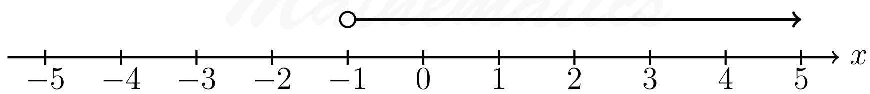
Dr Oliver Mathematics
GCSE Mathematics
2021 November Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. (a) Write down the inequality shown on this number line. (1)

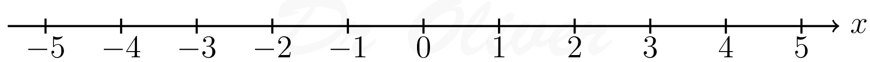


Solution

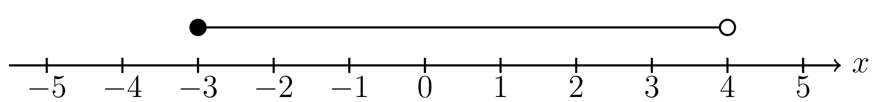
$$\underline{\underline{x > -1}}.$$

(b) On the number line below, show the inequality (2)

$$-3 \leqslant y < 4.$$



Solution



2. (a) Find the Highest Common Factor (HCF) of 60 and 84. (2)

Solution

$$\begin{array}{r|l} & 60 \\ \hline 2 & 30 \\ \hline 2 & 15 \\ \hline 3 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

So

$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5.$$

$$\begin{array}{r|l}
 & 84 \\ \hline
2 & 42 \\ \hline
2 & 21 \\ \hline
3 & 7 \\ \hline
7 & 1 \\ \hline
\end{array}$$

So

$$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7.$$

Hence, the HCF is

$$2^2 \times 3 = \underline{\underline{12}}.$$

(b) Find the Lowest Common Multiple (LCM) of 24 and 40. (2)

Solution

$$\begin{array}{r|l}
 & 24 \\ \hline
2 & 12 \\ \hline
2 & 6 \\ \hline
2 & 3 \\ \hline
3 & 1 \\ \hline
\end{array}$$

So

$$24 = 2 \times 2 \times 2 \times 5 = 2^3 \times 3.$$

$$\begin{array}{r|l}
 & 40 \\ \hline
2 & 20 \\ \hline
2 & 10 \\ \hline
2 & 5 \\ \hline
5 & 1 \\ \hline
\end{array}$$

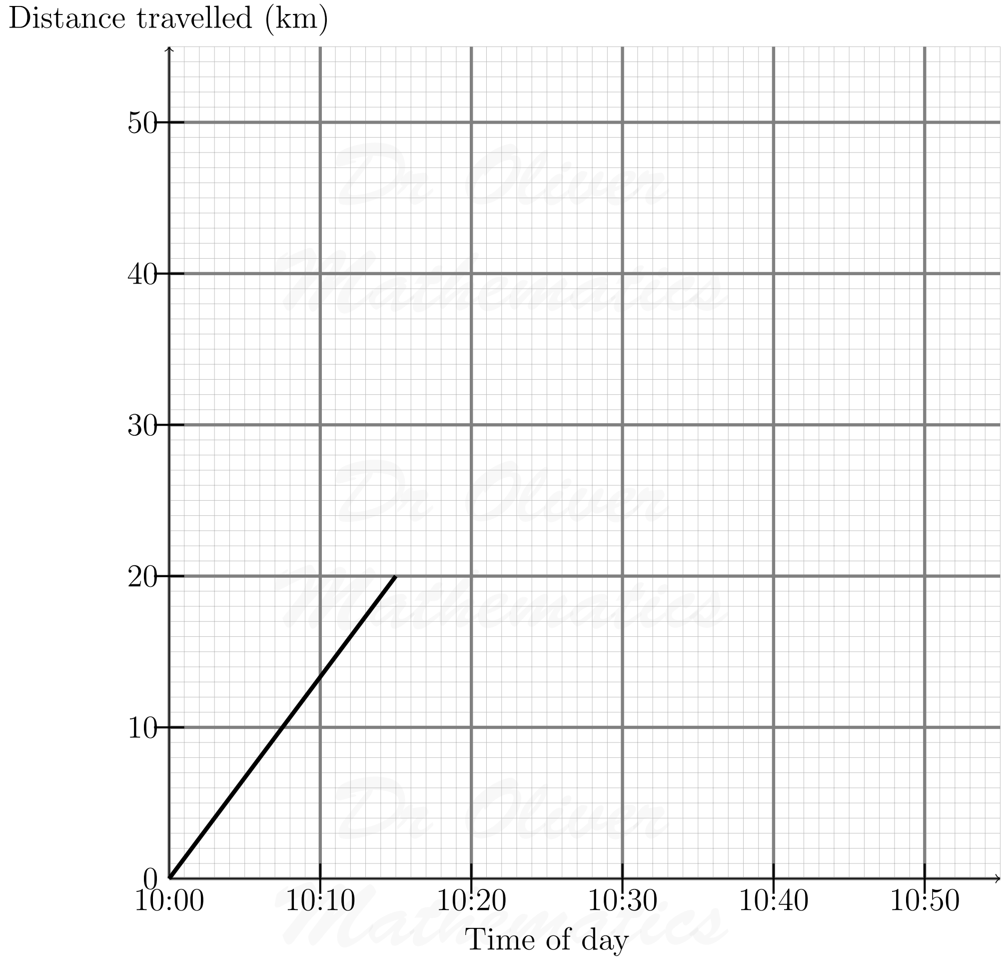
So

$$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5.$$

Hence, the LCM is

$$2^3 \times 3 \times 5 = \underline{\underline{120}}.$$

3. Sam drives his car on a journey.
Here is the travel graph for the first 15 minutes of his journey.



(a) Work out Sam's speed, in km/h, for the first 15 minutes of his journey. (2)

