Dr Oliver Mathematics
GCSE Mathematics
2012 June Paper 1H: Non-Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Sam wants to find out the types of film people like best.
   He is going to ask whether they like comedy films or action films or science fiction films or musicals best.

   (a) Design a suitable table for a data collection sheet he could use to collect this information. (2)

   > **Solution**
   >
   > A suitable question with a time frame, e.g., "Did you watch any movies last night/last week/last month? Tick the appropriate box."
   > At least six exhaustive and non-overlapping tick boxes: for example, Comedy, Action, Science Fiction, Musicals, Other, Don't Watch Movies.
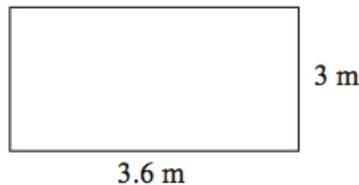   > Your table must include tally marks and frequency, totals, etc.

   Sam collects his data by asking 10 students in his class at school.
   This might **not** be a good way to find out the types of film people like best.

   (b) Give **one** reason why. (1)

   > **Solution**
   > E.g., it is a small sample, it is susceptible to bias, they are all at the same school.

2. The diagram shows a patio in the shape of a rectangle.

   The patio is 3.6 m long and 3 m wide.
   Matthew is going to cover the patio with paving slabs.
   Each paving slab is a square of side 60 cm.
   Matthew buys 32 of the paving slabs.

(a) Does Matthew buy enough paving slabs to cover the patio? (3)
You must show all your working.

> **Solution**
> $$\frac{360}{60} = 6 \text{ and } \frac{300}{60} = 5$$
> and
> $$5 \times 6 = 30;$$
> so, <u>yes</u>, Matthew buys enough.

The paving slabs cost £8.63 each.

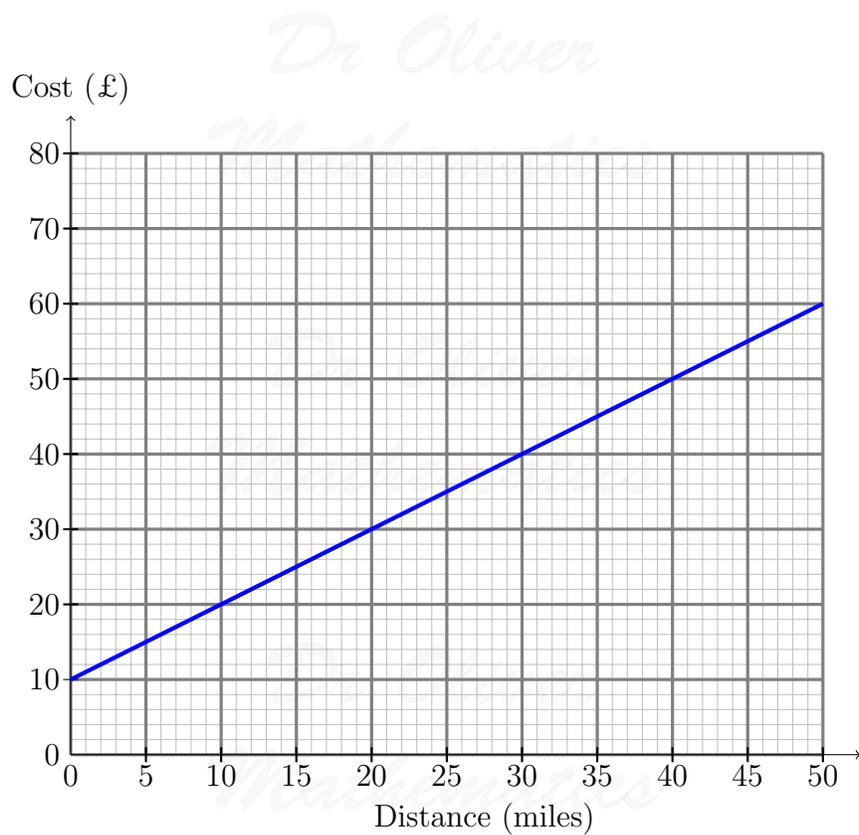(b) Work out the total cost of the 32 paving slabs. (3)

> **Solution**
>
> | × | 30 | 2 |
> |------|-----|------|
> | 8 | 240 | 16 |
> | 0.6 | 18 | 1.2 |
> | 0.03 | 0.9 | 0.06 |
>
> The cost is
> $$240 + 16 + 18 + 1.2 + 0.9 + 0.06 = 240 + 34 + 1.2 + 0.96$$
> $$= 274 + 2.16$$
> $$= \underline{\underline{£276.16}}.$$

3. Bill uses his van to deliver parcels.
   For each parcel Bill delivers there is a fixed charge plus £1.00 for each mile.
   You can use the graph to find the total cost of having a parcel delivered by Bill.

Cost (£) vs Distance (miles)

(a) How much is the fixed charge? (1)

> **Solution**
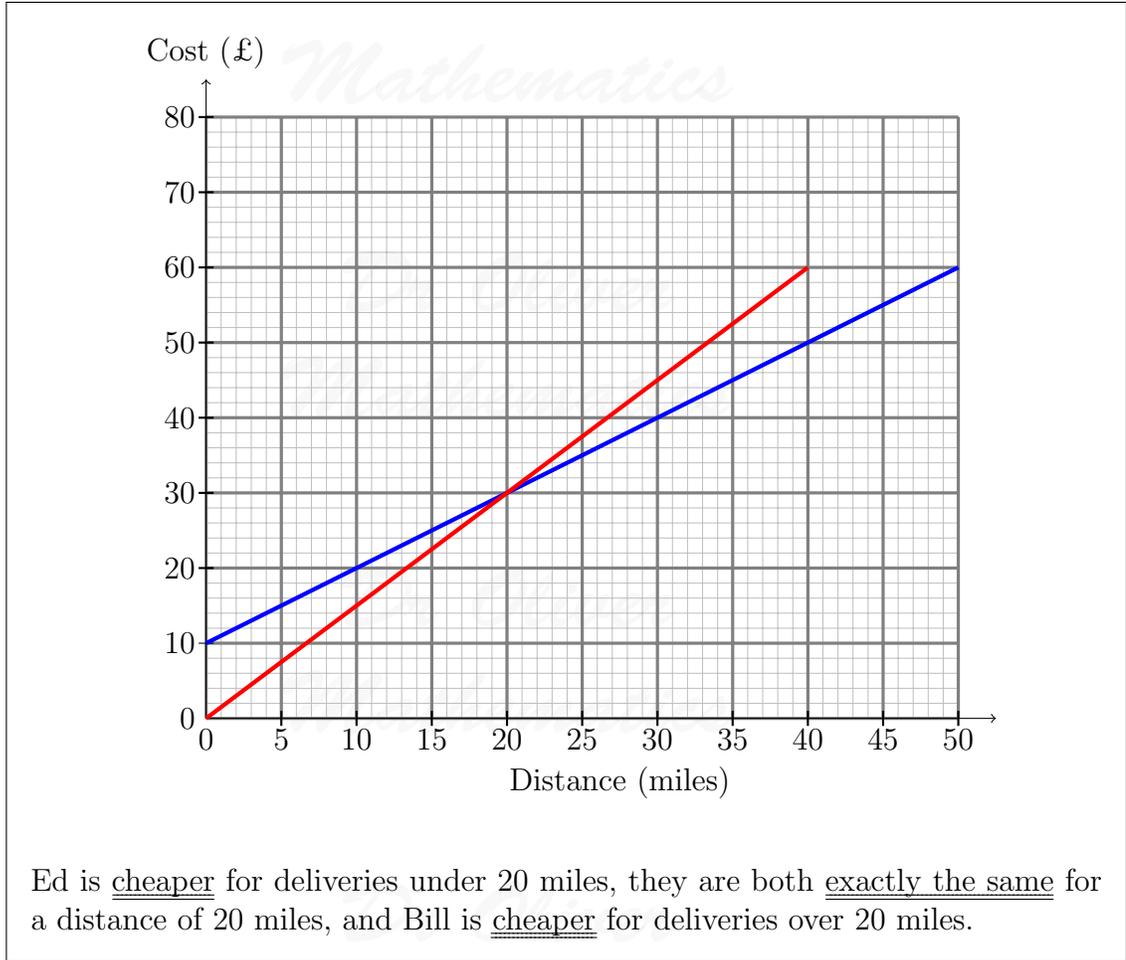> £10.

