Dr Oliver Mathematics
Mathematics
Integration Part 1
Past Examination Questions

This booklet consists of 44 questions across a variety of examination topics.
The total number of marks available is 190.

1. Find $\int \left(1 + 3\sqrt{x} - \frac{1}{x^2}\right)\,\mathrm{d}x$. (4)

Solution

$$\begin{aligned}\int \left(1 + 3\sqrt{x} - \frac{1}{x^2}\right)\,\mathrm{d}x &= \int \left(1 + 3x^{\frac{1}{2}} - x^{-2}\right)\,\mathrm{d}x \\ &= x + 3 \times \tfrac{2}{3}x^{\frac{3}{2}} - \frac{x^{-1}}{-1} + c \\ &= \underline{\underline{x + 2x^{\frac{3}{2}} + x^{-1} + c}}.\end{aligned}$$

2. The gradient of the curve C is given by (5)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (3x-1)^2.$$

The point $P(1, 4)$ lies on C. Find an equation for the curve C in the form $y = \mathrm{f}(x)$.

Solution

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = (3x-1)^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= 9x^2 - 6x + 1 \\ \Rightarrow y &= 9 \times \tfrac{1}{3}x^3 - 6 \times \tfrac{1}{2}x^2 + x + c \\ \Rightarrow y &= 3x^3 - 3x^2 + x + c.\end{aligned}$$

Now, it goes through the point $(1, 4)$:

$$4 = 3 - 3 + 1 + c \Rightarrow c = 3$$

and we have

$$\underline{\underline{y = 3x^3 - 3x^2 + x + 3}}.$$

3. Find $\displaystyle\int \left(6x - \frac{4}{x^2}\right)\,\mathrm{d}x$. (3)

Solution

$$\int \left(6x - \frac{4}{x^2}\right)\mathrm{d}x = \int \left(6x - 4x^{-2}\right)\mathrm{d}x$$
$$= 6 \times \tfrac{1}{2}x^2 - \frac{4x^{-1}}{-1} + c$$
$$= \underline{\underline{3x^2 + 4x^{-1} + c}}.$$

4. (a) Show that $\dfrac{(3-\sqrt{x})^2}{\sqrt{x}}$ can be written is $9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}$. (2)

Solution

$$\frac{(3-\sqrt{x})^2}{\sqrt{x}} = \frac{9 - 6\sqrt{x} + x}{\sqrt{x}}$$
$$= \frac{9 - 6x^{\frac{1}{2}} + x}{x^{\frac{1}{2}}}$$
$$= \underline{\underline{9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}}}.$$

Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(3-\sqrt{x})^2}{\sqrt{x}}$, $x > 0$, and that $y = \frac{2}{3}$ at $x = 1$,

(b) find y in terms of x. (6)

Solution

$$\int y\,\mathrm{d}x = \int \left(9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}\right)\mathrm{d}x$$
$$= 9 \times 2x^{\frac{1}{2}} - 6x + \tfrac{2}{3}x^{\frac{3}{2}} + c$$
$$= 18x^{\frac{1}{2}} - 6x + \tfrac{2}{3}x^{\frac{3}{2}} + c.$$

Now, $y = \frac{2}{3}$ at $x = 1$,

$$\tfrac{2}{3} = 18 - 6 + \tfrac{2}{3} + c \Rightarrow c = -12.$$

Hence

$$\underline{\underline{y = 18x^{\frac{1}{2}} - 6x + \tfrac{2}{3}x^{\frac{3}{2}} - 12}}.$$

5. Find $\displaystyle\int \left(2x^2 - \frac{6}{x^3}\right) \mathrm{d}x$. (3)

Solution

$$\begin{aligned}\int \left(2x^2 - \frac{6}{x^3}\right) \mathrm{d}x &= \int \left(2x^2 - 6x^{-3}\right) \mathrm{d}x \\ &= 2 \times \tfrac{1}{3}x^3 - 6 \times \left(\frac{x^{-2}}{-2}\right) + c \\ &= \underline{\underline{\tfrac{2}{3}x^3 + 3x^{-2} + c}}.\end{aligned}$$

6. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(1, 6)$. Given that (7)

$$\mathrm{f}'(x) = 3 + \frac{5x^2 + 2}{x^{\frac{1}{2}}},\ x > 0,$$

find $\mathrm{f}(x)$ and simplify your answer.