Solution

$$\text{Speed} = \frac{20-0}{0.25-0} = \underline{\underline{80 \text{ km/h}}}.$$

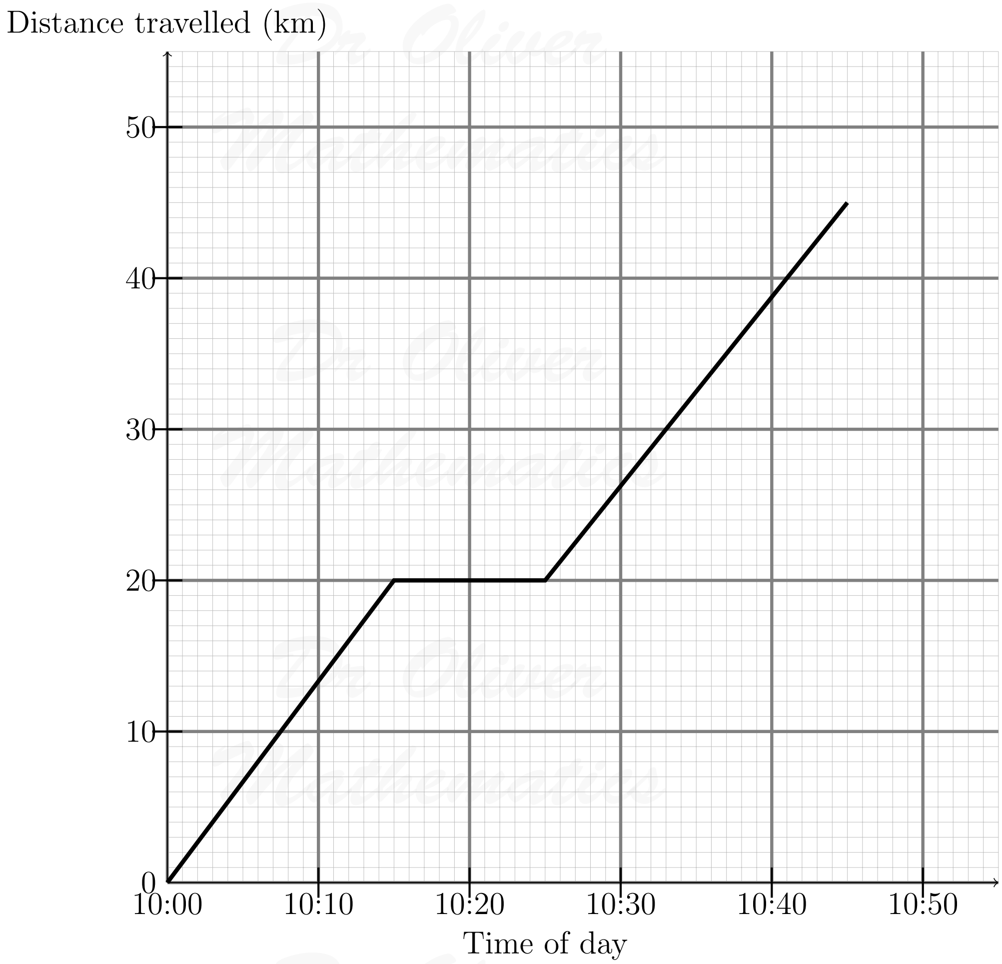
At 10:15 Sam stops for 10 minutes and then drives for 20 minutes at a speed of 75 km/h.

(b) On the grid, complete the travel graph for Sam's journey. (3)

Solution

Once he continues, he goes an extra

$$\tfrac{1}{3} \times 75 = 25 \text{ km}.$$



4. (a) Complete the table of values for (2)

$$y = x^2 - 2x + 2.$$

x	-2	-1	0	1	2	3	4
y	10		2			5	

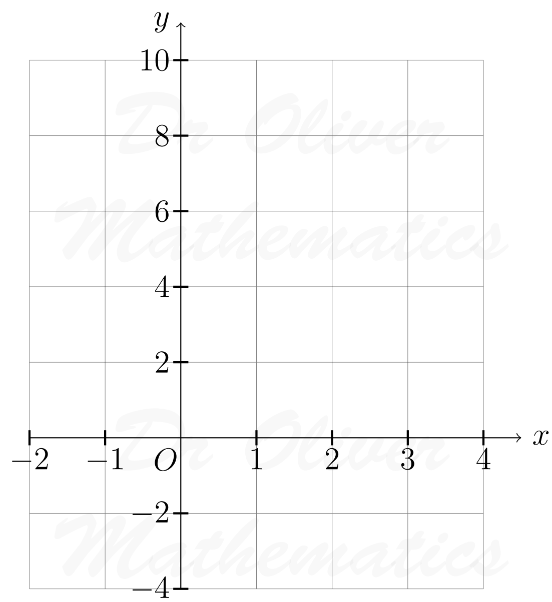
Solution

x	-2	-1	0	1	2	3	4
y	10	$\underline{\underline{5}}$	2	$\underline{\underline{1}}$	$\underline{\underline{2}}$	5	$\underline{\underline{\underline{10}}}$

