Dr Oliver Mathematics
Worked Examples
Probability 1

From: Edexcel 2018 November Paper 1H (Non-Calculator)

1. There are only green pens and blue pens in a box. (6)

   There are three more blue pens than green pens in the box.
   There are more than 12 pens in the box.

   Simon is going to take at random two pens from the box.
   The probability that Simon will take two pens of the same colour is $\frac{27}{55}$.

   Work out the number of green pens in the box.

Solution

Let n be the number of green pens. Then $(n + 3)$ is the number of blue pens and $(2n + 3)$ is the total number of pens.

Now, the two ways that Simon can take the same colour is GG or BB.

GG: n is the number of ways that Simon can choose a green pen and there are $(2n + 3)$ pens:

$$\tfrac{n}{2n+3} \times \ldots$$

If that happens, $(n - 1)$ is the number of ways that Simon can choose *another* green pen and there are

$$(2n + 3) - 1 = 2n + 2 \text{ pens:}$$

$$\tfrac{n}{2n+3} \times \tfrac{n-1}{2n+2}.$$

We can tidy the denominator of the second fraction:

$$\tfrac{n}{2n+3} \times \tfrac{n-1}{2(n+1)}.$$

BB: $(n + 3)$ is the number of ways that Simon can choose a blue pen and there are $(2n + 3)$ pens:

$$\tfrac{n+3}{2n+3} \times \ldots$$

If that happens,

$$(n + 3) - 1 = n + 2$$

is the number of ways that Simon can choose *another* blue pen and there are

$$(2n+3)-1 = 2n+2 = 2(n+1) \text{ pens}:$$

$$\tfrac{n}{2n+3} \times \tfrac{n-1}{2(n+1)}.$$

Hence,

$$\begin{aligned} \text{P(same colour)} &= \text{P}(GG) + \text{P}(BB) \\ &= \left(\tfrac{n}{2n+3} \times \tfrac{n-1}{2(n+1)}\right) + \left(\tfrac{n+3}{2n+3} \times \tfrac{n+2}{2(n+1)}\right) \\ &= \tfrac{n(n-1)}{2(n+1)(2n+3)} + \tfrac{(n+3)(n+2)}{2(n+1)(2n+3)} \\ &= \tfrac{n(n-1)+(n+3)(n+2)}{2(n+1)(2n+3)} \end{aligned}$$

Next,

$$\begin{aligned} \text{P(same colour)} = \tfrac{27}{55} &\Rightarrow \text{P}(GG) + \text{P}(BB) = \tfrac{27}{55} \\ &\Rightarrow \tfrac{n(n-1)+(n+3)(n+2)}{2(n+1)(2n+3)} = \tfrac{27}{55} \end{aligned}$$

Cross-multipling:

$$\Rightarrow 55[n(n-1)+(n+3)(n+2)] = 54(n+1)(2n+3)$$

Expand the brackets:

×	n	$+3$
n	n^2	$+3n$
$+2$	$+2n$	$+6$

×	$2n$	$+3$
n	$2n^2$	$+3n$
$+1$	$+2n$	$+3$

$$\begin{aligned} &\Rightarrow 55[(n^2-n)+(n^2+5n+6)] = 54(2n^2+5n+3) \\ &\Rightarrow 55(2n^2+4n+6) = 54(2n^2+5n+3) \\ &\Rightarrow 110n^2+220n+330 = 108n^2+270n+162 \\ &\Rightarrow 2n^2-50n+168 = 0 \\ &\Rightarrow 2(n^2-25n+84) = 0 \end{aligned}$$

Factorise the quadratic equation:

$$\begin{matrix} \text{add to:} & -25 \\ \text{multiply to:} & +84 \end{matrix} \Bigg\} -4, -21$$

$$\Rightarrow 2(n-4)(n-21) = 0$$
$$\Rightarrow n-4 \text{ or } n-21 = 0$$
$$\Rightarrow n = 4 \text{ or } n = 21;$$

because there are more than 12 pens (the third line of the question!) in the box, $\underline{\underline{n = 21}}$.