Solution

$$\begin{aligned}\mathrm{f}'(x) = 3 + \frac{5x^2 + 2}{x^{\frac{1}{2}}} &\Rightarrow f'(x) = 3 + \frac{5x^2}{x^{\frac{1}{2}}} + \frac{2}{x^{\frac{1}{2}}} \\ &\Rightarrow \mathrm{f}'(x) = 3 + 5x^{\frac{3}{2}} + 2x^{-\frac{1}{2}} \\ &\Rightarrow \mathrm{f}(x) = 3x + 5 \times \tfrac{2}{5}x^{\frac{5}{2}} + 2 \times 2x^{\frac{1}{2}} + c \\ &\Rightarrow \mathrm{f}(x) = 3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} + c.\end{aligned}$$

Now, it goes through the point $(1, 6)$:

$$6 = 3 + 2 + 4 + c \Rightarrow c = -3$$

and we have

$$\underline{\underline{\mathrm{f}(x) = 3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - 3}}.$$

7. Find $\int \left(6x^2 + 2 + x^{-\frac{1}{2}}\right)\,\mathrm{d}x$. (3)

Solution

$$\begin{aligned}\int \left(6x^2 + 2 + x^{-\frac{1}{2}}\right)\,\mathrm{d}x &= 6 \times \tfrac{1}{3}x^3 + 2x + 2x^{\frac{1}{2}} + c \\ &= \underline{\underline{2x^3 + 2x + 2x^{\frac{1}{2}} + c}}.\end{aligned}$$

8. The curve C with equation $y = \mathrm{f}(x)$, $x \neq 0$, passes through the point $(3, 7\frac{1}{2})$. Given that $\mathrm{f}'(x) = 2x + \dfrac{3}{x^2}$, find $\mathrm{f}(x)$. (5)

Solution

$$\begin{aligned}\mathrm{f}'(x) = 2x + \frac{3}{x^2} &\Rightarrow \mathrm{f}'(x) = 2x + 3x^{-2} \\ &\Rightarrow \mathrm{f}(x) = 2 \times \tfrac{1}{2}x^2 + 3 \times \left(\frac{x^{-1}}{-1}\right) + c \\ &\Rightarrow \mathrm{f}(x) = x^2 - 3x^{-1} + c.\end{aligned}$$

Now, it goes through the point $(3, 7\frac{1}{2})$:

$$7\tfrac{1}{2} = 9 - 1 + c \Rightarrow c = -\tfrac{1}{2}$$

and we have

$$\underline{\underline{\mathrm{f}(x) = x^2 - 3x^{-1} - \tfrac{1}{2}}}.$$

9. (a) Show that $(4 + 3\sqrt{x})^2$ can be written as $16 + k\sqrt{x} + 9x$, where k is a constant to be found. (2)

Solution

$$(4 + 3\sqrt{x})^2 = (4 + 3\sqrt{x})(4 + 3\sqrt{x}) = 16 + \underline{\underline{24}}\sqrt{x} + 9x.$$

(b) Find $\int (4 + 3\sqrt{x})^2\,\mathrm{d}x$. (3)

Solution

$$\begin{aligned}\int (4 + 3\sqrt{x})^2 \,\mathrm{d}x &= \int \left(16 + 24x^{\frac{1}{2}} + 9x\right) \,\mathrm{d}x \\ &= 16x + 24 \times \tfrac{2}{3}x^{\frac{3}{2}} + 9 \times \tfrac{1}{2}x^2 + c \\ &= \underline{\underline{16x + 16x^{\frac{3}{2}} + \tfrac{9}{2}x^2 + c}}.\end{aligned}$$

10. The curve C with equation $y = \mathrm{f}(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C. Given that (5)

$$\mathrm{f}'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

find $\mathrm{f}(x)$.

Solution

$$\begin{aligned}\mathrm{f}'(x) = 3x^2 - 6 - \frac{8}{x^2} &\Rightarrow \mathrm{f}'(x) = 3x^2 - 6 - 8x^{-2} \\ &\Rightarrow \mathrm{f}(x) = 3 \times \tfrac{1}{3}x^3 - 6x - 8 \times \left(\frac{x^{-1}}{-1}\right) + c \\ &\Rightarrow \mathrm{f}(x) = x^3 - 6x + 8x^{-1} + c.\end{aligned}$$

Now, it goes through the point $(2, 1)$:

$$1 = 8 - 12 + 4 + c \Rightarrow c = 1$$

and we have

$$\underline{\underline{\mathrm{f}(x) = x^3 - 6x + 8x^{-1} + 1}}.$$

11. Find $\int \left(3x^2 + 4\sqrt{x}\right) \,\mathrm{d}x$, $x > 0$. (3)

Solution

$$\int \left(3x^2 + 4\sqrt{x}\right)\, \mathrm{d}x = \int \left(3x^2 + 4x^{\frac{1}{2}}\right)\, \mathrm{d}x$$
$$= 3 \times \tfrac{1}{3}x^3 + 4 \times \tfrac{2}{3}x^{\frac{3}{2}} + c$$
$$= \underline{\underline{x^3 + \tfrac{8}{3}x^{\frac{3}{2}} + c}}.$$