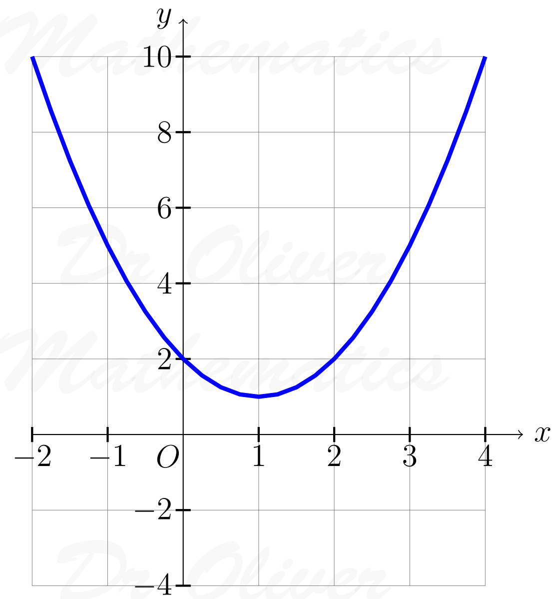
(b) On the grid, draw the graph of (2)

$$y = x^2 - 2x + 2,$$

for values of x from -2 to 4.



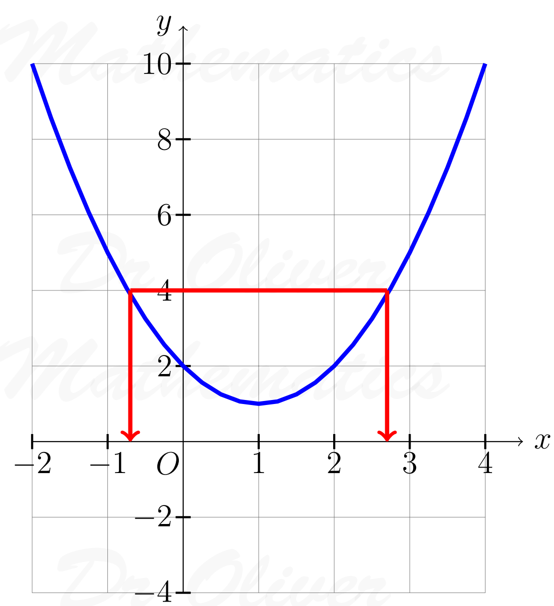
Solution



(c) Use your graph to find estimates of the solutions of the equation **(2)**

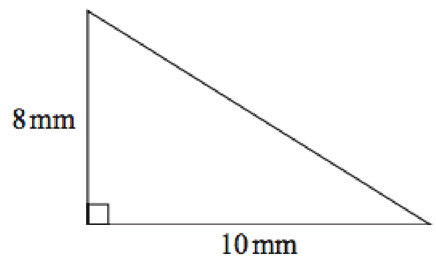
$$x^2 - 2x + 2 = 4.$$

Solution

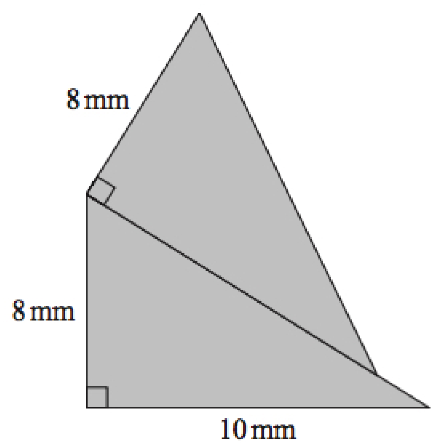


Correct read-off: approximately $x = -0.7$ or $x = 2.7$.

5. Here is a right-angled triangle. (4)



The shaded shape below is made from two of these triangles.



Work out the perimeter of the shaded shape.
Give your answer correct to 3 significant figures.

Solution

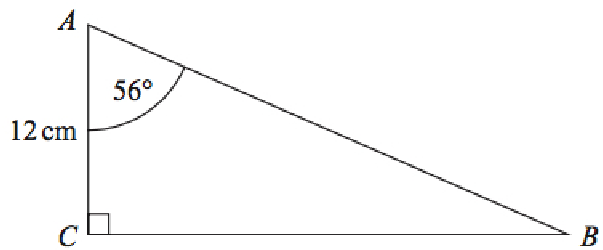
Let h cm the hypotenuse. Then

$$\begin{aligned} h &= \sqrt{8^2 + 10^2} \\ &= \sqrt{64 + 100} \\ &= 2\sqrt{41}. \end{aligned}$$

Finally,

$$\begin{aligned}\text{perimeter} &= 8 + 8 + 2\sqrt{41} + (2\sqrt{41} - 10) + 10 \\ &= 16 + 4\sqrt{41} \\ &= 41.612\,496\,95 \text{ (FCD)} \\ &= \underline{\underline{41.6 \text{ cm (3 sf)}}}.\end{aligned}$$

6. ABC is a right-angled triangle.

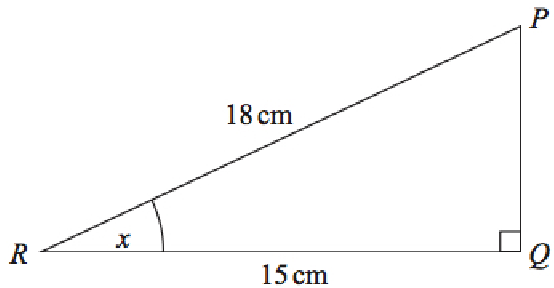


(a) Work out the length of BC. (2)
Give your answer correct to 1 decimal place.

