

Dr Oliver Mathematics
Advance Level Further Mathematics
Statistics 2: Calculator
1 hour 30 minutes

The total number of marks available is 75.

You must write down all the stages in your working.

1. In a call centre, the number of telephone calls, X , received during any 10-minute period follows a Poisson distribution with mean 9.

- (a) Find (4)
(i) $P(X > 5)$,
(ii) $P(4 \leq X < 10)$.

The length of a working day is 7 hours.

- (b) Using a suitable approximation, find the probability that there are fewer than 370 telephone calls in a randomly selected working day. (5)

A week, consisting of 5 working days, is selected at random.

- (c) Find the probability that in this week at least 4 working days have fewer than 370 telephone calls. (3)

2. A fair coin is spun 6 times and the random variable T represents the number of tails obtained.

- (a) Give two reasons why a binomial model would be a suitable distribution for modelling T . (2)
(b) Find $P(T = 5)$. (2)
(c) Find the probability of obtaining more tails than heads. (2)

A second coin is biased such that the probability of obtaining a head is $\frac{1}{4}$.

This second coin is spun 6 times.

- (d) Find the probability that, for the second coin, the number of heads obtained is greater than or equal to the number of tails obtained. (3)

3. The length of time, T , minutes, spent completing a particular task has probability density function

$$f(t) = \begin{cases} \frac{1}{2}(t - 1) & 1 < t \leq 2, \\ \frac{1}{16}(14t - 3t^2 - 8) & 2 < t \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Use algebraic integration to find $E(T)$. (4)

Given that $E(T^2) = \frac{267}{40}$,

(b) find $\text{Var}(T)$. (2)

(c) Find the cumulative distribution function $F(t)$. (5)

(d) Find the 20th percentile of the time taken to complete the task. (3)

(e) Find the probability that the time spent completing the task is more than 1.5 minutes. (2)

Given that a person has already spent 1.5 minutes on the task,

(f) find the probability that this person takes more than 3 minutes to complete the task. (2)

4. David aims to catch the train to work each morning. The scheduled departure time of the train is 0830.

The number of minutes after 0830 that the train departs may be modelled by the random variable X . Given that X has a continuous uniform distribution over $[\alpha, \beta]$ and that $E(X) = 4$ and $\text{Var}(X) = 12$,

(a) find the value of α and the value of β . (5)

Each morning, the probability that David oversleeps is 0.05.

If David oversleeps he will be late for work.

If he does not oversleep he will be in time to catch the train, but will be late for work if the train departs after 0835.

(b) Find the probability that David will be late for work. (3)

Given that David is late for work,

(c) find the probability that he overslept. (2)

5. Past records show that the proportion of customers buying organic vegetables from *Tesson* supermarket is 0.35.

During a particular day, a random sample of 40 customers from *Tesson* supermarket was taken and 18 of them bought organic vegetables.

(a) Test, at the 5% level of significance, whether or not this provides evidence that the proportion of customers who bought organic vegetables has increased. State your hypotheses clearly. (5)

The manager of *Tesson* supermarket claims that the proportion of customers buying organic eggs is different from the proportion of those buying organic vegetables. To test this claim the manager decides to take a random sample of 50 customers.

- (b) Using a 5% level of significance, find the critical region to enable the *Tesson* supermarket manager to test her claim. The probability for each tail of the region should be as close as possible to 2.5%. (3)

During a particular day, a random sample of 50 customers from *Tesson* supermarket is taken and 8 of them bought organic eggs.

- (c) Using your answer to part (b), state whether or not this sample supports the manager's claim. Use a 5% level of significance. (1)
- (d) State the actual significance level of this test. (1)

The proportion of customers who buy organic fruit from *Tesson* supermarket is 0.2.

During a particular day, a random sample of 200 customers from *Tesson* supermarket is taken. Using a suitable approximation, the probability that fewer than n of these customers bought organic fruit is 0.0465, correct to 4 decimal places.

- (e) Find the value of n . (6)

6. The continuous random variable X has the following cumulative distribution function

$$F(x) = \begin{cases} 0 & x \leq 1, \\ \frac{4}{15}(x-1) & 1 < x \leq 2, \\ k \left(\frac{ax^3}{3} - \frac{x^4}{4} \right) + b & 2 < x \leq 4, \\ 1 & x > 4, \end{cases}$$

where k , a , and b are constants.

Given that the mode of X is $\frac{8}{3}$,

- (a) show that $a = 4$. (4)
- (b) Find $P(X < 2.5)$, giving your answer to 3 significant figures. (6)