12. The curve C with equation $y = \mathrm{f}(x)$ passes through the point $(5, 65)$. Given that $\mathrm{f}'(x) = 6x^2 - 10x - 12$, use integration to find $\mathrm{f}(x)$. (4)

Solution

$$\mathrm{f}'(x) = 6x^2 - 10x - 12 \Rightarrow \mathrm{f}(x) = 6 \times \tfrac{1}{3}x^3 - 10 \times \tfrac{1}{2}x^2 - 12x + c$$
$$\Rightarrow \mathrm{f}(x) = 2x^3 - 5x^2 - 12x + c.$$

Now, it goes through the point $(5, 65)$:

$$65 = 250 - 125 - 60 + c \Rightarrow c = 0$$

and we have

$$\underline{\underline{\mathrm{f}(x) = 2x^3 - 5x^2 - 12x}}.$$

13. Find $\int \left(3x^2 + 4x^5 - 7\right)\, \mathrm{d}x$.

Solution

$$\int \left(3x^2 + 4x^5 - 7\right)\, \mathrm{d}x = 3 \times \tfrac{1}{3}x^3 + 4 \times \tfrac{1}{6}x^6 - 7x + c$$
$$= \underline{\underline{x^3 + \tfrac{2}{3}x^6 - 7x + c}}.$$

14. The curve C with equation $y = \mathrm{f}(x)$, $x > 0$, and $\mathrm{f}'(x) = 4x - 6\sqrt{x} + \dfrac{8}{x^2}$. Given that the point $P(4, 1)$ lies on C, find $\mathrm{f}(x)$ and simplify your answer. (4)

Solution

$$\mathrm{f}'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2} \Rightarrow \mathrm{f}'(x) = 4x - 6x^{\frac{1}{2}} + 8x^{-2}$$
$$\Rightarrow \mathrm{f}(x) = 4 \times \tfrac{1}{2}x^2 - 6 \times \tfrac{2}{3}x^{\frac{3}{2}} + 8 \times \left(\frac{x^{-1}}{-1}\right) + c$$
$$\Rightarrow \mathrm{f}(x) = 2x^2 - 4x^{\frac{3}{2}} - 8x^{-1} + c.$$

Now, it goes through the point $(4, 1)$:

$$1 = 32 - 32 - 2 + c \Rightarrow c = 3$$

and we have

$$\underline{\underline{\mathrm{f}(x) = 2x^2 - 4x^{\frac{3}{2}} - 8x^{-1} + 3}}.$$

15. Find $\int \left(2 + 5x^2\right)\,\mathrm{d}x$.

Solution

$$\int \left(2 + 5x^2\right)\,\mathrm{d}x = 2x + 5 \times \tfrac{1}{3}x^3 + c$$
$$= \underline{\underline{2x + \tfrac{5}{3}x^3 + c}}.$$

16. The gradient of a curve C is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x^2+3)^2}{x^2}$, $x \neq 0$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 6 + 9x^{-2}$. (2)

Solution

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x^2+3)^2}{x^2} = \frac{x^4 + 6x^2 + 9}{x^2}$$
$$= \underline{\underline{x^2 + 6 + 9x^{-2}}}.$$

The point $(3, 20)$ lies on C.

(b) Find an equation for curve C in the form $y = \mathrm{f}(x)$. (6)

Solution

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 6 + 9x^{-2} \Rightarrow y = \tfrac{1}{3}x^3 + 6x + 9\left(\frac{x^{-1}}{-1}\right) + c$$
$$\Rightarrow y = \tfrac{1}{3}x^3 + 6x - 9x^{-1} + c.$$

Now, it goes through the point $(3, 20)$:

$$20 = 9 + 18 - 3 + c \Rightarrow c = -4$$

and we have

$$\underline{\underline{y = \tfrac{1}{3}x^3 + 6x - 9x^{-1} - 4}}.$$

17. Find $\int \left(12x^5 - 8x^3 + 3\right)\,\mathrm{d}x$, giving each term in its simplest form.

Solution

$$\int \left(12x^5 - 8x^3 + 3\right)\,\mathrm{d}x = 12 \times \tfrac{1}{6}x^6 - 8 \times \tfrac{1}{4}x^4 + 3x + c$$
$$= \underline{\underline{2x^6 - 2x^4 + 3x + c}}.$$

18. The curve has equation $y = \mathrm{f}(x)$ and passes through the point $(4, 22)$. Given that

$$\mathrm{f}'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find $\mathrm{f}(x)$, giving each term in its simplest form.