Ed uses a van to deliver parcels.
For each parcel Ed delivers it costs £1.50 for each mile.
There is **no** fixed charge.

(b) Compare the cost of having a parcel delivered by Bill with the cost of having a parcel delivered by Ed. (3)

> **Solution**

Ed is <u>cheaper</u> for deliveries under 20 miles, they are both <u>exactly the same</u> for a distance of 20 miles, and Bill is <u>cheaper</u> for deliveries over 20 miles.

4. Here are the speeds, in miles per hour, of 16 cars. (3)

$$31 \quad 52 \quad 43 \quad 49 \quad 36 \quad 35 \quad 33 \quad 29$$
$$54 \quad 43 \quad 44 \quad 46 \quad 42 \quad 39 \quad 55 \quad 48$$

Draw an ordered stem and leaf diagram for these speeds.

Solution

$$
\begin{array}{c|ccccccc}
2 & 9 \\
3 & 1 & 3 & 5 & 6 & 9 \\
4 & 2 & 3 & 3 & 4 & 6 & 8 & 9 \\
5 & 2 & 4 & 5 \\
\end{array}
$$

Key: 2|9 means 29 mph.

5. You can work out the amount of medicine, c ml, to give to a child by using the formula (2)

$$c = \frac{ma}{150}.$$

m is the age of the child, in months.
a is an adult dose, in ml.
A child is 30 months old.
An adult's dose is 40 ml.
Work out the amount of medicine you can give to the child.

Solution

$$c = \frac{30 \times 40}{150}$$
$$= \frac{1\,200}{150}$$
$$= \underline{\underline{8 \text{ ml}}}.$$

6. Here are the ingredients needed to make 12 shortcakes.

Shortcakes

Makes **12** shortcakes

50 g of sugar
200 g of butter
200 g of flour
10 ml of milk

Liz makes some shortcakes.
She uses 25 ml of milk.

(a) How many shortcakes does Liz make? (2)

Solution

$$\frac{25}{10} \times 12 = 2.5 \times 12 = \underline{\underline{30 \text{ cookies}}}.$$

Robert has 500 g of sugar, 1 000 g of butter, 1 000 g of flour, and 500 ml of milk.

(b) Work out the greatest number of shortcakes Robert can make. (2)

Solution

Sugar:
$$\frac{500}{50} = 10.$$

Butter:
$$\frac{1\,000}{200} = 5.$$

Flour:
$$\frac{1\,000}{200} = 5.$$

Milk:
$$\frac{500}{10} = 50.$$

He can make
$$5 \times 12 = \underline{\underline{60 \text{ cookies}}}.$$

7. Buses to Acton leave a bus station every 24 minutes. (3)
Buses to Barton leave the same bus station every 20 minutes.
A bus to Acton and a bus to Barton both leave the bus station at 9:00 am.
When will a bus to Acton and a bus to Barton next leave the bus station at the same time?

Solution

	24
2	12
2	6
2	3
3	1

and so
$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3.$$

	20
2	10
2	5
5	1

and so
$$20 = 2 \times 2 \times 5 = 2^2 \times 5.$$
Hence,
$$\text{LCM}(24,\ 20) = 2^3 \times 3 \times 5 = 120$$
and so it will be $\underline{\underline{11:00\ \text{am}}}$ that they leave the bus station at the same time.

8. (a) Expand $3(2y-5)$. (1)

> **Solution**
> $$3(2y-5) = \underline{\underline{6y-15}}.$$

(b) Factorise completely (2)
$$8x^2 + 4xy.$$

> **Solution**
> $$8x^2 + 4xy = \underline{\underline{4x(2x+y)}}.$$
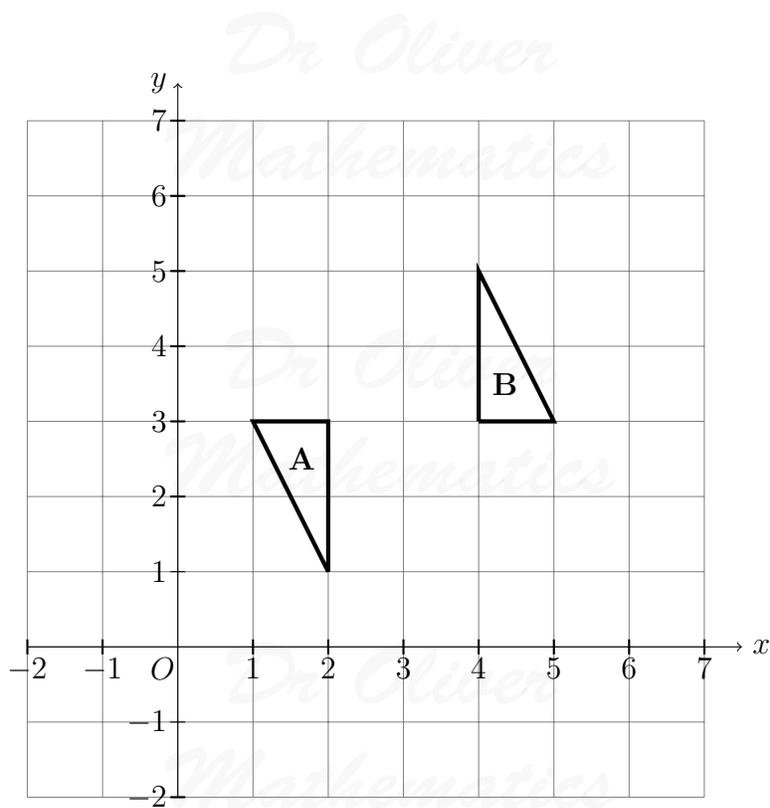
(c) Make h the subject of the formula (2)
$$t = \frac{gh}{10}.$$

> **Solution**
> $$t = \frac{gh}{10} \Rightarrow 10t = gh$$
> $$\Rightarrow h = \underline{\underline{\frac{10t}{g}}}.$$

