Dr Oliver Mathematics
Applied Mathematics: Differentiation

The total number of marks available is 49.
You must write down all the stages in your working.

1. Differentiate, and simplify as appropriate,
   (a) $f(x) = \exp(\tan \frac{1}{2}x)$, where $-\pi < x < \pi$, (3)
   (b) $g(x) = (x^3 + 1)\ln(x^3 + 1)$, where $x > 0$. (3)

2. Given that (5)
$$y = \ln(1 + \sin x),$$
   where $0 < x < \pi$, show that
$$\frac{d^2y}{dx^2} = \frac{-1}{1 + \sin x}.$$

3. (a) Given (2)
$$f(x) = x \tan 2x$$
   for $-\frac{1}{4}\pi < x < \frac{1}{4}\pi$, obtain an expression for $f'(x)$.
   (b) Show that (3)
$$f''(x) = 4 \sec^2 2x(1 + 2x \tan 2x).$$
   (c) Hence find the exact value of (4)
$$\int_0^{\frac{1}{6}\pi} \frac{1 + 2x \tan 2x}{\cos^2 2x} \, dx.$$

4. Differentiate the following, simplifying your answers as appropriate.
   (a) $f(x) = e^{2x} \tan x$, $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$. (3)
   (b) $g(x) = \dfrac{\cos 2x}{x^3}$. (4)

5. Differentiate the following, simplifying where possible.
   (a) $f(x) = \dfrac{1 + \sin x}{1 + 2\sin x}$, $0 \leqslant x \leqslant \pi$, (3)
   (b) $g(x) = \ln(1 + e^{2x})$. (2)

6. Given the curve (3)
$$y = \frac{x}{x^2 + 4},$$
   calculate the gradient when $x = 2$.

7. Given that (2)

$$y = \sin(e^{5x}),$$

find $\dfrac{dy}{dx}$.

8. Find the gradient of the tangent to the curve (4)

$$y = 2x\sqrt{x-1}$$

at the point where $x = 10$.

9. Given that (3)

$$y = e^{5x}\tan 2x,$$

find $\dfrac{dy}{dx}$.

10. A curve is defined by (5)

$$y = \frac{\sin x}{2 - \cos x} \text{ for } 0 \leqslant x \leqslant \pi.$$

Find the exact values of the coordinates of the stationary point of this curve.