Solution

$$\mathrm{f}'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7 \Rightarrow \mathrm{f}(x) = 3 \times \tfrac{1}{3}x^3 - 3 \times \tfrac{2}{3}x^{\frac{3}{2}} - 7x + c$$
$$\Rightarrow \mathrm{f}(x) = x^3 - 2x^{\frac{3}{2}} - 7x + c.$$

Now, it goes through the point $(4, 22)$:

$$22 = 64 - 16 - 28 + c \Rightarrow c = 2$$

and we have

$$\underline{\underline{\mathrm{f}(x) = x^3 - 2x^{\frac{3}{2}} - 7x + 2.}}$$

19. Given that $y = 2x^3 + \dfrac{3}{x^2}$, $x \neq 0$, find $\displaystyle\int y \, \mathrm{d}x$, simplifying each term. (3)

Solution

$$\begin{aligned}\int \left(2x^3 + \frac{3}{x^2}\right) \mathrm{d}x &= \int \left(2x^3 + 3x^{-2}\right) \mathrm{d}x \\ &= 2 \times \tfrac{1}{4}x^4 + 3 \times \left(\frac{x^{-1}}{-1}\right) + c \\ &= \underline{\underline{\tfrac{1}{2}x^4 - 3x^{-1} + c}}.\end{aligned}$$

20. (7)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 5x^{-\frac{1}{2}} + x\sqrt{x}.$$

Given that $y = 35$ at $x = 4$, find y in terms of x, given each term in its simplest form.

Solution

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = 5x^{-\frac{1}{2}} + x\sqrt{x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= 5x^{-\frac{1}{2}} + x^{\frac{3}{2}} \\ \Rightarrow y &= 5 \times 2x^{\frac{1}{2}} + \tfrac{2}{5}x^{\frac{5}{2}} + c \\ \Rightarrow y &= 10x^{\frac{1}{2}} + \tfrac{2}{5}x^{\frac{5}{2}} + c.\end{aligned}$$

Now, it goes through the point $(4, 35)$:

$$35 = 20 + 12.8 + c \Rightarrow c = 2.2$$

and we have

$$\underline{\underline{y = 10x^{\frac{1}{2}} + \tfrac{2}{5}x^{\frac{5}{2}} + 2.2.}}$$

21. Find (4)

$$\int \left(8x^3 + 6x^{\frac{1}{2}} - 5\right) \mathrm{d}x,$$

given each term in its simplest form.

Solution

$$\begin{aligned}\int \left(8x^3 + 6x^{\frac{1}{2}} - 5\right) \mathrm{d}x &= 8 \times \tfrac{1}{4}x^4 + 6 \times \tfrac{2}{3}x^{\frac{3}{2}} - 5x + c \\ &= \underline{\underline{2x^4 + 4x^{\frac{3}{2}} - 5x + c}}.\end{aligned}$$

22. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where (5)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x - \frac{5}{\sqrt{x}} - 2.$$

Given that the point $P(4, 5)$ lies on C, find $\mathrm{f}(x)$.

Solution

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = 3x - \frac{5}{\sqrt{x}} - 2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= 3x - 5x^{-\frac{1}{2}} - 2 \\ \Rightarrow y &= 3 \times \tfrac{1}{2}x^2 - 5 \times 2x^{\frac{1}{2}} - 2x + c \\ \Rightarrow y &= \tfrac{3}{2}x^2 - 10x^{\frac{1}{2}} - 2x + c.\end{aligned}$$

Now, it goes through the point $(4, 5)$:

$$5 = 24 - 20 - 8 + c \Rightarrow c = 9$$

and we have

$$\underline{\underline{y = \tfrac{3}{2}x^2 - 10x^{\frac{1}{2}} - 2x + 9}}.$$

23. Find (5)

$$\int \left(12x^5 - 3x^2 + 4x^{\frac{1}{3}}\right) \mathrm{d}x,$$

given each term in its simplest form.

Solution

$$\int \left(12x^5 - 3x^2 + 4x^{\frac{1}{3}}\right) \mathrm{d}x = 12 \times \tfrac{1}{6}x^6 - 3 \times \tfrac{1}{3}x^3 + 4 \times \tfrac{3}{4}x^{\frac{4}{3}} + c$$
$$= \underline{\underline{2x^6 - x^3 + 3x^{\frac{4}{3}} + c}}.$$

24. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(-1, 0)$. Given that (5)

$$\mathrm{f}'(x) = 12x^2 - 8x + 1,$$

find $\mathrm{f}(x)$.