9. Describe fully the single transformation that maps triangle **A** onto triangle **B**. (3)

Solution
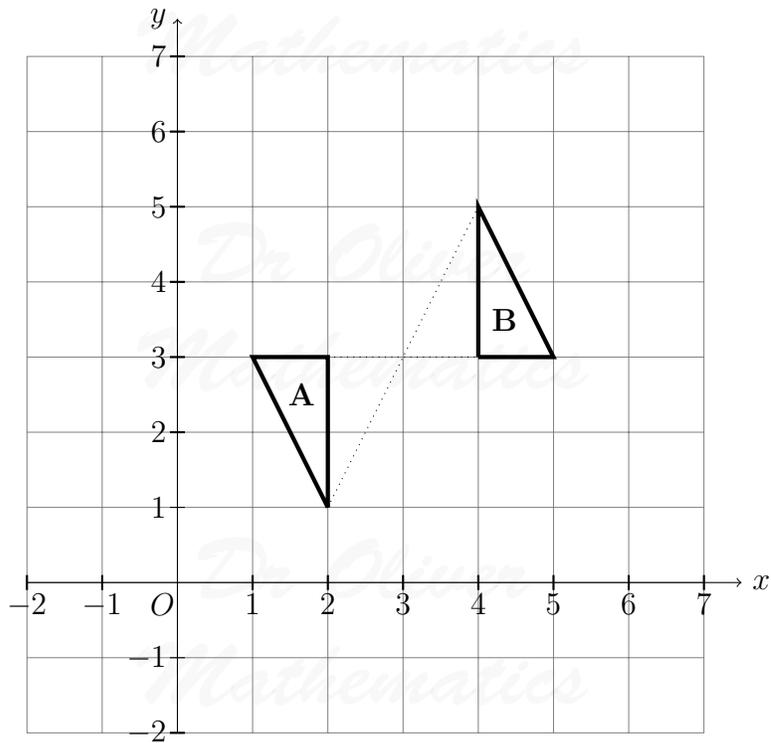
Hence, it is a <u>rotation</u>, centre $(3, 3)$, about <u>180°</u>.

10. Railtickets and Cheaptrains are two websites selling train tickets.   (4)
    Each of the websites adds a credit card charge and a booking fee to the ticket price.

Railtickets

Credit card charge: 2.25% of ticket price

Booking fee: 80 pence

Cheaptrains

Credit card charge: 1.5% of ticket price

Booking fee: £1.90

Nadia wants to buy a train ticket.
The ticket price is £60 on each website.
Nadia will pay by credit card.
Will it be cheaper for Nadia to buy the train ticket from Railtickets or from Cheaptrains?

Solution

Railtickets:

×	1	0.02	0.002	0.0005
60	60	1.2	0.12	0.03

The final cost is

$$60 \times 1.0225 + 0.8 = 61.35 + 0.8$$
$$= \pounds 62.15.$$

Cheaptrains:

×	1	0.01	0.005
60	60	0.6	0.3

The final cost is

$$60 \times 1.015 + 1.9 = 60.9 + 1.9$$
$$= \pounds 62.80.$$

Hence, it will be cheaper by <u>Railtickets</u>.
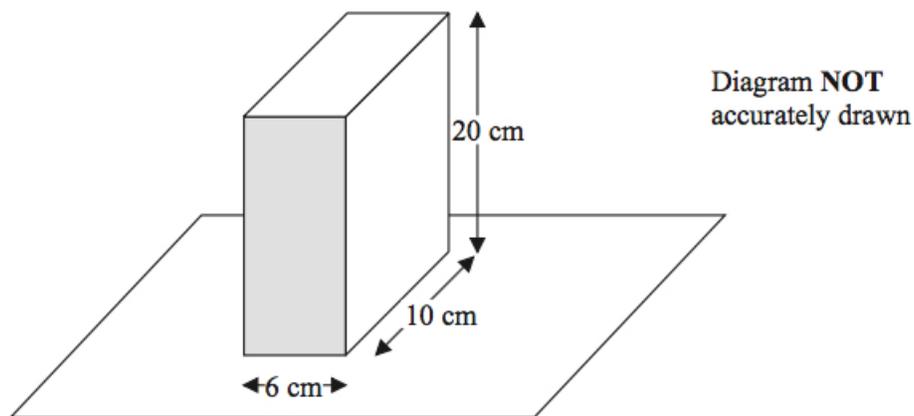
11. The diagram shows a parallelogram. (3)
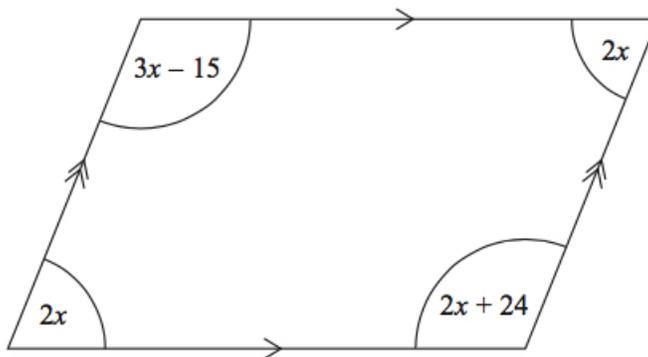
The sizes of the angles, in degrees, are $2x$, $(3x - 15)$, $2x$, and $(2x + 24)$.
Work out the value of x.

Solution

$$2x + (3x - 15) + 2x + (2x + 24) = 360 \Rightarrow 9x + 9 = 360$$
$$\Rightarrow 9x = 351$$
$$\Rightarrow \underline{\underline{x = 39}}.$$

12. Jane has a carton of orange juice. (3)
    The carton is in the shape of a cuboid.

Diagram **NOT** accurately drawn

20 cm

10 cm

6 cm

The depth of the orange juice in the carton is 8 cm.
Jane closes the carton.
Then she turns the carton over so that it stands on the shaded face.
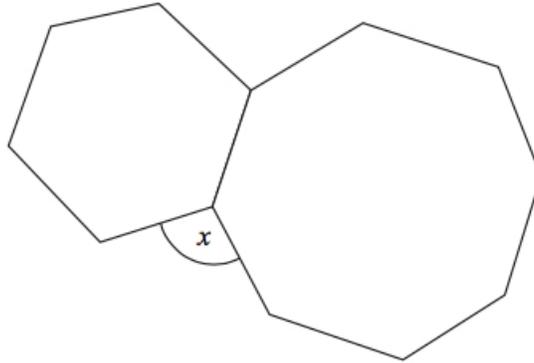Work out the depth, in cm, of the orange juice now.

Solution

$$\text{Volume} = 6 \times 10 \times 8$$
$$= 6 \times 80$$
$$= 480 \text{ cm}^3.$$

Now, let us say that depth of the orange juice in the carton is x cm:

$$6 \times 20 \times x = 480 \Rightarrow 120x = 480$$
$$\Rightarrow \underline{\underline{x = 4}}.$$

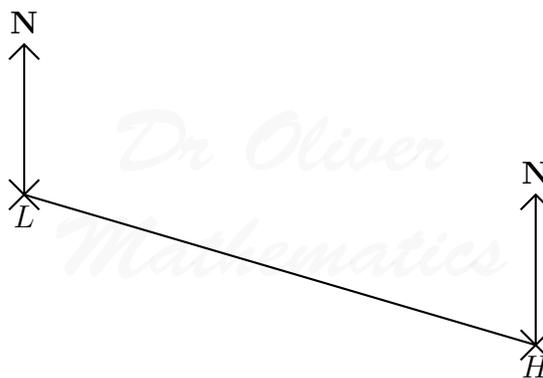13. The diagram shows a regular hexagon and a regular octagon. (4)

Calculate the size of the angle marked x.
You must show all your working.