Solution

$$\begin{aligned}\tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan 56^\circ = \frac{BC}{12} \\ &\Rightarrow BC = 12\tan 56^\circ \\ &\Rightarrow BC = 17.790\,731\,62 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{BC = 17.8 \text{ cm (1 dp)}}}.\end{aligned}$$

PQR is a right-angled triangle.



(b) Work out the size of the angle marked x. (2)
Give your answer correct to 1 decimal place.

Solution

$$\begin{aligned}\cos = \frac{\text{adj}}{\text{hyp}} &\Rightarrow \cos x = \tfrac{15}{18} \\ &\Rightarrow x = 33.557\,309\,76 \\ &\Rightarrow \underline{\underline{x = 33.6^\circ \text{ (1 dp)}}}.\end{aligned}$$

7. Liquid **A** has a density of 1.8 g/cm^3. (3)
Liquid **B** has a density of 1.2 g/cm^3.

80 cm^3 of liquid **A** is mixed with 40 cm^3 of liquid **B** to make 120 cm^3 of liquid **C**.

Work out the density of liquid **C**.

Solution

For liquid **A**,

$$\begin{aligned}\text{density} = \frac{\text{mass}}{\text{volume}} &\Rightarrow 1.8 = \frac{\text{mass}}{80} \\ &\Rightarrow \text{mass} = 1.8 \times 80 \\ &\Rightarrow \text{mass} = 144.\end{aligned}$$

For liquid **B**,

$$\begin{aligned}\text{density} = \frac{\text{mass}}{\text{volume}} &\Rightarrow 1.2 = \frac{\text{mass}}{40} \\ &\Rightarrow \text{mass} = 1.2 \times 40 \\ &\Rightarrow \text{mass} = 48.\end{aligned}$$

Hence, their combined mass is

$$144 + 48 = 192$$

and their combined volume is 120. Finally,

$$\begin{aligned}\text{density of liquid } \mathbf{C} &= \frac{192}{120} \\ &= \underline{\underline{1.6 \text{ g/cm}^3}}.\end{aligned}$$

8. The grouped frequency table gives information about the time, in minutes, taken by 50 people to solve a puzzle. (1)

Time (t minutes)	Frequency
$0 < t \leqslant 10$	5
$10 < t \leqslant 20$	8
$20 < t \leqslant 30$	12
$30 < t \leqslant 40$	15
$40 < t \leqslant 50$	7
$50 < t \leqslant 60$	3

Brian was asked to draw a cumulative frequency table for this information.

This is the table that Brian drew.

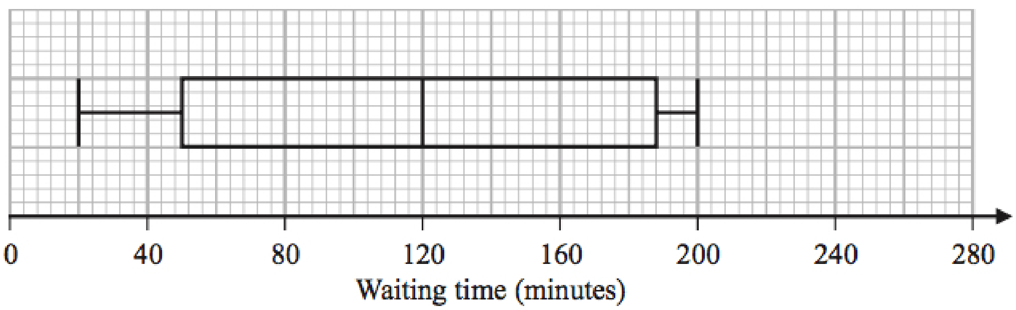
Time (t minutes)	Cumulative Frequency
$0 < t \leqslant 10$	5
$10 < t \leqslant 20$	13
$20 < t \leqslant 30$	25
$30 < t \leqslant 40$	40
$40 < t \leqslant 50$	47
$50 < t \leqslant 60$	50

Write down **one** thing that is wrong with this cumulative frequency table.

Solution

E.g., Brian has not used $0 < t \leqslant \dots$ for his cumulative frequency table.

9. The box plot shows information about the length of time, in minutes, some people waited to see a doctor at a hospital on Monday.



(a) Work out the interquartile range of the information in the box plot. (2)

Solution

$$\begin{aligned} \text{IQR} &= 188 - 50 \\ &= \underline{\underline{138 \text{ minutes}}}. \end{aligned}$$

Becky says, "50% of the people waited for at least 2 hours."

(b) Is Becky correct? (1)
Explain why.

Solution

$\underline{\underline{\text{Yes}}}$: the median is at 120 minutes.