Solution

$$\mathrm{f}'(x) = 12x^2 - 8x + 1 \Rightarrow \mathrm{f}(x) = 12 \times \tfrac{1}{3}x^3 - 8 \times \tfrac{1}{2}x^2 + x + c$$
$$\Rightarrow \mathrm{f}(x) = 4x^3 - 4x^2 + x + c.$$

Now, it goes through the point $(-1, 0)$:

$$0 = -4 - 4 - 1 + c \Rightarrow c = 9$$

and we have

$$\underline{\underline{\mathrm{f}(x) = 4x^3 - 4x^2 + x + 9}}.$$

25. Given that $y = 2x^5 + 7 + \dfrac{1}{x^3}$, $x \neq 0$, find, in its simplest form, $\displaystyle\int y \, \mathrm{d}x$. (5)

Solution

$$\int \left(2x^5 + 7 + \frac{1}{x^3}\right) \mathrm{d}x = \int \left(2x^5 + 7 + x^{-3}\right) \mathrm{d}x$$
$$= 2 \times \tfrac{1}{6}x^6 + 7x + \frac{x^{-2}}{-2} + c$$
$$= \underline{\underline{\tfrac{1}{3}x^6 + 7x - \tfrac{1}{2}x^{-2} + c}}.$$

26. Given that $\dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$,

(a) write down the value of p and write down the value of q. (2)

Solution

$$\frac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}} = \frac{6x + 3x^{\frac{5}{2}}}{x^{\frac{1}{2}}} = \underline{\underline{6x^{\frac{1}{2}} + 3x^2}}.$$

Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$, and that $y = 90$ when $x = 4$,

(b) find y in terms of x, simplifying the coefficient of each terms. (5)

Solution

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} &= 6x^{\frac{1}{2}} + 3x^2 \\ \Rightarrow y &= 6 \times \tfrac{2}{3}x^{\frac{3}{2}} + 3 \times \tfrac{1}{3}x^3 + c \\ \Rightarrow y &= 4x^{\frac{3}{2}} + x^3 + c.\end{aligned}$$

Now, it goes through the point $(4, 90)$:

$$90 = 32 + 64 + c \Rightarrow c = -6$$

and we have

$$\underline{\underline{y = 4x^{\frac{3}{2}} + x^3 - 6}}.$$

27. Given that $y = x^4 + 6x^{\frac{1}{2}}$, find, in its simplest form, $\displaystyle\int y \, \mathrm{d}x$. (3)

Solution

$$\begin{aligned}\int \left(x^4 + 6x^{\frac{1}{2}}\right) \mathrm{d}x &= \tfrac{1}{5}x^5 + 6 \times \tfrac{2}{3}x^{\frac{3}{2}} + c \\ &= \underline{\underline{\tfrac{1}{5}x^5 + 4x^{\frac{3}{2}} + c}}.\end{aligned}$$

28. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(2, 10)$. Given that (5)

$$\mathrm{f}'(x) = 3x^2 - 3x + 5,$$

find $\mathrm{f}(1)$.

Solution

$$\begin{aligned}\mathrm{f}'(x) = 3x^2 - 3x + 5 &\Rightarrow \mathrm{f}(x) = 3 \times \tfrac{1}{3}x^3 - 3 \times \tfrac{1}{2}x^2 + 5x + c \\ &\Rightarrow \mathrm{f}(x) = x^3 - \tfrac{3}{2}x^2 + 5x + c.\end{aligned}$$

Now, it goes through the point $(2, 10)$:

$$10 = 8 - 6 + 10 + c \Rightarrow c = -2$$

and we have

$$\mathrm{f}(x) = x^3 - \tfrac{3}{2}x^2 + 5x - 2.$$

Now,

$$\mathrm{f}(1) = 1 - \tfrac{3}{2} + 5 - 2 = \underline{\underline{2\tfrac{1}{2}}}.$$

29. Find (4)

$$\int \left(6x^2 + \frac{2}{x^2} + 5\right)\,\mathrm{d}x,$$

giving each term in its simplest form.

