

**Dr Oliver Mathematics**  
**GCSE Mathematics**  
**2019 Paper 3H: Calculator**  
**1 hour 30 minutes**

The total number of marks available is 80.

You must write down all the stages in your working.

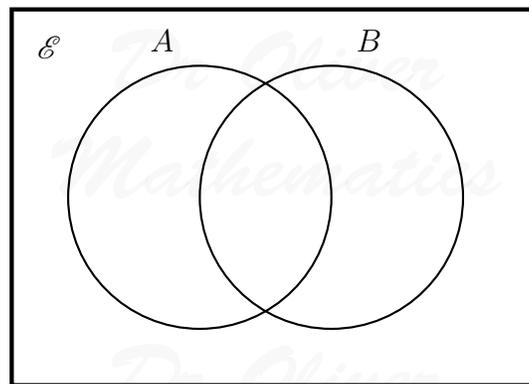
1.  $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ .

$A = \{1, 5, 6, 8, 9\}$ .

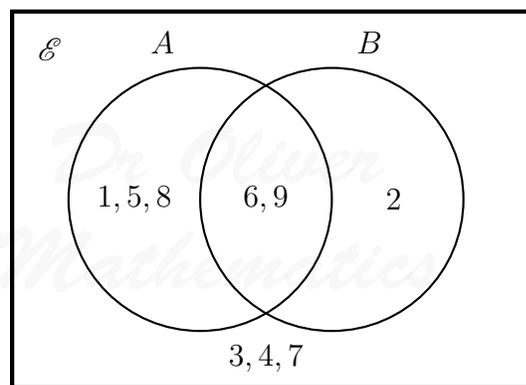
$B = \{2, 6, 9\}$ .

(a) Complete the Venn diagram to represent this information.

(2)



**Solution**



A number is chosen at random from the universal set  $\mathcal{E}$ .

(b) Find the probability that the number is in the set  $A \cap B$ .

(3)

**Solution**

$$P(A \cap B) = \underline{\underline{\frac{2}{9}}}$$

2. Katy invests £200 000 in a savings account for 4 years. (3)  
The account pays compound interest at a rate of 1.5% per annum.

Calculate the total amount of interest Katy will get at the end of 4 years.

**Solution**

$$\begin{aligned} \text{Interest} &= (200\,000 \times 1.015^4) - 200\,000 \\ &= 212\,272.710\,1 - 200\,000 \text{ (FCD)} \\ &= 12\,272.710\,13 \text{ (FCD)} \\ &= \underline{\underline{\pounds 12\,272.71}} \text{ (2 dp)}. \end{aligned}$$

3. The table shows information about the heights of 80 plants.

Height ( $h$ cm)	Frequency
$10 < h \leq 20$	7
$20 < h \leq 30$	13
$30 < h \leq 40$	14
$40 < h \leq 50$	12
$50 < h \leq 60$	16
$60 < h \leq 70$	18

- (a) Find the class interval that contains the median. (1)

**Solution**

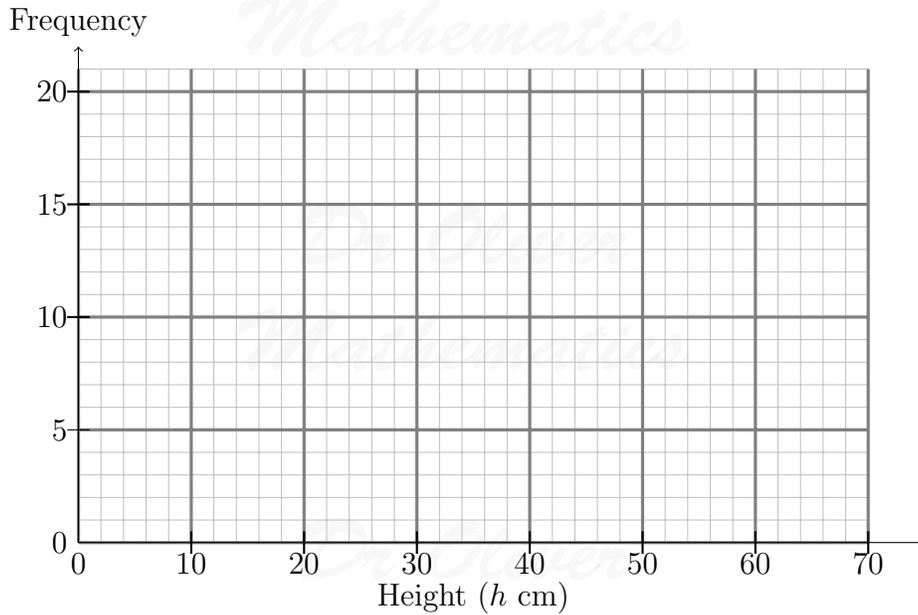
Height ( $h$ cm)	Frequency	Cumulative Frequency
$10 < h \leq 20$	7	7
$20 < h \leq 30$	13	$13 + 7 = 20$
$30 < h \leq 40$	14	$14 + 20 = 34$
$40 < h \leq 50$	12	$12 + 34 = 46$
$50 < h \leq 60$	16	$16 + 46 = 62$
$60 < h \leq 70$	18	$18 + 62 = 80$