The table gives information about the length of time, in minutes, some people waited to see a doctor at the same hospital on Tuesday.

	Waiting time (minutes)
Shortest time	20
Lower quartile	50
Median	100
Upper quartile	140
Longest time	210

Becky was asked to compare the distribution of the lengths of times people waited on Monday with the distribution of the lengths of times people waited on Tuesday.

She wrote, "People had to wait longer on Tuesday than on Monday."

(c) Give **one** reason why Becky may be wrong. (1)

Solution
E.g., the median is lower on Tuesday.

10. Louise invests £x in Better Investments for 3 years. (4)
Sadiq invests £x in County Bank for 3 years.

Better Investments

Compound Interest

2.5% per annum

County Bank

Compound Interest

2% per annum for the first two years
3.5% per annum for each extra year

At the end of the 3 years, the value of Louise's investment is £344 605.

Work out the value of Sadiq's investment at the end of the 3 years.

Solution

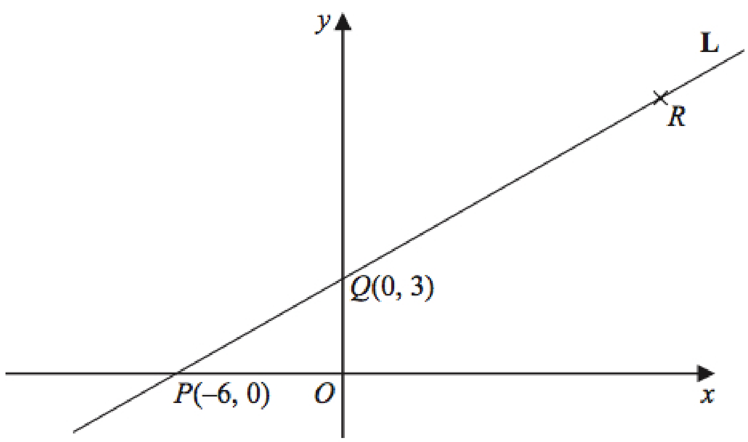
Well,

$$344\,605 = x \times (1.025)^3 \Rightarrow x = \frac{344\,605}{(1.025)^3}$$
$$\Rightarrow x = 320\,000.$$

Finally,

$$\begin{aligned}\text{Sadiq's investment} &= 320\,000 \times (1.02)^2 \times 1.035 \\ &= \underline{\underline{£344\,580.48}}\end{aligned}$$

11. Here is a sketch of the line **L**.



The points $P(-6, 0)$ and $Q(0, 3)$ are points on the line **L**.

The point R is such that PQR is a straight line and

$$PQ : QR = 2 : 3.$$

(a) Find the coordinates of R. (2)

Solution
Well,

$$2 + 3 = 5$$

and

$$\begin{pmatrix} -6 \\ 0 \end{pmatrix} + \tfrac{5}{2}\begin{pmatrix} 0-(-6) \\ 3-0 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \end{pmatrix} + \tfrac{5}{2}\begin{pmatrix} 6 \\ 3 \end{pmatrix}$$
$$= \begin{pmatrix} -6 \\ 0 \end{pmatrix} + \begin{pmatrix} 15 \\ 7.5 \end{pmatrix}$$
$$= \begin{pmatrix} 9 \\ 7.5 \end{pmatrix};$$

hence, $\underline{\underline{R(9, 7.5)}}$.

(b) Find an equation of the line that is perpendicular to $\mathbf{L}$ and passes through Q. (3)

Solution

$$-\frac{1}{0.5} = -2$$

and an equation of the line is

$$y - 3 = -2(x - 0) \Rightarrow \underline{\underline{y = -2x + 3}}.$$

12. Expand and simplify (3)

$$(x-2)(3x+2)(2x+3).$$

Solution

$\times$	x	-2
$3x$	$3x^2$	$-6x$
$+2$	$+2x$	-4

So

$$(x-2)(3x+2) = 3x^2 - 4x - 4.$$

$\times$	$3x^2$	$-4x$	-4
$2x$	$6x^3$	$-8x^2$	$-8x$
$+3$	$+9x^2$	$-12x$	-12

Hence,

$$(3x^2 - 4x - 4)(2x + 3) = \underline{\underline{6x^3 + x^2 - 20x - 12}}.$$

13. In a school there are 16 teachers and 220 students. (2)
Of these students 120 are girls and 100 are boys.

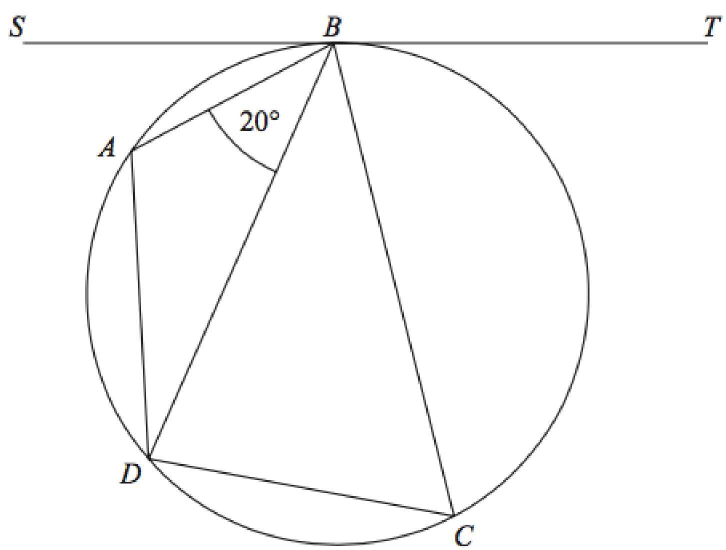
One teacher, one girl, and one boy are going to be chosen to represent the school.