Solution

$$\begin{aligned}\int \left(6x^2 + \frac{2}{x^2} + 5\right)\,\mathrm{d}x &= \int \left(6x^2 + 2x^{-2} + 5\right)\,\mathrm{d}x \\ &= 6 \times \tfrac{1}{3}x^3 + 2\left(\frac{x^{-1}}{-1}\right) + 5x + c \\ &= \underline{\underline{2x^3 - 2x^{-1} + 5x + c}}.\end{aligned}$$

30. The point $P(4, -1)$ lies on the curve C with equation $y = \mathrm{f}(x)$, $x > 0$, and (4)

$$\mathrm{f}'(x) = \tfrac{1}{2}x - \frac{6}{\sqrt{x}} + 3.$$

Find $\mathrm{f}(x)$.

Solution

$$\mathrm{f}'(x) = \tfrac{1}{2}x - \frac{6}{\sqrt{x}} + 3 \Rightarrow \mathrm{f}'(x) = \tfrac{1}{2}x - 6x^{-\frac{1}{2}} + 3$$
$$\Rightarrow \mathrm{f}(x) = \tfrac{1}{2} \times \tfrac{1}{2}x^2 - 6 \times 2x^{\frac{1}{2}} + 3x + c$$
$$\Rightarrow \mathrm{f}(x) = \tfrac{1}{4}x^2 - 12x^{\frac{1}{2}} + 3x + c.$$

Now, it goes through the point $(4, -1)$:

$$-1 = 4 - 24 + 12 + c \Rightarrow c = 7$$

and we have

$$\underline{\underline{\mathrm{f}(x) = \tfrac{1}{4}x^2 - 12x^{\frac{1}{2}} + 3x + 7}}.$$

31. (6)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -x^3 + \frac{4x - 5}{2x^3},\ x \neq 0.$$

Given that $y = 7$ at $x = 1$, find y in terms of x, giving each term in its simplest form.

Solution

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -x^3 + \frac{4x - 5}{2x^3} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -x^3 + 2x^{-2} - \tfrac{5}{2}x^{-3}$$
$$\Rightarrow y = -\tfrac{1}{4}x^4 + 2\left(\frac{x^{-1}}{-1}\right) - \tfrac{5}{2}\left(\frac{x^{-2}}{-2}\right) + c$$
$$\Rightarrow y = -\tfrac{1}{4}x^4 - 2x^{-1} + \tfrac{5}{4}x^{-2} + c.$$

Now, it goes through the point $(1, 7)$:

$$7 = -\tfrac{1}{4} - 2 + \tfrac{5}{4} + c \Rightarrow c = 8$$

and we have

$$\underline{\underline{y = -\tfrac{1}{4}x^4 - 2x^{-1} + \tfrac{5}{4}x^{-2} + 8}}.$$

32. Find (4)

$$\int \left(10x^4 - 4x - \frac{3}{\sqrt{x}}\right) \mathrm{d}x,$$

giving each term in its simplest form.

Solution

$$\begin{aligned}\int \left(10x^4 - 4x - \frac{3}{\sqrt{x}}\right)\,\mathrm{d}x &= \int \left(10x^4 - 4x - 3x^{-\frac{1}{2}}\right)\,\mathrm{d}x \\ &= 10 \times \tfrac{1}{5}x^5 - 4 \times \tfrac{1}{2}x^2 - 3 \times 2x^{\frac{1}{2}} + c \\ &= \underline{\underline{2x^5 - 2x^2 - 6x^{\frac{1}{2}} + c}}.\end{aligned}$$

33.

$$\mathrm{f}'(x) = \frac{(3-x^2)^2}{x^2},\ x \neq 0.$$

(a) Show that (3)

$$\mathrm{f}'(x) = 9x^{-2} + A + Bx^2,$$

where A and B are constants to be found.

Solution

$$\mathrm{f}'(x) = \frac{(3-x^2)^2}{x^2} = \frac{9 - 6x^2 + x^4}{x^2} = \underline{\underline{9x^{-2} - 6 + x^2}}.$$

Given that the point $(-3, 10)$ lies on the curve with equation $y = \mathrm{f}(x)$,

(b) find $\mathrm{f}(x)$. (5)

Solution

$$\begin{aligned}\mathrm{f}'(x) = \frac{(3-x^2)^2}{x^2} &\Rightarrow \mathrm{f}'(x) = 9x^{-2} - 6 + x^2 \\ &\Rightarrow \mathrm{f}(x) = 9 \times \left(\frac{x^{-1}}{-1}\right) - 6x + \tfrac{1}{3}x^3 + c \\ &\Rightarrow \mathrm{f}(x) = -9x^{-1} - 6x + \tfrac{1}{3}x^3 + c.\end{aligned}$$

Now, it goes through the point $(-3, 10)$:

$$10 = 3 + 18 - 9 + c \Rightarrow c = -2$$

and we have

$$\underline{\underline{\mathrm{f}(x) = -9x^{-1} - 6x + \tfrac{1}{3}x^3 - 2}}.$$

34. Find (4)

$$\int \left(3x^2 - \frac{4}{x^2}\right)\, \mathrm{d}x,$$

giving each term in its simplest form.