Solution

The angle in a regular hexagon is $120°$ (why?) and the angle in a regular octagon is $135°$ (why?). Finally,

$$x + 120 + 135 = 360 \Rightarrow x + 255 = 360$$
$$\Rightarrow \underline{\underline{x = 105}}.$$

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14. The diagram shows the position of a lighthouse L and a harbour H.

The scale of the diagram is 1 cm represents 5 km.

(a) Work out the real distance between L and H. (1)

Solution

$$5 \times 7 = \underline{\underline{35 \text{ km}}}.$$

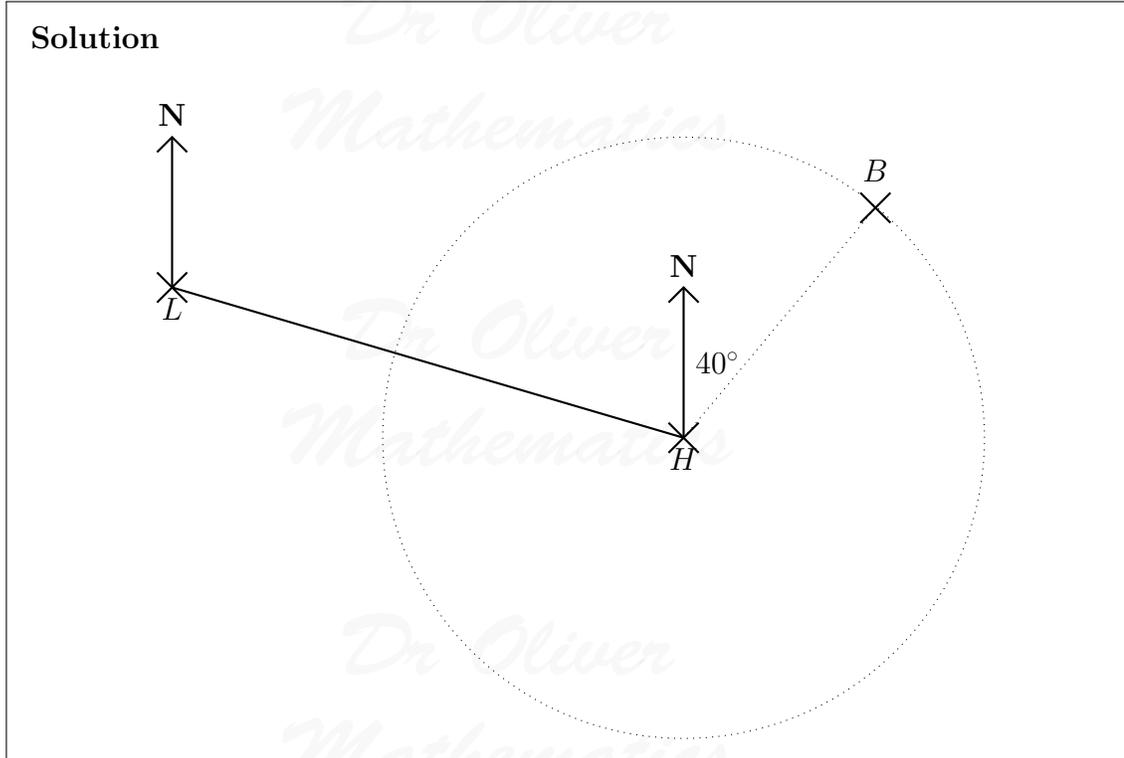(b) Measure the bearing of H from L. (1)

Solution

$$90 + 16 = \underline{\underline{106°}}.$$

A boat B is 20 km from H on a bearing of $040°$.

(c) On the diagram, mark the position of boat B with a cross ($\times$). (2)
Label it B.

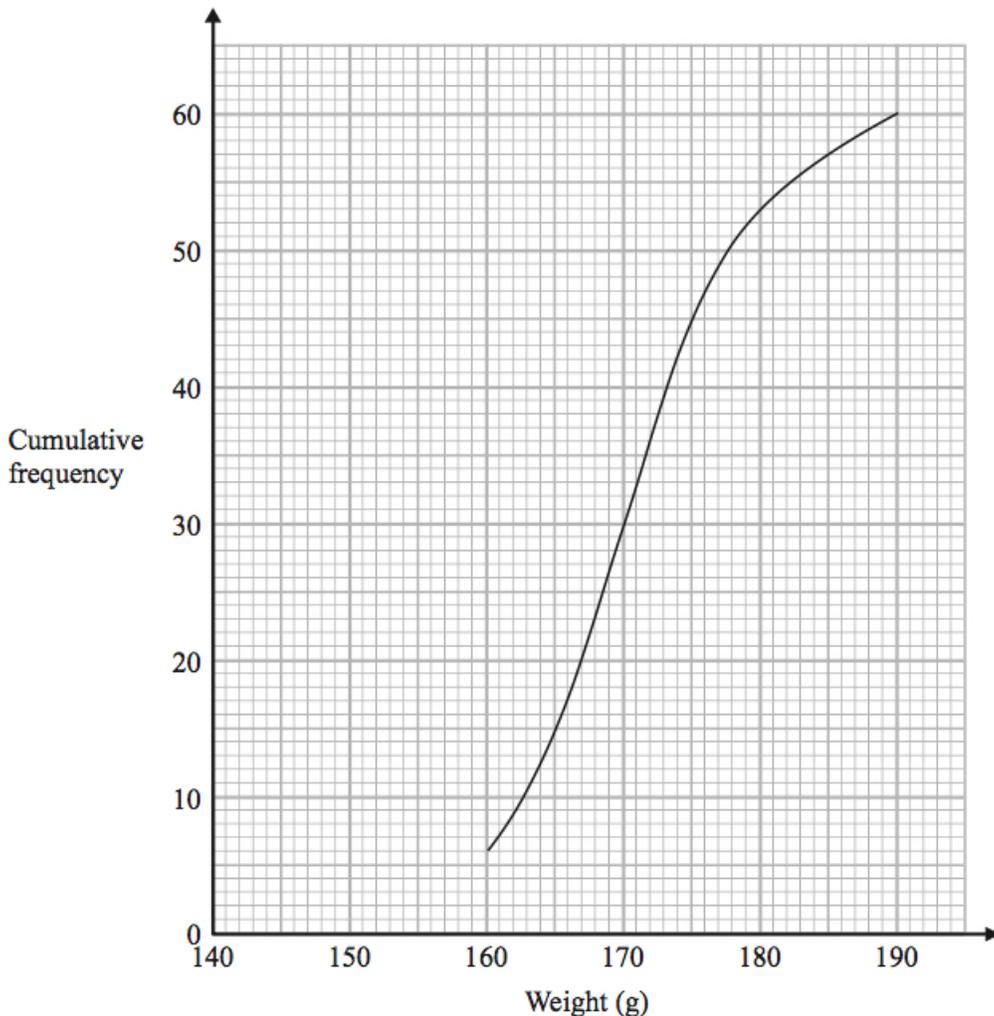**Solution**

15. Harry grows tomatoes.
This year he put his tomato plants into two groups, group A and group B.
Harry gave fertiliser to the tomato plants in group A.
He did not give fertiliser to the tomato plants in group B.
Harry weighed 60 tomatoes from group A.
The cumulative frequency graph shows some information about these weights.