Now, the median is the

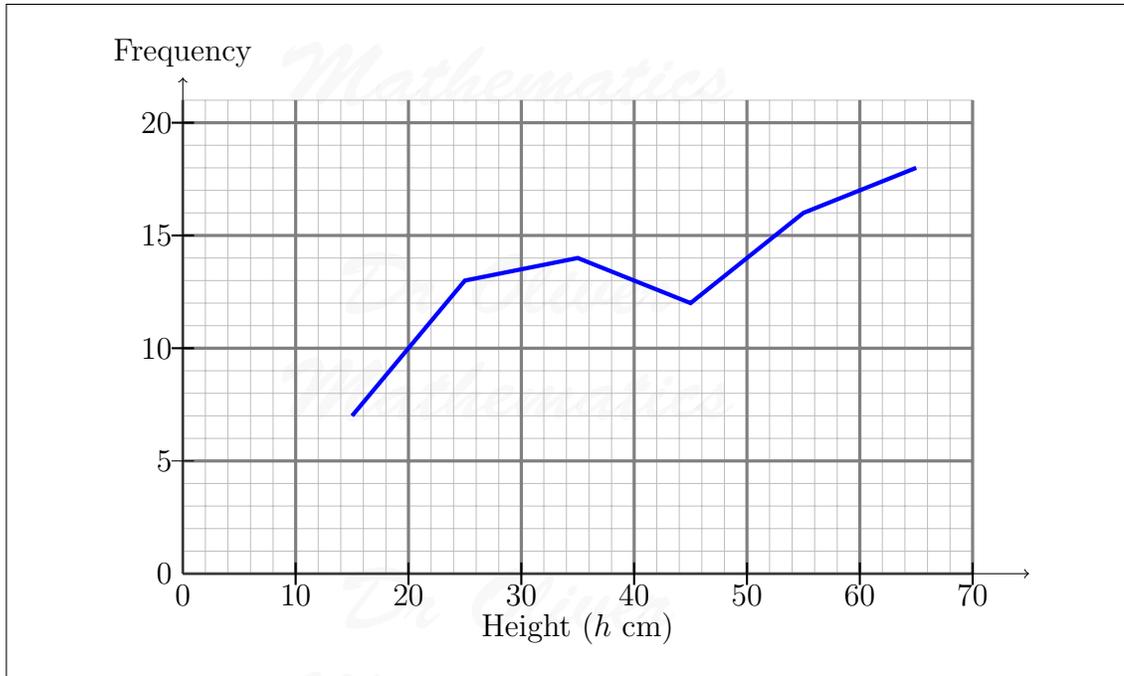
$$\frac{80 + 1}{2} = 40\frac{1}{2}\text{th}$$

place which means the the median is  $40 < h \leq 50$ .

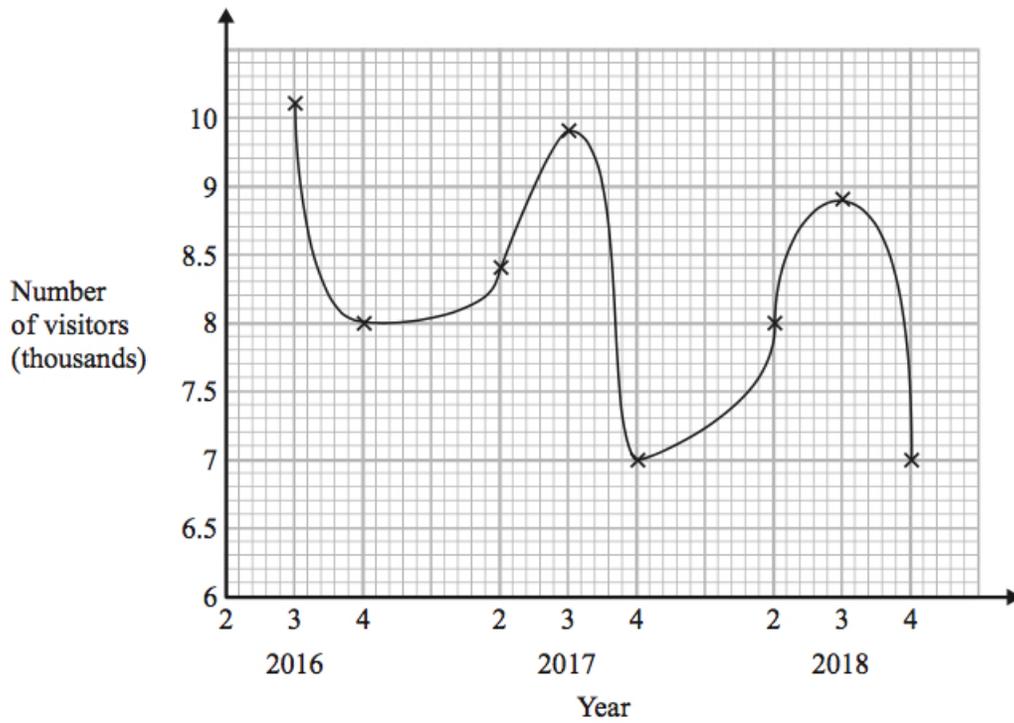
(b) On the grid, draw a frequency polygon for the information in the table. (2)