Solution

$$\begin{aligned}\int \left(3x^2 - \frac{4}{x^2}\right) \mathrm{d}x &= \int \left(3x^2 - 4x^{-2}\right) \mathrm{d}x \\ &= 3 \times \tfrac{1}{3}x^3 - 4\left(\frac{x^{-1}}{-1}\right) + c \\ &= \underline{\underline{x^3 + 4x^{-1} + c}}.\end{aligned}$$

35. The curve with equation $y = \mathrm{f}(x)$ passes through the point $P(9, 0)$. Given that (6)

$$\mathrm{f}'(x) = \frac{x+9}{\sqrt{x}},\ x > 0,$$

find $\mathrm{f}(x)$.

Solution

$$\begin{aligned}\mathrm{f}'(x) = \frac{x+9}{\sqrt{x}} &\Rightarrow \mathrm{f}'(x) = \frac{x+9}{x^{\frac{1}{2}}} \\ &\Rightarrow \mathrm{f}'(x) = x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} \\ &\Rightarrow \mathrm{f}(x) = \tfrac{2}{3}x^{\frac{3}{2}} + 9 \times 2x^{\frac{1}{2}} + c \\ &\Rightarrow \mathrm{f}(x) = \tfrac{2}{3}x^{\frac{3}{2}} + 18x^{\frac{1}{2}} + c.\end{aligned}$$

Now, it goes through the point $(9, 0)$:

$$0 = 18 + 54 + c \Rightarrow c = -72$$

and we have

$$\underline{\underline{\mathrm{f}(x) = \tfrac{2}{3}x^{\frac{3}{2}} + 18x^{\frac{1}{2}} - 72.}}$$

36. Find (3)

$$\int \left(8x^3 + 4\right)\, \mathrm{d}x,$$

giving each term in its simplest form.

Solution

$$\int \left(8x^3 + 4)\right)\, \mathrm{d}x = 8 \times \tfrac{1}{4}x^4 + 4x + c$$
$$= \underline{\underline{2x^4 + 4x + c}}.$$

37. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(4, 25)$. Given that (5)

$$\mathrm{f}'(x) = \tfrac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1,\ x > 0,$$

find $\mathrm{f}(x)$, simplifying each term.

Solution

$$\mathrm{f}'(x) = \tfrac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1 \Rightarrow \mathrm{f}(x) = \tfrac{3}{8} \times \tfrac{1}{3}x^3 - 10 \times 2x^{\frac{1}{2}} + x + c$$
$$\Rightarrow \mathrm{f}(x) = \tfrac{1}{8}x^3 - 20x^{\frac{1}{2}} + x + c.$$

Now, it goes through the point $(4, 25)$:

$$25 = 8 - 40 + 4 + c \Rightarrow c = 53$$

and we have

$$\underline{\underline{\mathrm{f}(x) = \tfrac{1}{8}x^3 - 20x^{\frac{1}{2}} + x + 53.}}$$

38. Given that $y = 2x^5 + \dfrac{6}{\sqrt{x}}$, $x > 0$, find, in their simplest form, $\displaystyle\int y\, \mathrm{d}x$. (3)

Solution

$$\int \left(2x^5 + \frac{6}{\sqrt{x}}\right)\,\mathrm{d}x = \int \left(2x^5 + 6x^{-\frac{1}{2}}\right)\,\mathrm{d}x$$
$$= 2 \times \tfrac{1}{6}x^6 + 6 \times 2x^{\frac{1}{2}} + c$$
$$= \underline{\underline{\tfrac{1}{3}x^6 + 12x^{\frac{1}{2}} + c.}}$$

39. (6)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^{-\frac{1}{2}} + x\sqrt{x},\ x > 0.$$

Given that $y = 37$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

Solution

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^{-\frac{1}{2}} + x\sqrt{x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 6x^{-\frac{1}{2}} + x^{\frac{3}{2}}$$
$$\Rightarrow y = 6 \times 2x^{\frac{1}{2}} + \tfrac{2}{5}x^{\frac{5}{2}} + c$$
$$\Rightarrow y = 12x^{\frac{1}{2}} + \tfrac{2}{5}x^{\frac{5}{2}} + c.$$

