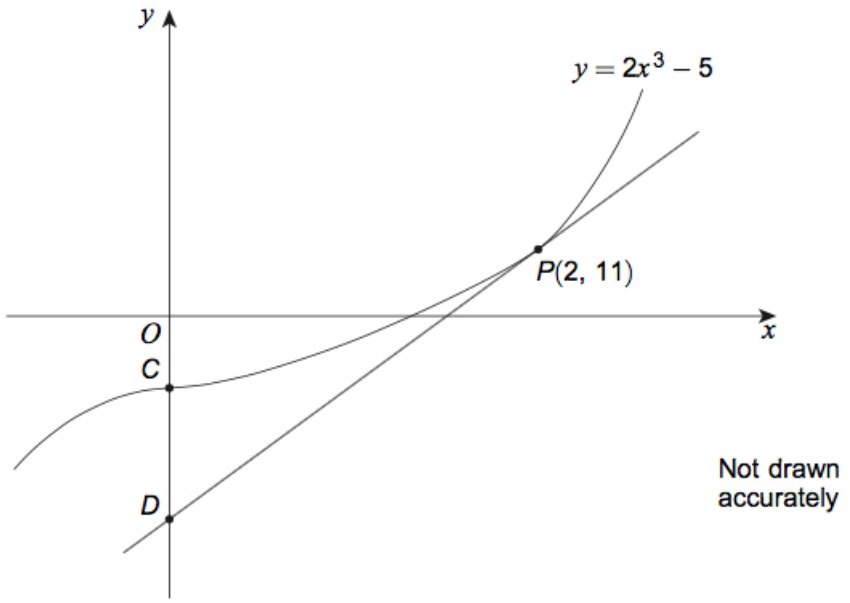
$$\mathrm{f}(x+1) - \mathrm{f}(x).$$

Give your answer as a fraction in its simplest form.
You **must** show your working.

25. The curve (6)

$$y = 2x^3 - 5$$

intersects the y-axis at C.
The tangent to the curve at $P(2, 11)$ intersects the y-axis at D.



Work out the length CD.

26. (a) Prove that (2)

$$\sin^2 x - 3\cos^2 x \equiv 4\sin^2 x - 3.$$

(b) Hence, or otherwise, work out the values of x between 0° and 360° for which (4)

$$\sin^2 x - 3\cos^2 x = 0.$$

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