**Solution**



4. Sean has drawn a time series graph to show the numbers, in thousands, of visitors to a fun park. (2)



Write down two things that are wrong or could be misleading with this graph.

**Solution**

E.g., No 1s on the 2017 and 2018 axis: maybe it is closed then but we would have a zero.

It should be joined with straight lines.

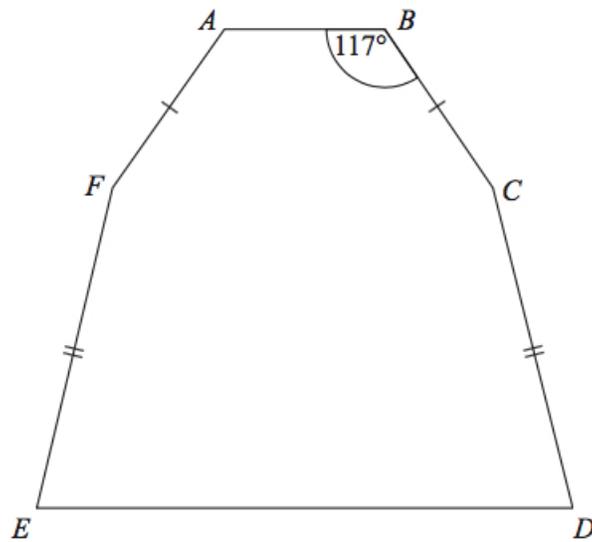
9.5 missing from vertical axes.

The vertical scale starts off at 6 rather than 0.

There is lack of clarity about what the numbers on the  $x$ -axis represent.

5. The diagram shows a hexagon.  
The hexagon has one line of symmetry.

(4)



$$FA = BC.$$

$$EF = CD.$$

$$\text{Angle } ABC = 117^\circ.$$

$$\text{Angle } BCD = 2 \times \text{angle } CDE.$$

Work out the size of angle  $AFE$ .

You must show all your working.

**Solution**

The line of symmetry goes through the midpoint of  $AB$  and the midpoint of  $DE$  (why?). Now, the total angles is

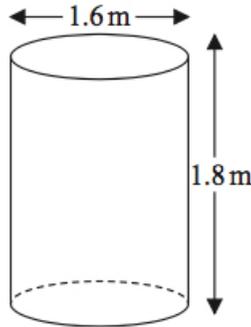
$$(6 - 2) \times 180 = 720^\circ$$

and

$$\begin{aligned}2(117 + \angle AFE + \angle CDE) &= 720 \Rightarrow 2(117 + \angle AFE + \frac{1}{2}\angle AFE) = 720 \\ &\Rightarrow 117 + \frac{3}{2}\angle AFE = 360 \\ &\Rightarrow \frac{3}{2}\angle AFE = 243 \\ &\Rightarrow \underline{\underline{\angle AFE = 162^\circ}}.\end{aligned}$$

6. Jeremy has to cover 3 tanks completely with paint. Each tank is in the shape of a cylinder with a top and a bottom.

(5)



The tank has a diameter of 1.6 m and a height of 1.8 m.

Jeremy has 7 tins of paint.

Each tin of paint covers  $5 \text{ m}^2$ .

Has Jeremy got enough paint to cover completely the 3 tanks?

You must show how you get your answer.

**Solution**

$$\begin{aligned}\text{Paint} &= 3[(2 \times \pi \times 0.8 \times 1.8) + 2(\pi \times 0.8^2)] \\ &= 39.207\ 076\ 32 \text{ (FCD)}.\end{aligned}$$

Now,

$$7 \times 5 = 35 \text{ m}^2$$

and, no, he does not have enough.

7. Work out

(2)

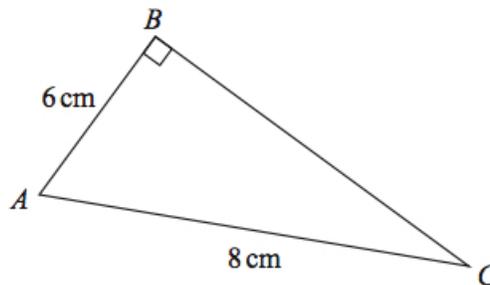
$$\sqrt{\frac{2.5 \times \sin 43^\circ}{8.2^2 - 50.5}}$$

Give your answer correct to 3 significant figures.

