Dr Oliver Mathematics
Mathematics: Higher
2019 Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.

1. Find the x-coordinates of the stationary points on the curve with equation (4)

$$y = \tfrac{1}{2}x^4 - 2x^3 + 6.$$

2. The equation (3)

$$x^2 + (k-5)x + 1 = 0$$

has equal roots.

Determine the possible values of k.

3. Circle C_1 has equation (2)

$$x^2 + y^2 - 6x - 2y - 26 = 0.$$

Circle C_2 has centre $(4, -2)$.
The radius of C_2 is equal to the radius of C_1.
Find the equation of circle C_2.

4. A sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c,$$

where the first three terms of the sequence are 6, 9, and 11.

(a) Find the values of m and c. (3)

(b) Hence, calculate the fourth term of the sequence. (1)

5. (a) Show that the points $A(1, 5, -3)$, $B(4, -1, 0)$, and $C(8, -9, 4)$ are collinear. (3)

(b) State the ratio in which B divides AC. (1)

6. Given that (3)

$$y = \frac{1}{(1-3x)^5},\ x \neq 1,$$

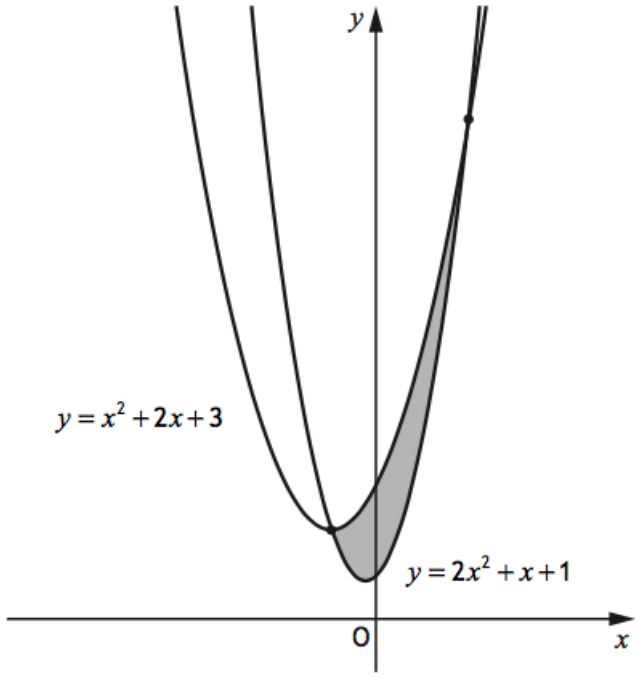
find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

7. The line, L, makes an angle of 30° with the positive direction of the x-axis. (4)
Find the equation of the line perpendicular to L, passing through $(0, -4)$.

8. The graphs of

$$y = x^2 + 2x + 3 \text{ and } y = 2x^2 + x + 1$$

are shown below.



The graphs intersect at the points where $x = -1$ and $x = 2$.

(a) Express the shaded area, enclosed between the curves, as an integral. (1)

(b) Evaluate the shaded area. (3)

9. Vectors $\mathbf{u}$ and $\mathbf{v}$ have components

$$\mathbf{u} = \begin{pmatrix} p \\ -2 \\ 4 \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} 2p + 16 \\ -3 \\ 6 \end{pmatrix}, \; p \in \mathbb{R}.$$

(a) (i) Find an expression for $\mathbf{u}.\mathbf{v}$. (1)

(ii) Determine the values of p for which $\mathbf{u}$ and $\mathbf{v}$ are perpendicular. (3)

(b) Determine the value of p for which $\mathbf{u}$ and $\mathbf{v}$ are parallel. (2)