Work out the number of different ways there are to choose one teacher, one girl, and one boy.

Solution

$$16 \times 120 \times 100 = \underline{\underline{192\,000}}.$$

14. A, B, C, and D are four points on a circle. (4)



SBT is a tangent to the circle.
Angle $ABD = 20^\circ$.

The size of angle BAD : the size of angle $BCD = 3 : 1$.

Find the size of angle SBA.
Give a reason for each stage of your working.

Solution

Well, $\angle BAD$ and $\angle BCD$ add up to $180°$, using opposite angles in a cyclic quadrilateral). Now,

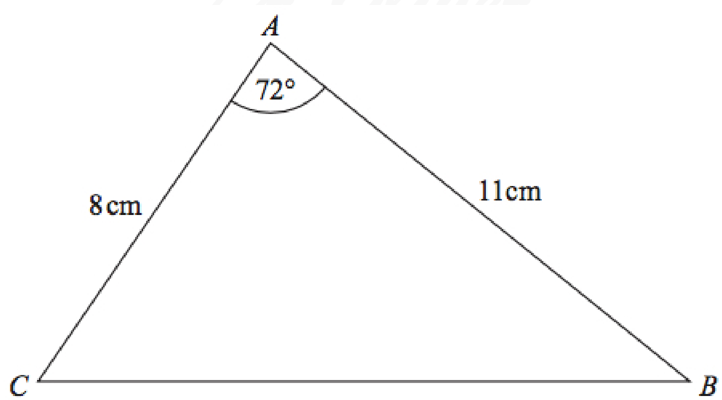
$$\angle BCD = \tfrac{1}{4} \times 180 = 45°$$

and so

$$\angle SBA = 45 - 20 = \underline{\underline{25°}},$$

using the alternate segment theorem.

15. Here is triangle ABC.



(a) Find the length of BC. (3)
Give your answer correct to 3 significant figures.

Solution

Cosine rule:

$$\begin{aligned} & BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos BAC \\ \Rightarrow\ & BC^2 = 11^2 + 8^2 - 2 \times 11 \times 8 \times \cos 72° \\ \Rightarrow\ & BC^2 = 130.613\,009 \text{ (FCD)} \\ \Rightarrow\ & BC = 11.428\,604\,86 \text{ (FCD)} \\ \Rightarrow\ & \underline{\underline{BC = 11.4 \text{ cm (3 sf)}}}. \end{aligned}$$

(b) Find the area of triangle ABC. (2)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}\text{Area} &= \tfrac{1}{2} \times AB \times AC \times \sin BAC \\ &= \tfrac{1}{2} \times 11 \times 8 \times \sin 72^\circ \\ &= 41.846\,486\,72 \text{ (FCD)} \\ &= \underline{\underline{41.8 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

16. (a) Use the iteration formula (3)

$$x_{n+1} = \sqrt[3]{10 - 2x_n},$$

to find the values of x_1, x_2, and x_3.
Start with $x_0 = 2$.

Solution

$$\begin{aligned}x_1 &= \sqrt[3]{10 - 2 \times 10} \\ &= \underline{\underline{1.817\,120\,593 \text{ (FCD)}}} \\ x_2 &= \sqrt[3]{10 - 2 \times 1.817\ldots} \\ &= \underline{\underline{1.853\,318\,496 \text{ (FCD)}}} \\ x_3 &= \sqrt[3]{10 - 2 \times 1.853\ldots} \\ &= \underline{\underline{1.846\,265\,953 \text{ (FCD)}}}.\end{aligned}$$

The values of x_1, x_2, and x_3 found in part (a) are estimates of the solution of an equation of the form

$$x^3 + ax + b = 0,$$

where a and b are integers.

(b) Find the value of a and the value of b. (1)

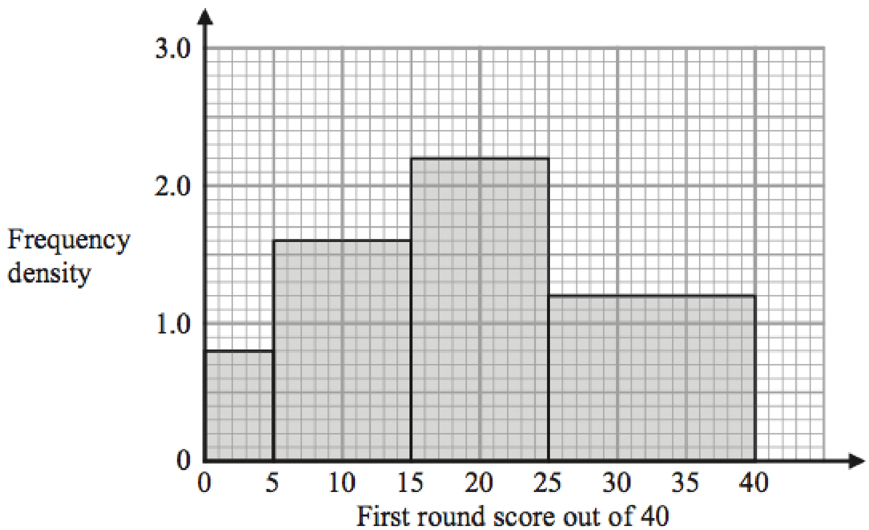
Solution

$$\begin{aligned}x = \sqrt[3]{10 - 2x} &\Rightarrow x^3 = 10 - 2x \\ &\Rightarrow x^3 + 2x - 10 = 0;\end{aligned}$$