Now, it goes through the point $(4, 37)$:

$$37 = 24 + 12\tfrac{4}{5} + c \Rightarrow c = \tfrac{1}{5}$$

and we have

$$\underline{\underline{y = 12x^{\frac{1}{2}} + \tfrac{2}{5}x^{\frac{5}{2}} + \tfrac{1}{5}}}.$$

40. Given that $y = 4x^3 - \dfrac{5}{x^2}$, $x > 0$, find, in their simplest form, $\int y\,\mathrm{d}x$. (3)

Solution

$$\int \left(4x^3 - \frac{5}{x^2}\right)\,\mathrm{d}x = \int \left(4x^3 - 5x^{-2}\right)\,\mathrm{d}x$$
$$= 4 \times \tfrac{1}{4}x^4 - 5\left(\frac{x^{-1}}{-1}\right) + c$$
$$= \underline{\underline{x^4 + 5x^{-1} + c}}.$$

41. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(4, 9)$. Given that (5)

$$\mathrm{f}'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2,\ x > 0,$$

find $\mathrm{f}(x)$, simplifying each term.

Solution

$$\begin{aligned}\mathrm{f}'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2 &\Rightarrow \mathrm{f}'(x) = \tfrac{3}{2}x^{\frac{1}{2}} - \tfrac{9}{4}x^{-\frac{1}{2}} + 2 \\ &\Rightarrow \mathrm{f}(x) = \tfrac{3}{2} \times \tfrac{2}{3}x^{\frac{3}{2}} - \tfrac{9}{4} \times 2x^{\frac{1}{2}} + 2x + c \\ &\Rightarrow \mathrm{f}(x) = x^{\frac{3}{2}} - \tfrac{9}{2}x^{\frac{1}{2}} + 2x + c.\end{aligned}$$

Now, it goes through the point $(4, 9)$:

$$9 = 8 - 9 + 8 + c \Rightarrow c = 2$$

and we have

$$\underline{\underline{\mathrm{f}(x) = x^{\frac{3}{2}} - \tfrac{9}{2}x^{\frac{1}{2}} + 2x + 2}}.$$

42. Find (4)

$$\int \left(2x^4 - \frac{4}{\sqrt{x}} + 3\right) \mathrm{d}x,$$

giving each term in its simplest form.

Solution

$$\begin{aligned}\int \left(2x^4 - \frac{4}{\sqrt{x}} + 3\right) \mathrm{d}x &= \int \left(2x^4 - 4x^{-\frac{1}{2}} + 3\right) \mathrm{d}x \\ &= 2 \times \tfrac{1}{5}x^5 - 4 \times 2x^{\frac{1}{2}} + 3x + c \\ &= \underline{\underline{\tfrac{2}{5}x^5 - 8x^{\frac{1}{2}} + 3x + c}}.\end{aligned}$$

43. Find (4)

$$\int \left(2x^5 - \frac{1}{4x^3} - 5\right) \mathrm{d}x,$$

giving each term in its simplest form.

Solution

$$\int \left(2x^5 - \frac{1}{4x^3} - 5\right)\,\mathrm{d}x = \int \left(2x^5 - \tfrac{1}{4}x^{-3} - 5\right)\,\mathrm{d}x$$
$$= 2 \times \tfrac{1}{6}x^4 - \tfrac{1}{4} \times \left(-\tfrac{1}{2}x^{-2}\right) - 5x + c$$
$$= \underline{\underline{\tfrac{1}{3}x^4 + \tfrac{1}{8}x^{-2} - 5x + c}}.$$

44. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where (5)

$$\mathrm{f}'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}.$$

Given that the point $P(4, -8)$ lies on C, find $\mathrm{f}(x)$, giving each term in its simplest form.

Solution

$$\mathrm{f}'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}} \Rightarrow \mathrm{f}'(x) = 30 + 6x^{-\frac{1}{2}} - 5x^{\frac{3}{2}}$$
$$\Rightarrow \mathrm{f}(x) = 30x + 6 \times 2x^{\frac{1}{2}} - 5 \times \tfrac{2}{5}x^{\frac{5}{2}} + c$$
$$\Rightarrow \mathrm{f}(x) = 30x + 12x^{\frac{1}{2}} - 2x^{\frac{5}{2}} + c.$$

Now, it goes through the point $(4, -8)$:

$$-8 = 120 + 24 - 64 + c \Rightarrow c = -88$$

and we have

$$\underline{\underline{\mathrm{f}(x) = 30x + 12x^{\frac{1}{2}} - 2x^{\frac{5}{2}} - 88}}.$$