**Solution**

$$\begin{aligned}\sqrt{\frac{2.5 \times \sin 43^\circ}{8.2^2 - 50.5}} &= 0.319\,141\,985\,5 \text{ (FCD)} \\ &= \underline{\underline{0.319}} \text{ (3 sf)}.\end{aligned}$$

8.  $ABC$  is a right-angled triangle.



Here is Sarah's method to find the length of  $BC$ .

$$\begin{aligned}BC^2 &= AB^2 + AC^2 \\ &= 6^2 + 8^2 \\ &= 100 \\ BC &= 10.\end{aligned}$$

(a) What mistake has Sarah made in her method?

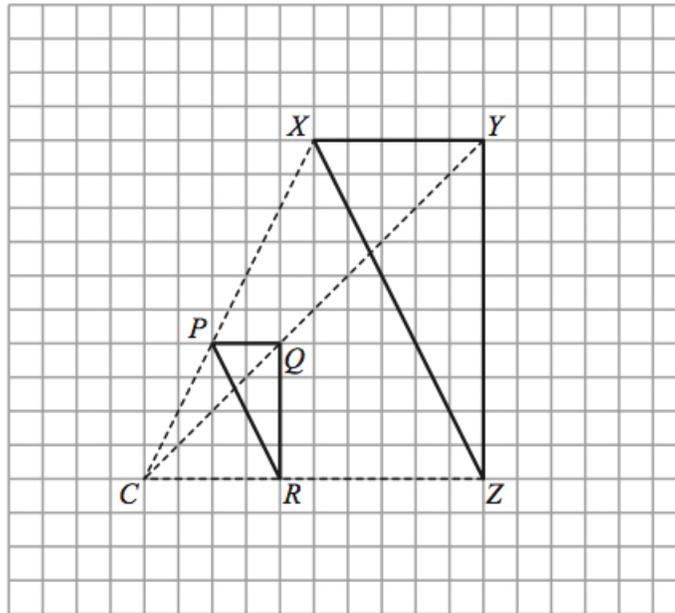
(1)

**Solution**

It should be

$$\underline{\underline{BC^2 = AB^2 - AC^2.}}$$

Roy is going to enlarge triangle  $PQR$  with centre  $C$  and scale factor  $1\frac{1}{2}$ .



He draws triangle  $XYZ$ .

- (b) Explain why Roy's diagram is **not** correct. (1)

**Solution**

It is enlarged by a factor of  $2\frac{1}{2}$ , not  $1\frac{1}{2}$ .

9. A company has to make a large number of boxes. (3)

The company has 6 machines.

All the machines work at the same rate.

When all the machines are working, they can make all the boxes in 9 days.

The table gives the number of machines working each day.

	Day 1	Day 2	Day 3	All other days
Number of machines working	3	4	5	6

Work out the total number of days taken to make all the boxes.

**Solution**

Let the total number of days taken to make all the boxes be  $x$ . Then,

$$9 \times 6 = 54$$

and

$$\begin{aligned} 3 + 4 + 5 + (6 \times x) &= 54 \Rightarrow 12 + 6x = 54 \\ &\Rightarrow 6x = 42 \\ &\Rightarrow x = 7. \end{aligned}$$

Hence, the total number of days taken to make all the boxes is

$$7 + 3 = \underline{10}.$$

10. Marie invests £8 000 in an account for one year. At the end of the year, interest is added to her account.

(3)

Marie pays tax on this interest at a rate of 20%. She pays £28.80 tax.

Work out the percentage interest rate for the account.

**Solution**

Let  $x$  be the amount the Marie pays before tax:

$$\begin{aligned} x \times \frac{20}{100} &= 28.80 \Rightarrow x = \frac{28.80 \times 100}{20} \\ &\Rightarrow x = 144 \end{aligned}$$

and so Marie end up with £8 144. Finally,

$$\frac{8\,144}{8\,000} = 1.018$$

and the percentage interest rate for the account is 1.8%.

11. In May 2019, the distance between Earth and Mars was  $3.9 \times 10^7$  km.

In May 2019, a signal was sent from Earth to Mars.  
Assuming that the signal sent from Earth to Mars travelled at a speed of  $3 \times 10^5$  km per second,

- (a) how long did the signal take to get to Mars? (2)

**Solution**

$$\frac{3.9 \times 10^7}{3 \times 10^5} = \underline{130 \text{ seconds}}.$$

The speed of the signal sent from Earth to Mars in May 2019 was actually less than  $3 \times 10^5$  km per second.

- (b) How will this affect your answer to part (a)? (1)

**Solution**

It will be longer than 130 seconds.

12. Patrick has to work out the exact value of  $64^{\frac{1}{4}}$ . (1)  
Patrick says, “ $\frac{1}{4}$  of 64 is 16 so  $64^{\frac{1}{4}} = 16$ .”  
Explain what is wrong with what Patrick says.