hence,

$$\underline{\underline{a = 2}} \text{ and } \underline{\underline{b = -10}}.$$

17. Some people took part in the first round of a competition. (4)
The histogram gives information about the scores of these people in the first round.



20% of the people got a score high enough for them to qualify for the second round.

Work out an estimate for the score needed to qualify for the second round.
You must show all your working.

Solution

We will make up a table:

First Round	Frequency Density	Width	Frequency	Cum. Freq.
$0 - 5$	0.8	5	$0.8 \times 5 = 4$	4
$5 - 15$	1.6	10	$1.6 \times 10 = 16$	20
$15 - 25$	2.2	10	$2.2 \times 10 = 22$	42
$25 - 40$	1.2	15	$1.2 \times 15 = 18$	60

20% people qualify:

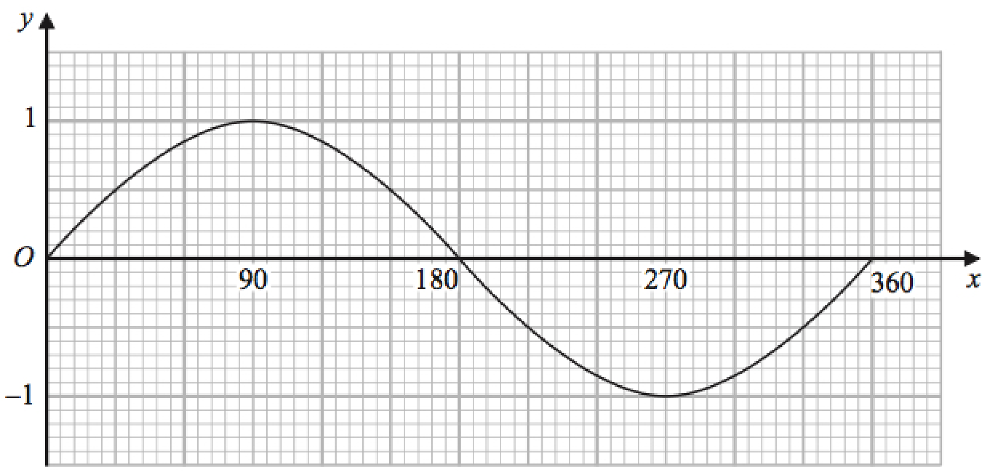
$$0.2 \times 60 = 12.$$

But we have, in the $25-40$, 18 people ... so we just divide that in to thirds! Hence, you need 30 points.

18. Here is a graph of

$$y = \sin x^\circ,$$

for $0 \leqslant x \leqslant 360$.



(a) Using this graph, find estimates of all **four** solutions of (2)

$$\sin x^\circ = 0.6$$

for $0 \leqslant x \leqslant 720$.

Solution

$$x = 37, 143, 397, \text{ or } 503.$$

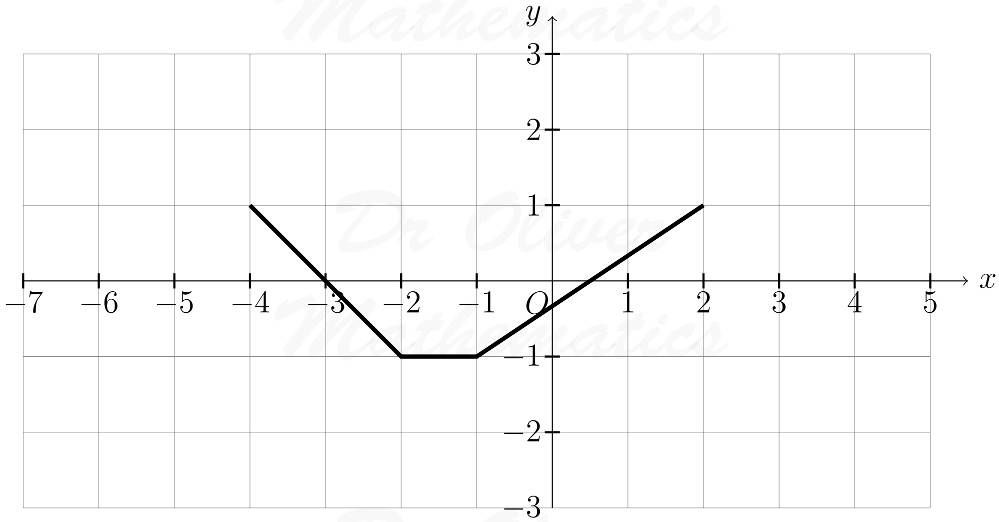
The graph of $y = \sin x^\circ$ is reflected in the x-axis.