13

(a) Use the graph to find an estimate for the median weight. (1)

Solution

170 g.

The 60 tomatoes from group A had a minimum weight of 153 grams and a maximum weight of 186 grams.

(b) Use this information and the cumulative frequency graph to draw a box plot for the (3)
60 tomatoes from group A.

Group A

140   150   160   170   180   190

Solution
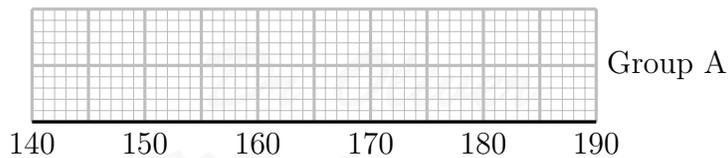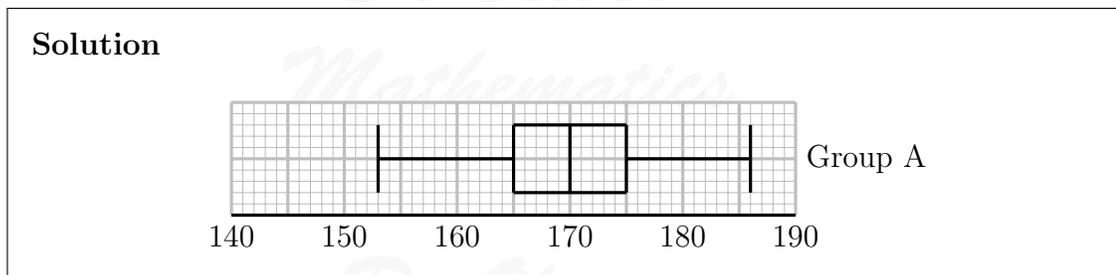
Group A

Harry did not give fertiliser to the tomato plants in group B.
Harry weighed 60 tomatoes from group B.
He drew this box plot for his results.
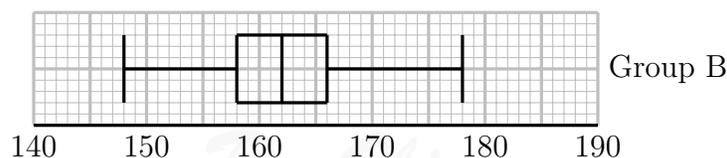
Group B

(c) Compare the distribution of the weights of the tomatoes from group A with the distribution of the weights of the tomatoes from group B. (2)

Solution

Average

Since the median for group A (170) is higher than the median for group B (163), group A weigh more on average.

Spread

Since the range for group B ($178 - 148 = 30$) is smaller than the range for group A ($186 - 153 = 33$), the marks were more consistent in group B.

OR

Since the IQR for group B ($166 - 158 = 8$) is smaller than the IQR for group A ($175 - 165 = 10$), the marks were more consistent in group B.

16. (a) Simplify (1)
$$(m^{-2})^5.$$

Solution
$$(m^{-2})^5 = \underline{\underline{m^{-10}}}.$$

15

(b) Factorise (2)

$$x^2 + 3x - 10.$$

Solution

$$\left.\begin{array}{ll} \text{add to:} & +3 \\ \text{multiply to:} & -10 \end{array}\right\} + 5, -2$$

$$x^2 + 3x - 10 = \underline{\underline{(x + 5)(x - 2)}}.$$

17. (a) Write down the value of 10^0. (1)

Solution

$$10^0 = \underline{\underline{1}}.$$

(b) Write 6.7×10^{-5} as an ordinary number. (1)

Solution

$$6.7 \times 10^{-5} = \underline{\underline{0.000\,067}}.$$

(c) Work out the value of (2)
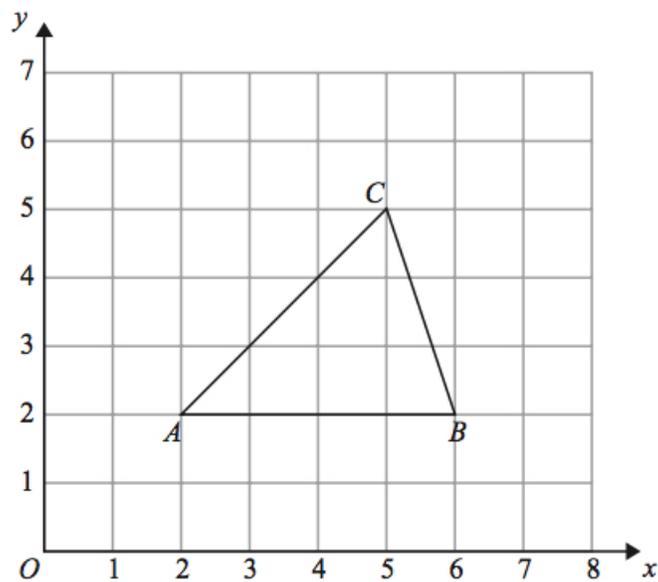
$$(3 \times 10^7) \times (9 \times 10^6).$$

Give your answer in standard form.

Solution

$$(3 \times 10^7) \times (9 \times 10^6) = (3 \times 9) \times (10^7 \times 10^6)$$
$$= 27 \times 10^{13}$$
$$= \underline{\underline{2.7 \times 10^{14}}}.$$

18. Triangle ABC is drawn on a centimetre grid. (3)

A is the point $(2, 2)$.
B is the point $(6, 2)$.
C is the point $(5, 5)$.
Triangle PQR is an enlargement of triangle ABC with scale factor $\frac{1}{2}$ and centre $(0, 0)$.
Work out the area of triangle PQR.