**Solution**

We want the fourth root of 64:

$$\sqrt[4]{64} = \sqrt{\sqrt{64}} = \sqrt{8} = 2\sqrt{2}.$$

13. The density of ethanol is  $1.09 \text{ g/cm}^3$ . (4)  
The density of propylene is  $0.97 \text{ g/cm}^3$ .

60 litres of ethanol are mixed with 128 litres of propylene to make 188 litres of antifreeze.

Work out the density of the antifreeze.

Give your answer correct to 2 decimal places.

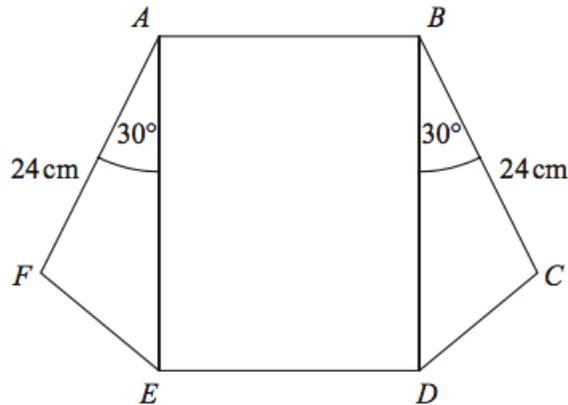
**Solution**

$$(60 \times 1.09) + (128 \times 0.97) = 65.4 + 124.16 \\ = 189.56$$

and the density of the antifreeze is

$$\frac{189.56}{188} = 1.008297872 \text{ (FCD)} \\ = \underline{\underline{1.01 \text{ g/cm}^3}} \text{ (2 dp).}$$

14. The diagram shows a rectangle,  $ABDE$ , and two congruent triangles,  $AFE$  and  $BCD$ . (4)



Area of rectangle  $ABDE$  = area of triangle  $AFE$  + area of triangle  $BCD$ .

$AB : AE = 1 : 3$ .

Work out the length of  $AE$ .

**Solution**

$$AB \times AE = 2 \times \frac{1}{2} \times 24 \times AE \times \sin 30^\circ \Rightarrow AB = 2 \times \frac{1}{2} \times 24 \times \sin 30^\circ \\ \Rightarrow AB = 12$$

and, hence,

$$AE = 3 \times 12 = \underline{\underline{36 \text{ cm}}}.$$

15. The graph of the curve  $C$  with equation  $y = f(x)$  is transformed to give the graph of the curve  $S$  with equation  $y = f(-x) - 3$ . (2)

The point on  $C$  with coordinates  $(7, 2)$  is mapped to the point  $Q$  on  $S$ .

Find the coordinates of  $Q$ .

**Solution**

$$y = f(-x): (-7, 2).$$

$$y = f(-x) - 3: \underline{\underline{(-7, -1)}}.$$

16. Here are the first six terms of a quadratic sequence.

(3)

$$-1 \quad 5 \quad 15 \quad 29 \quad 47 \quad 69$$

Find an expression, in terms of  $n$ , for the  $n$ th term of this sequence.

**Solution**

Let the

$$n\text{th term} = an^2 + bn + c.$$

We only need the second line of differences (why?):

$$\begin{array}{ccccccc}
 & & -1 & & 5 & & 15 \\
 & & & 6 & & 10 & \\
 & & & & 4 & & \\
 a + b + c & & 4a + 2b + c & & 9a + 3b + c & & \\
 3a + b & & 5a + b & & & & \\
 & & 2a & & & & 
 \end{array}$$

We compare terms:

$$2a = 4 \Rightarrow a = 2,$$

$$3a + b = 6 \Rightarrow 3 \times 2 + b = 6$$

$$\Rightarrow b = 0,$$

and

$$a + b + c = -1 \Rightarrow 2 + 0 + c = -1$$

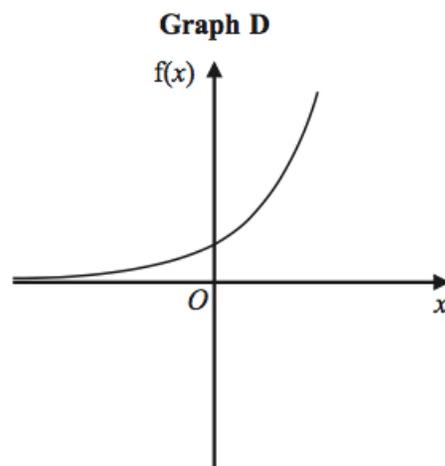
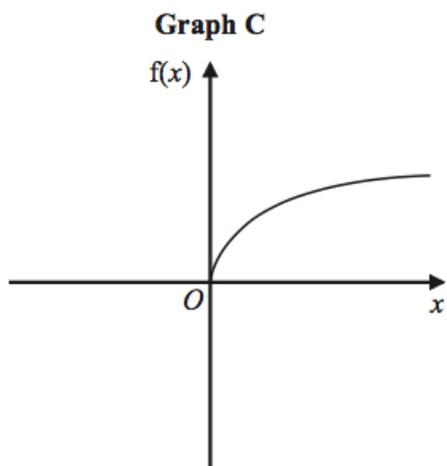
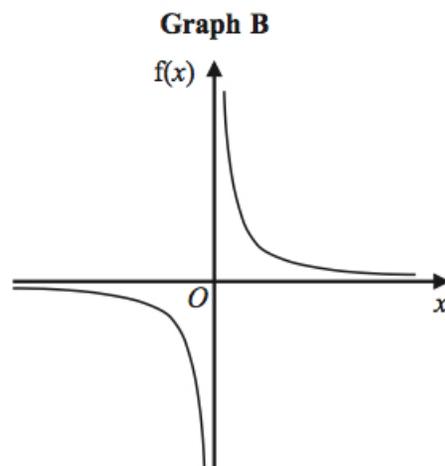
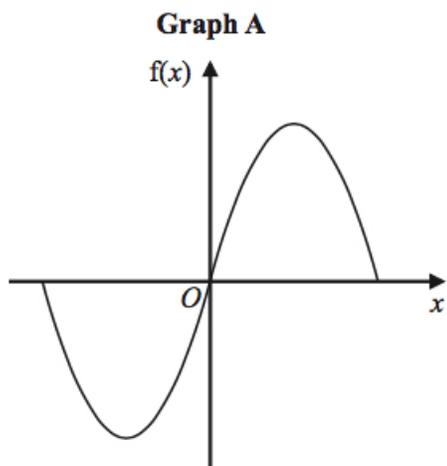
$$\Rightarrow c = -3;$$

hence,

$$n\text{th term} = \underline{\underline{2n^2 - 3}}.$$

17. Here are four graphs.

(2)



The graphs represent four different types of function  $f$ .

Match each description of the function in the table to the letter of its graph.

Description of function	Graph
$f(x)$ is inversely proportional to $x$	
$f(x)$ is a trigonometrical function	
$f(x)$ is an exponential function	
$f(x)$ is directly proportional to $\sqrt{x}$	

**Solution**

Description of function	Graph
$f(x)$ is inversely proportional to $x$	<u><u>B</u></u>
$f(x)$ is a trigonometrical function	<u><u>A</u></u>
$f(x)$ is an exponential function	<u><u>D</u></u>
$f(x)$ is directly proportional to $\sqrt{x}$	<u><u>C</u></u>

18. (a) Show that

$$(2x + 1)(x + 3)(3x + 7)$$

(3)

can be written in the form

$$ax^3 + bx^2 + cx + d,$$

where  $a, b, c,$  and  $d$  are integers.

**Solution**

$\times$	$2x$	$+1$
$x$	$2x^2$	$+x$
$+3$	$+6x$	$+3$

$$(2x + 1)(x + 3) = 2x^2 + 7x + 3.$$

$\times$	$2x^2$	$+7x$	$+3$
$3x$	$6x^3$	$+21x^2$	$+9x$
$+7$	$+14x^2$	$+49x$	$+21$

So

$$(2x + 1)(x + 3)(3x + 7) = \underline{\underline{6x^3 + 35x^2 + 58x + 21}};$$

hence,  $a = 6$ ,  $b = 35$ ,  $c = 58$ , and  $d = 21$ .

(b) Solve

$$(1 - x)^2 < \frac{9}{25}.$$

(3)

**Solution**

$$\begin{aligned}(1-x)^2 < \frac{9}{25} &\Rightarrow -\frac{3}{5} < 1-x < \frac{3}{5} \\ &\Rightarrow \frac{3}{5} > x-1 > -\frac{3}{5} \\ &\Rightarrow \underline{\underline{\frac{13}{5} > x > \frac{2}{5}}}\end{aligned}$$

19.

$$D = \frac{u^2}{2a}.$$

$u = 26.2$ , correct to 3 significant figures.

$a = 4.3$ , correct to 2 significant figures

- (a) Calculate the upper bound for the value of  $D$ . (3)  
Give your answer correct to 6 significant figures.  
You must show all your working.

**Solution**

$$26.15 \leq u < 26.25$$

and

$$4.25 \leq a < 4.35.$$

So, the upper bound for the value of  $D$  is

$$\begin{aligned}\frac{26.25^2}{2 \times 4.25} &= 81.066\,176\,47 \text{ (FCD)} \\ &= \underline{\underline{81.066\,2 \text{ (6 sf)}}}\end{aligned}$$

The lower bound for the value of  $D$  is 78.600 3 correct to 6 significant figures.

- (b) By considering bounds, write down the value of  $D$  to a suitable degree of accuracy. (2)  
You must give a reason for your answer.

**Solution**

	Lower bound	Upper bound	Agree?
1 sf	80	80	Yes
2 sf	79	81	No

The value of  $D$  is 80 because the lower bound and upper bound agree to 1 significant figure but not to 2 significant figures.