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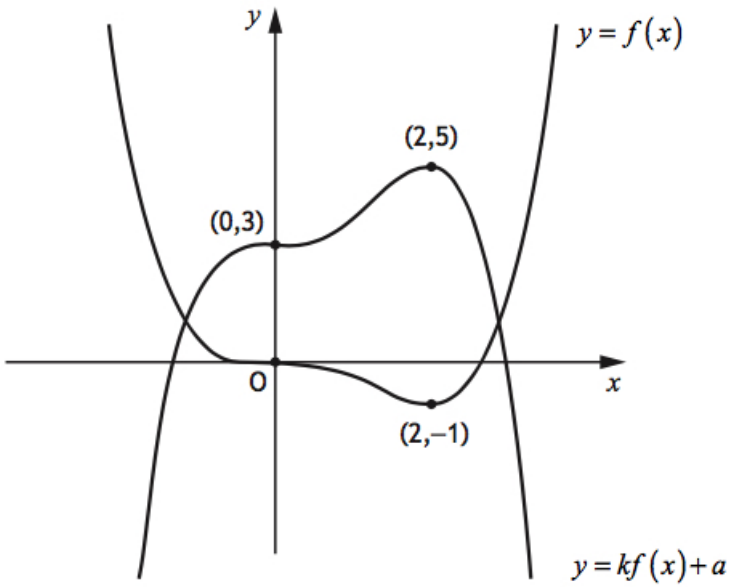
10. The diagram shows the graphs with equations

$$y = \mathrm{f}(x) \text{ and } y = k\,\mathrm{f}(x) + a.$$



(a) State the value of a. (1)

(b) Find the value of k. (1)

11. Evaluate (4)

$$\int_0^{\frac{1}{9}\pi} \cos(3x - \tfrac{1}{6}\pi)\,\mathrm{d}x.$$

12. Functions f and g are defined by

- $\mathrm{f}(x) = \dfrac{1}{\sqrt{x}}$, where $x > 0$ and
- $\mathrm{g}(x) = 5 - x$, where $x \in \mathbb{R}$.

(a) Determine an expression for $\mathrm{f}(\mathrm{g}(x))$. (2)

(b) State the range of values of x for which $\mathrm{f}(\mathrm{g}(x))$ is undefined. (1)

13. Triangles ABC and ADE are both right-angled.
Angles p and q are as shown in the diagram.

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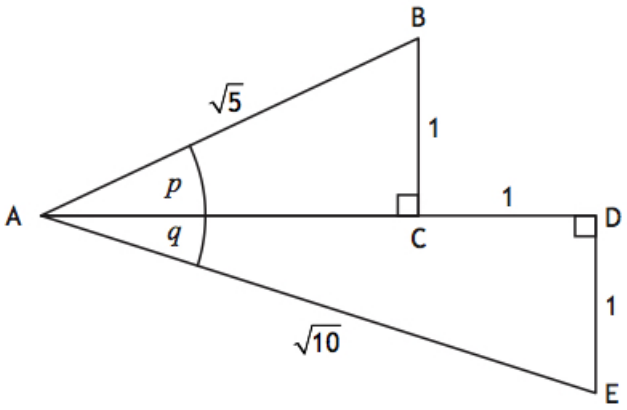

(a) Determine the value of

(i) $\cos p$ and (1)

(ii) $\cos q$. (1)

(b) Hence determine the value of $\sin(p + q)$. (3)

14. (a) Evaluate (3)

$$\log_{10} 4 + 2 \log_{10} 5.$$

(b) Solve (3)

$$\log_2(7x - 2) - \log_2 3 = 5, \; x \geqslant 1.$$

15. (a) Solve the equation (4)

$$\sin 2x^\circ + 6 \cos x^\circ = 0$$

for $0 \leqslant x < 360$.

(b) Hence solve (1)

$$\sin 4x^\circ + 6 \cos 2x^\circ = 0$$

for $0 \leqslant x < 360$.

16. The point P has coordinates $(4, k)$.
C is the centre of the circle with equation

$$(x - 1)^2 + (y + 2)^2 = 25.$$

(a) Show that the distance between the points P and C is given by (2)

$$\sqrt{k^2 + 4k + 13}.$$

(b) Hence, or otherwise, find the range of values of k such that P lies outside the circle. (4)

17. (a) Express (3)

$$(\sin x - \cos x)^2$$

in the form

$$p + q \sin rx,$$

where p, q, and r are integers.

(b) Hence, find (2)

$$\int (\sin x - \cos x)^2 \, \mathrm{d}x.$$

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