Solution

$$\begin{aligned}
\text{Area} &= \tfrac{1}{2} \times \left(\tfrac{1}{2} \times 4 \times 3\right) \\
&= \tfrac{1}{4} \times 6 \\
&= 1\tfrac{1}{2} \ \text{cm}^2.
\end{aligned}$$

19. Wendy goes to a fun fair.
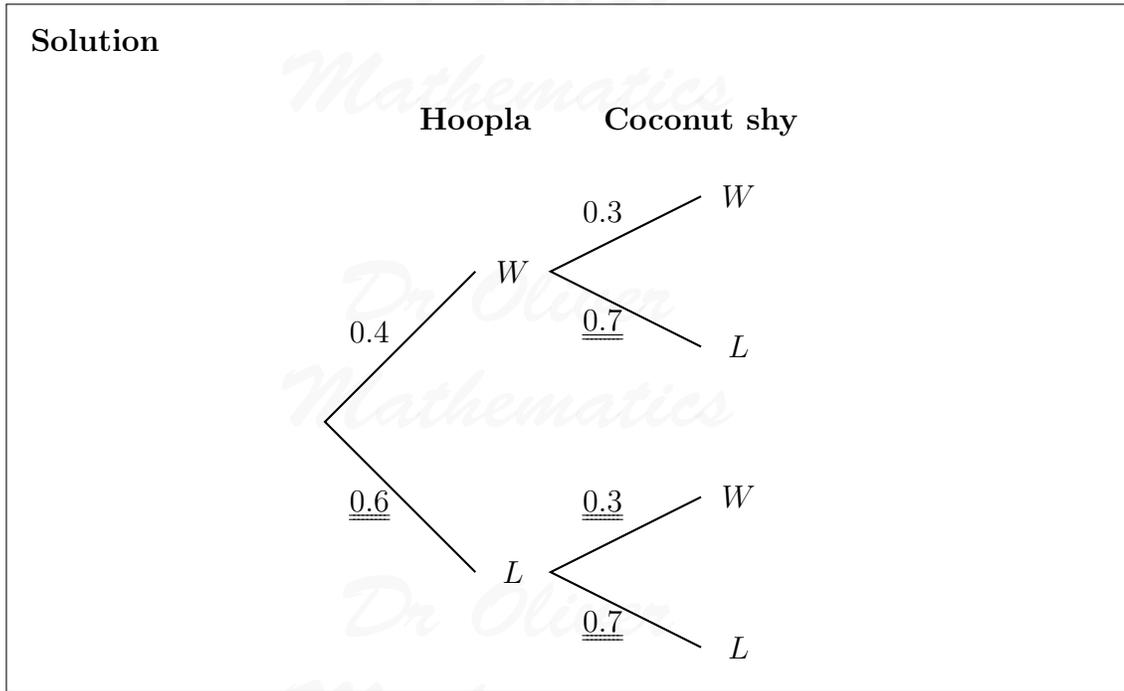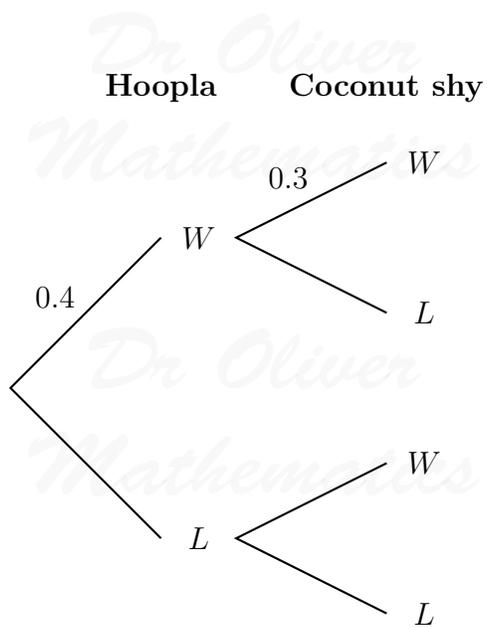   She has one go at Hoopla.
    She has one go on the Coconut shy.
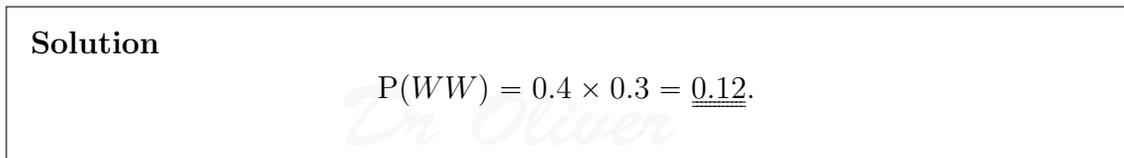   The probability that she wins at Hoopla is 0.4.
    The probability that she wins on the Coconut shy is 0.3.

    (a) Complete the probability tree diagram. (2)

17

Hoopla   Coconut shy

0.3 —— W

W

0.4

L —— W

L —— L

Solution

Hoopla   Coconut shy

0.3 —— W

W

0.4 $\underline{0.7}$ —— L

$\underline{0.6}$ $\underline{0.3}$ —— W

L

$\underline{0.7}$ —— L

(b) Work out the probability that Wendy wins at Hoopla and also wins on the Coconut shy. (2)

Solution

$$\text{P}(WW) = 0.4 \times 0.3 = \underline{0.12}.$$

20. Solve the simultaneous equations (4)

$$5x + 2y = 11$$
$$4x - 3y = 18.$$

Solution

$$5x + 2y = 11 \quad (1)$$
$$4x - 3y = 18 \quad (2)$$

$$4 \times (1): \quad 20x + 8y = 44 \quad (3)$$
$$5 \times (2): \quad 20x - 15y = 90 \quad (4)$$

$(3) - (4)$:

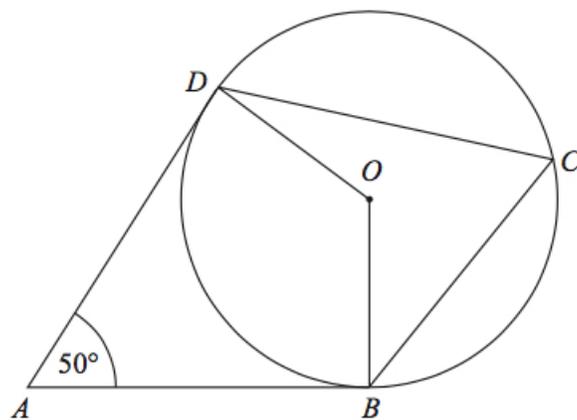$$23y = -46 \Rightarrow \underline{\underline{y = -2}}$$
$$\Rightarrow 5x - 4 = 11$$
$$\Rightarrow 5x = 15$$
$$\Rightarrow \underline{\underline{x = 3}}.$$

21. B, C, and D are points on the circumference of a circle, centre O. (4)

AB and AD are tangents to the circle.
Angle $DAB = 50°$.

Work out the size of angle BCD.
Give a reason for each stage in your working.