(b) Write down an equation of the reflected graph. (1)

Solution

E.g, $y = -\sin x^\circ$.

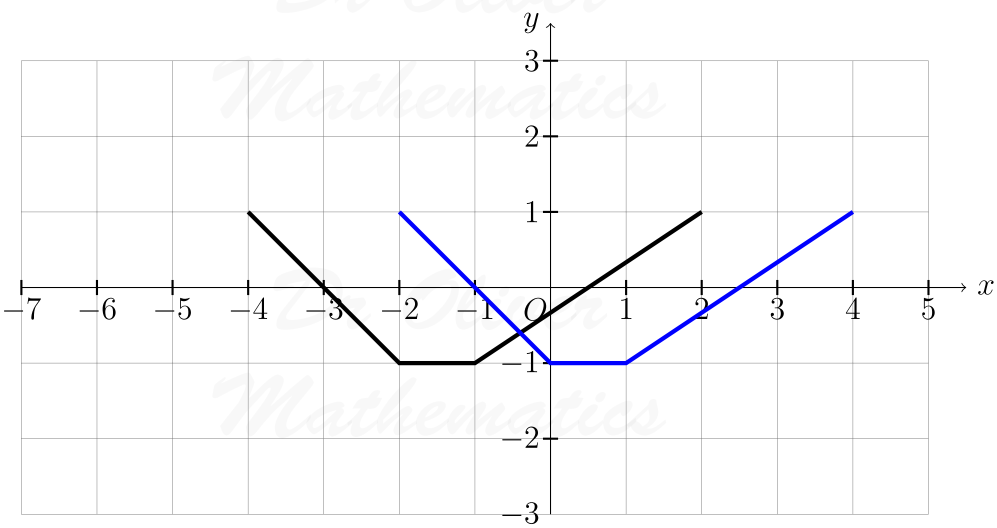
Here is a graph of $y = \mathrm{f}(x)$.



(c) On the grid, draw the graph of (1)

$$y = \mathrm{f}(x - 2).$$

Solution



19. **A**, **B**, and **C** are three spheres. (3)

The volume of sphere **A** is 125 cm^3.

The volume of sphere **B** is 27 cm^3.

The ratio of the radius of sphere **B** to the radius of sphere **C** is 1 : 2.

Work out the ratio of the surface area of sphere **A** to the surface area of sphere **C**.

Solution

Going from **A**, to **B**, the volume scale ratio (VSR) is

$$125 : 27 = 5^3 : 3^3$$

which means the length scale ratio (LSR) is

$$5 : 3$$

and the area scale ratio (ASR) is

$$5^2 : 3^2 = 25 : 9.$$

Given that the ratio of the radius of sphere **B** to the radius of sphere **C** is 1 : 2, the area scale ratio (ASR) is

$$1^2 : 2^2 = 1 : 4 = 9 : 36.$$

Finally, the ratio of the surface area of sphere **A** to the surface area of sphere **C** is

$$\underline{\underline{25 : 36}}.$$

20. In a village, (4)

- if it rains on one day, the probability that it will rain on the next day is 0.8 and
- if it does not rain on one day, the probability that it will rain on the next day is 0.6.

A weather forecaster says, "There is a 70% chance that it will rain in the village on Monday."

Work out an estimate for the probability that it will rain in the village on Wednesday. You must show all your working.

Solution

$$\begin{aligned}
\text{P(rain on Wednesday)} &= \text{P(dry, dry, rain)} + \text{P(dry, rain, rain)} \\
&\qquad\qquad + \text{P(rain, dry, rain)} + \text{P(rain, rain, rain)} \\
&= (0.3 \times 0.4 \times 0.6) + (0.3 \times 0.6 \times 0.8) + (0.7 \times 0.2 \times 0.6) \\
&\qquad\qquad + (0.7 \times 0.8 \times 0.8) \\
&= 0.072 + 0.144 + 0.084 + 0.448 \\
&= \underline{\underline{0.748}}.
\end{aligned}$$

21. The time period, T seconds, of a simple pendulum of length l cm is given by the formula (4)

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

Katie uses a simple pendulum in an experiment to find an estimate for the value of g.

Here are her results.

$l = 52.0$, correct to 3 significant figures.
$T = 1.45$, correct to 3 significant figures.

Work out the upper bound and the lower bound for the value of g.
Use $\pi = 3.142$.
You must show all your working.

Solution

Well,

$$\begin{aligned}
T = 2\pi\sqrt{\frac{l}{g}} &\Rightarrow \frac{T}{2\pi} = \sqrt{\frac{l}{g}} \\
&\Rightarrow \frac{T^2}{4\pi^2} = \frac{l}{g} \\
&\Rightarrow g = \frac{4\pi^2 l}{T^2}
\end{aligned}$$

and

$$\begin{aligned}
51.95 \leqslant l < 52.05 \\
1.445 \leqslant T < 1.455.
\end{aligned}$$

Now, the upper bound is

$$\frac{4 \times 3.142^2 \times 52.05}{1.445^2} = \underline{\underline{984.367\,785\,3 \text{ (FCD)}}}$$

and the lower bound is

$$\frac{4 \times 3.142^2 \times 51.95}{1.455^2} = \underline{\underline{969.018\,164\,3 \text{ (FCD)}}}.$$