20. Solve algebraically the simultaneous equations:

(5)

$$x^2 - 4y^2 = 9$$

$$3x + 4y = 7.$$

**Solution**

$$3x + 4y = 7 \Rightarrow 4y = 7 - 3x$$

$$\Rightarrow y = \frac{1}{4}(7 - 3x)$$

and substitute into quadratic equation:

$$x^2 - 4\left[\frac{1}{4}(7 - 3x)\right]^2 = 9 \Rightarrow x^2 - \frac{1}{4}(7 - 3x)^2 = 9$$

×	7	-3x
7	49	-21x
-3x	-21x	+9x <sup>2</sup>

$$\Rightarrow x^2 - \frac{1}{4}(49 - 42x + 9x^2) = 9$$

$$\Rightarrow 4x^2 - (49 - 42x + 9x^2) = 36$$

$$\Rightarrow 5x^2 - 42x + 85 = 0$$

$$\left. \begin{array}{l} \text{add to:} \qquad \qquad \qquad -42 \\ \text{multiply to: } (+5) \times (+85) = +425 \end{array} \right\} -25, -17$$

$$\Rightarrow 5x^2 - 25x - 17x + 85 = 0$$

$$\Rightarrow 5x(x - 5) - 17(x - 5) = 0$$

$$\Rightarrow (5x - 17)(x - 5) = 0$$

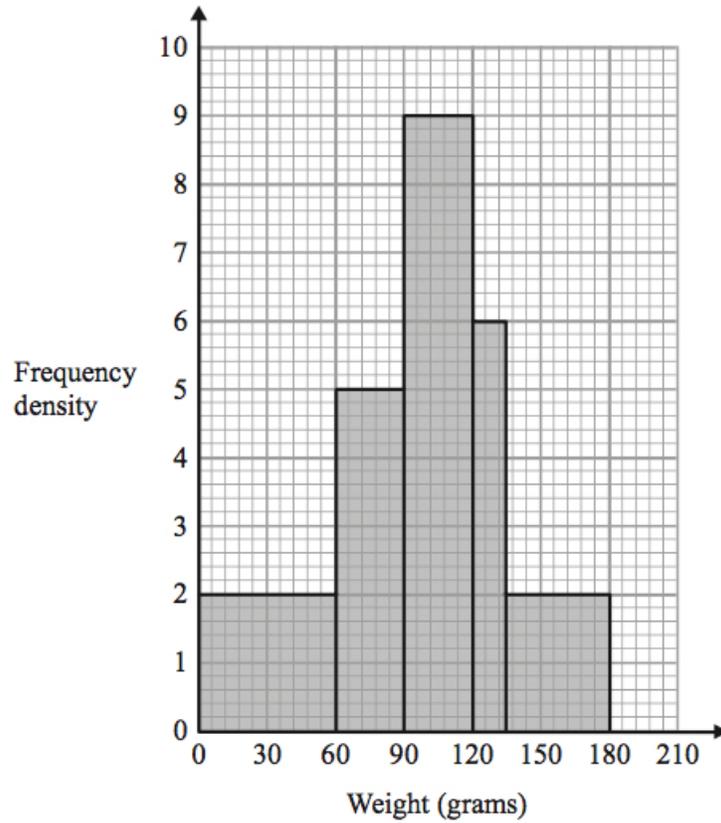
$$\Rightarrow x = 3\frac{2}{5} \text{ or } x = 5$$

$$\Rightarrow y = -\frac{4}{5} \text{ or } y = -2;$$

hence,  $x = 3\frac{2}{5}, y = -\frac{4}{5}$  or  $x = 5, y = -2$ .

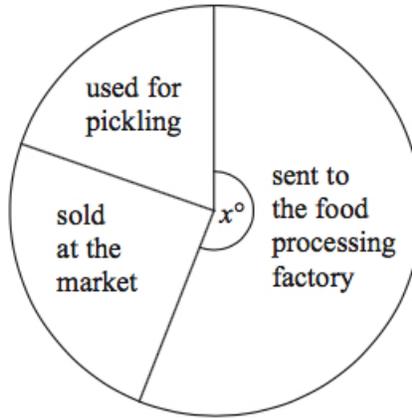
21. The histogram gives information about the distribution of the weights of some onions grown by a farmer.

(4)



Onions less than 60 grams in weight are used for pickling.  
Onions greater than 120 grams in weight are sold at the market.  
The rest of the onions are sent to a food processing factory.

A pie chart is drawn using the information opposite to show what the farmer does with the onions he grows.



The angle of the sector for the onions sent to the food processing factory is  $x^\circ$ .

Work out the value of  $x$ .