> **Solution**
>
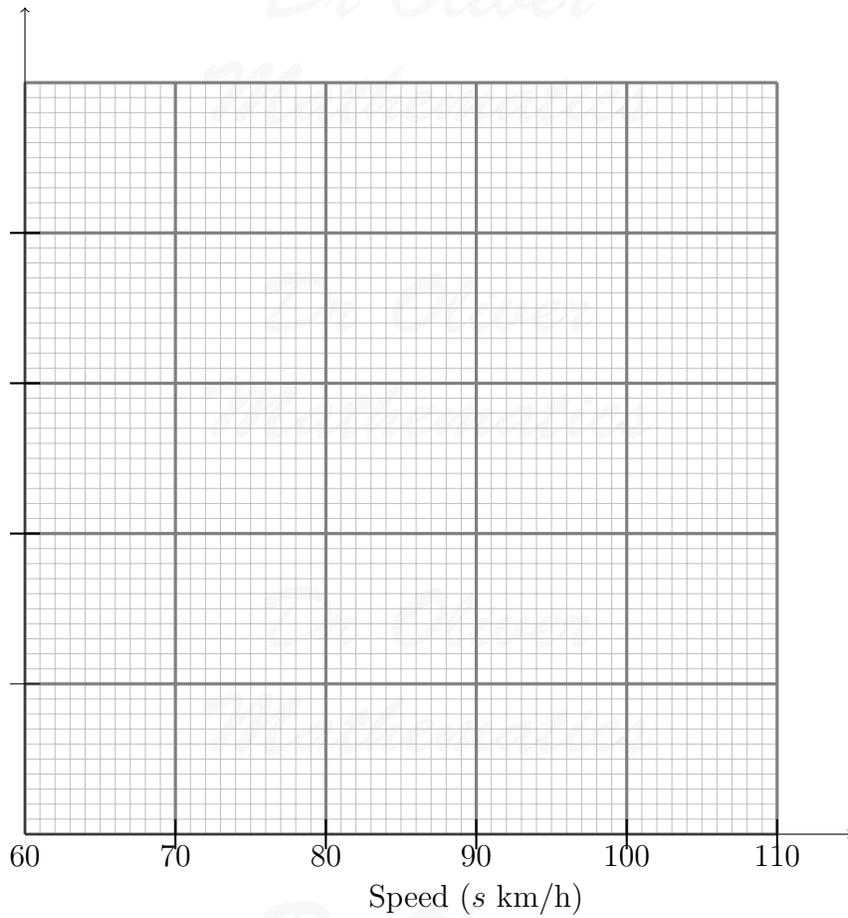> Angle $DOB = 360 - 90 - 90 - 50 = 130°$ because $\angle ADO$ and $\angle ABO$ are right angles.
> Angle $BCD = 130 \div 2 = \underline{\underline{65°}}$ because the angle at the circumference is twice the angle at the centre.

22. The table gives some information about the speeds, in km/h, of 100 cars.

Speed (s km/h)	Frequency
$60 < s \leqslant 65$	15
$65 < s \leqslant 70$	25
$70 < s \leqslant 80$	36
$80 < s \leqslant 100$	24

(a) On the grid, draw a histogram for the information in the table. (3)

Speed (s km/h)

Solution

Speed (s km/h)	Frequency	Width	Frequency Density
$60 < s \leqslant 65$	15	5	$\frac{15}{5} = 3$
$65 < s \leqslant 70$	25	5	$\frac{15}{5} = 5$
$70 < s \leqslant 80$	36	10	$\frac{3.6}{10} = 3.6$
$80 < s \leqslant 100$	24	20	$\frac{24}{20} = 1.2$

Frequency Density

Speed (s km/h)
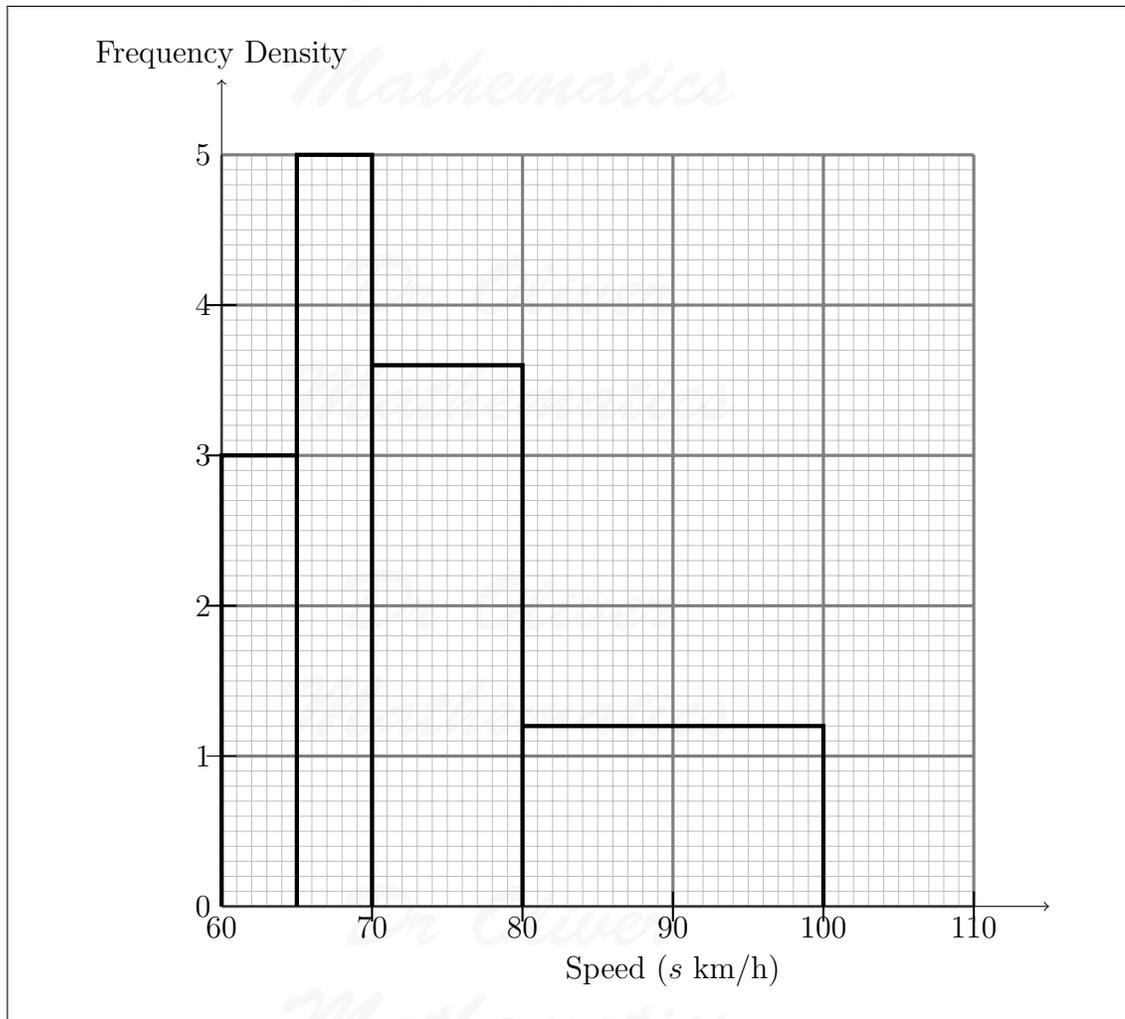
(b) Work out an estimate for the number of cars with a speed of more than 85 km/h. (2)

Solution

$$\tfrac{3}{4} \times 24 = \underline{18 \text{ cars}}.$$

23. (a) Simplify fully (3)

$$\frac{x^2 + 3x - 4}{2x^2 - 5x + 3}.$$

Solution

$$\left.\begin{array}{ll}\text{add to:} & +3 \\ \text{multiply to:} & -4\end{array}\right\} + 4, \ -1$$

$$x^2 + 3x - 4 = (x + 4)(x - 1).$$

$$\text{add to:} \qquad -5$$
$$\left.\text{multiply to:} \quad (+2) \times (+3) = +6 \right\} - 2, \ -3$$

$$2x^2 - 5x + 3 = 2x^2 - 2x - 3x + 3$$
$$= 2x(x - 1) - 3(x - 1)$$
$$= (2x - 3)(x - 1).$$

Hence,

$$\frac{x^2 + 3x - 4}{2x^2 - 5x + 3} = \frac{(x + 4)(x - 1)}{(2x - 3)(x - 1)}$$
$$= \frac{x + 4}{2x - 3}.$$

(b) Write $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (3)

$$\frac{4}{x + 2} + \frac{3}{x - 2}$$

as a single fraction in its simplest form.

Solution

$$\frac{4}{x + 2} + \frac{3}{x - 2} = \frac{4(x - 2)}{(x + 2)(x - 2)} + \frac{3(x + 2)}{(x + 2)(x - 2)}$$
$$= \frac{4(x - 2) + 3(x + 2)}{(x + 2)(x - 2)}$$
$$= \frac{4x - 8 + 3x + 6}{(x + 2)(x - 2)}$$
$$= \frac{7x - 2}{(x + 2)(x - 2)}.$$

24. Express the recurring decimal $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (3)

$$0.2\dot{8}\dot{1}$$

as a fraction in its simplest form.

23

Solution

Let $x = 0.28\dot{1}\dot{1}$. Then

$$10x = 2.8\dot{1}\dot{1} \quad (1)$$
$$1\,000x = 281.8\dot{1}\dot{1} \quad (2)$$
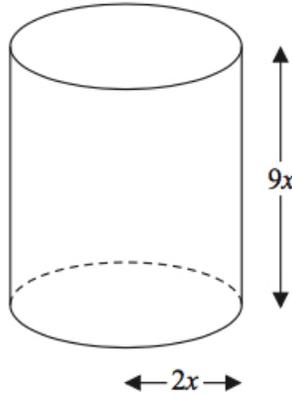
Now, do $(2) - (1)$:

$$990x = 279 \Rightarrow 110x = 31$$
$$\Rightarrow x = \tfrac{31}{110}.$$

25. The diagram shows a solid metal cylinder. (3)

The cylinder has base radius $2x$ and height $9x$.
The cylinder is melted down and made into a sphere of radius r.
Find an expression for r in terms of x.

Solution

$$\tfrac{4}{3}\pi r^3 = \pi \times (2x)^2 \times 9x \Rightarrow \tfrac{4}{3}r^3 = 4x^2 \times 9x$$
$$\Rightarrow \tfrac{4}{3}r^3 = 36x^3$$
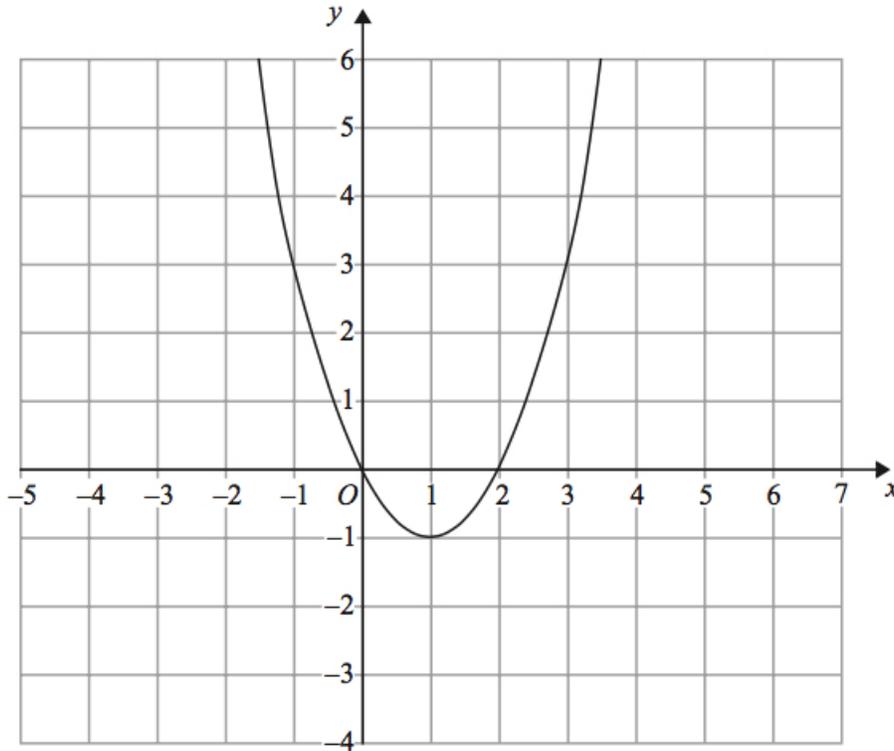$$\Rightarrow \tfrac{1}{3}r^3 = 9x^3$$
$$\Rightarrow r^3 = 27x^3$$
$$\Rightarrow r = 3x.$$

26. The graph of $y = f(x)$ is shown on each of the grids.

   (a) On this grid, sketch the graph of $y = f(x - 3)$.   (2)

> **Solution**
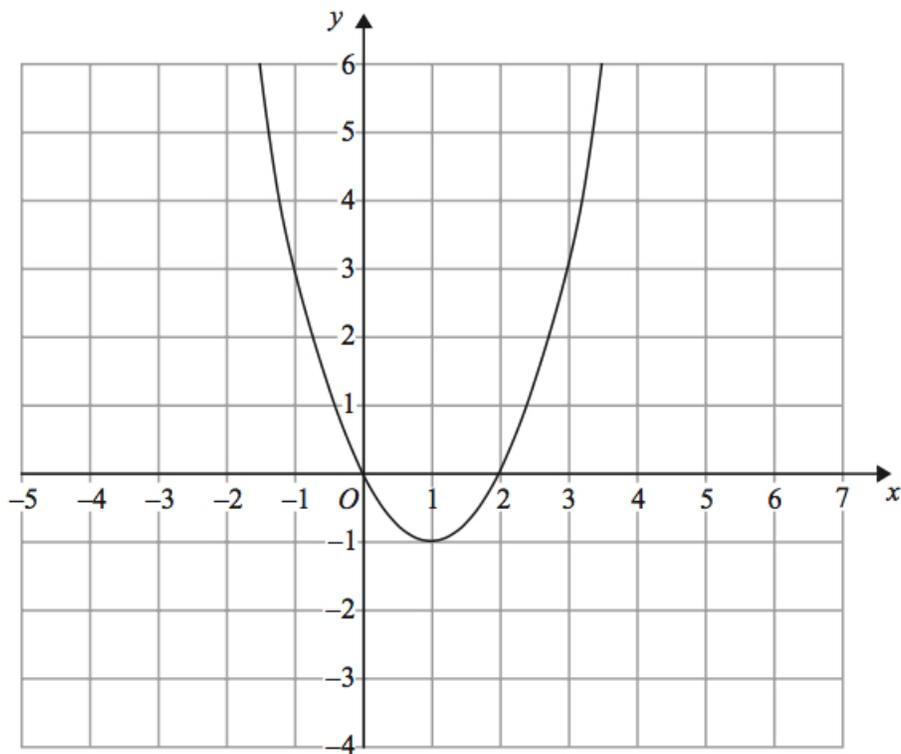> The curve goes through $(2, 3)$, $(3, 0)$, $(4, -1)$, $(5, 0)$, and $(6, 3)$.

   (b) On this grid, sketch the graph of $y = 2\,f(x)$.   (2)

Solution

The curve goes through $(-1, 6)$, $(0, 0)$, $(1, -2)$, $(2, 0)$, and $(3, 6)$.