### Solution

Weight	Frequency	Width	Frequency Density
$0 < w \leq 60$	120	60	$\frac{120}{60} = 2$
$60 < w \leq 90$	150	30	$\frac{150}{30} = 5$
$90 < w \leq 120$	270	30	$\frac{270}{30} = 9$
$120 < w \leq 135$	90	15	$\frac{90}{15} = 6$
$135 < w \leq 180$	90	45	$\frac{90}{45} = 2$

Now,

$$120 + 150 + 270 + 90 + 90 = 720.$$

Next, the angle of the sector used for pickling is

$$\frac{120}{720} \times 360 = 60$$

and the angle of the sector sold at the market is

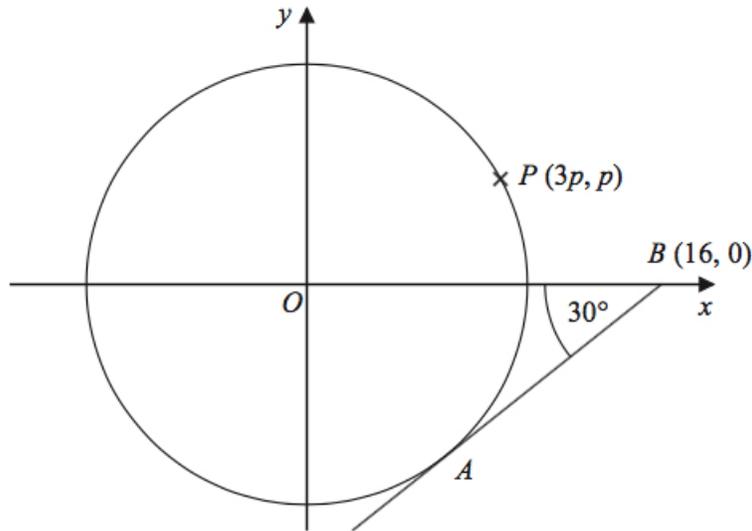
$$\frac{(90 + 90)}{720} \times 360 = 90.$$

Finally, angle of the sector for the onions sent to the food processing factory is

$$360 - 60 - 90 = \underline{\underline{210}}.$$

22. The diagram shows a circle, centre  $O$ .

(4)



$AB$  is the tangent to the circle at the point  $A$ .  
Angle  $OBA = 30^\circ$ .

Point  $B$  has coordinates  $(16, 0)$ .

Point  $P$  has coordinates  $(3p, p)$ .

Find the value of  $p$ .

Give your answer correct to 1 decimal place.

You must show all your working.

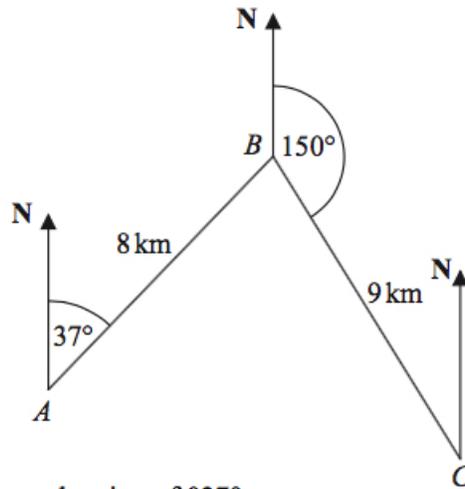
### Solution

$$\begin{aligned}\sin &= \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin 30^\circ = \frac{OA}{16} \\ &\Rightarrow OA = 16 \sin 30^\circ \\ &\Rightarrow OA = 8.\end{aligned}$$

Now,

$$\begin{aligned}(3p)^2 + p^2 &= 8^2 \Rightarrow 10p^2 = 64 \\ &\Rightarrow p^2 = 6.4 \\ &\Rightarrow p = 2.529\ 822\ 128 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{p = 2.5 \text{ (1 dp)}}}.\end{aligned}$$

23. The diagram shows the positions of three towns, Acton ( $A$ ), Barston ( $B$ ), and Chorlton ( $C$ ). (5)



Barston is 8 km from Acton on a bearing of 037°.  
Chorlton is 9 km from Barston on a bearing of 150°.

Find the bearing of Chorlton from Acton.

Give your answer correct to 1 decimal place.

You must show all your working.

### Solution

$$\overrightarrow{AB} = \begin{pmatrix} 8 \sin 37^\circ \\ 8 \cos 37^\circ \end{pmatrix}$$

and

$$\overrightarrow{BC} = \begin{pmatrix} 9 \sin 30^\circ \\ -9 \cos 30^\circ \end{pmatrix}.$$

Now,

$$\overrightarrow{AC} = \begin{pmatrix} 8 \sin 37^\circ + 9 \sin 30^\circ \\ 8 \cos 37^\circ - 9 \cos 30^\circ \end{pmatrix}.$$

Now,  $8 \cos 37^\circ - 9 \cos 30^\circ < 0$  and

$$90 + \tan^{-1} \left( \frac{-8 \cos 37^\circ + 9 \cos 30^\circ}{8 \sin 37^\circ + 9 \sin 30^\circ} \right) = 98.578\ 685\ 21 \text{ (FCD)}$$

and, hence, the bearing of Chorlton from Acton is

$$\underline{\underline{098.6^\circ \text{ (1 dp)}}}.$$