Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2005 June Paper 2: Calculator
2 hours

The total number of marks available is 80.
You must write down all the stages in your working.

1. A curve has the equation
$$y = \frac{8}{2x-1}.$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. (3)

Solution
Well,
$$\begin{aligned} y = \frac{8}{2x-1} &\Rightarrow y = 8(2x-1)^{-1} \\ &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 8 \cdot (-2)(2x-1)^{-2} \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = -16(2x-1)^{-2}}}. \end{aligned}$$

(b) Given that y is increasing at a rate of 0.2 units per second when $x = -0.5$, find the corresponding rate of change of x. (2)

Solution
Well,
$$x = -0.5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -4$$
and
$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} &\Rightarrow 0.2 = -4\frac{\mathrm{d}x}{\mathrm{d}t} \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}x}{\mathrm{d}t} = -0.05}}. \end{aligned}$$

2. A flower show is held over a three-day period – Thursday, Friday, and Saturday.

The table below shows the entry price per day for an adult and for a child, and the number of adults, and children attending on each day.

	Thursday	Friday	Saturday
Price (£) - Adult	12	10	10
Price (£) - Child	5	4	4
Number of adults	300	180	400
Number of children	40	40	150

(a) Write down two matrices such that their product will give the amount of entry money paid on Thursday and hence calculate this product. (2)

Solution

E.g.,

$$\begin{pmatrix} 300 & 40 \end{pmatrix} \begin{pmatrix} 12 \\ 5 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3\,800 \end{pmatrix}}}.$$

(b) Write down two matrices such that the elements of their product give the amount of entry money paid for each of Friday and Saturday and hence calculate this product. (2)

Solution

Well,

$$\begin{pmatrix} 180 & 40 \\ 400 & 150 \end{pmatrix} \text{ and } \begin{pmatrix} 10 \\ 4 \end{pmatrix}$$

and their product is

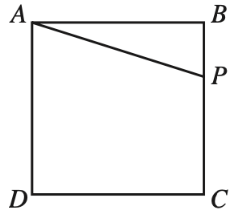
$$\begin{pmatrix} 180 & 40 \\ 400 & 150 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1\,960 \\ 4\,600 \end{pmatrix}}}.$$

(c) Calculate the total amount of entry money paid over the three-day period. (1)

Solution

$$3\,800 + 1\,960 + 4\,600 = \underline{\underline{£10\,360}}.$$

3. The diagram shows a square $ABCD$ of area 60 m^2. (5)



The point P lies on BC and the sum of the lengths of AP and BP is 12 m.

Given that the lengths of AP and BP are x m and y m respectively, form two equations in x and y and hence find the length of BP.

Solution

Well,

$$x + y = 12 \quad (1)$$

and Pythagoras' theorem

$$\begin{aligned} AB^2 + BP^2 = AP^2 &\Rightarrow 60 + y^2 = x^2 \\ &\Rightarrow x^2 - y^2 = 60 \quad (2). \end{aligned}$$

Now,

$$x^2 - y^2 = 60 \Rightarrow x^2 - (12 - x)^2 = 60$$

$\times$	12	$-x$
12	144	$-12x$
$-x$	$-12x$	$+x^2$

$$\begin{aligned} &\Rightarrow x^2 - (144 - 24x + x^2) = 60 \\ &\Rightarrow -144 + 24x = 60 \\ &\Rightarrow 24x = 204 \\ &\Rightarrow x = 8\tfrac{1}{2} \\ &\Rightarrow \underline{\underline{y = 3\tfrac{1}{2}}}. \end{aligned}$$

4. The functions f and g are defined by (5)

$$\mathrm{f} : x \mapsto \sin x,\ 0 \leqslant x \leqslant \tfrac{1}{2}\pi,$$
$$\mathrm{g} : x \mapsto 2x - 3,\ x \in \mathbb{R}.$$

Solve the equation

$$\mathrm{g}^{-1}\,\mathrm{f}(x) = \mathrm{g}^2(2.75).$$

Solution

Now,

$$\begin{aligned} y = 2x - 3 &\Rightarrow y + 3 = 2x \\ &\Rightarrow \tfrac{1}{2}(y+3) = x \end{aligned}$$

and so

$$\mathrm{g}^{-1}(x) = \tfrac{1}{2}(x+3)$$

and

$$\begin{aligned} \mathrm{g}^{-1}\,\mathrm{f}(x) &= \mathrm{g}^{-1}(\mathrm{f}(x)) \\ &= \mathrm{g}^{-1}(\sin x) \\ &= \tfrac{1}{2}(\sin x + 3). \end{aligned}$$

Next,

$$\begin{aligned} \mathrm{g}^2(2.75) &= \mathrm{g}(\mathrm{g}(2.75)) \\ &= \mathrm{g}(2.5) \\ &= 2. \end{aligned}$$

Finally,

$$\begin{aligned} \mathrm{g}^{-1}\,\mathrm{f}(x) = \mathrm{g}^2(2.75) &\Rightarrow \tfrac{1}{2}(\sin x + 3) = 2 \\ &\Rightarrow \sin x + 3 = 4 \\ &\Rightarrow \sin x = 1 \\ &\Rightarrow \underline{\underline{x = \tfrac{1}{2}\pi}}. \end{aligned}$$

5. (a) Differentiate (2)

$$x \ln x - x$$

with respect to x.

Solution

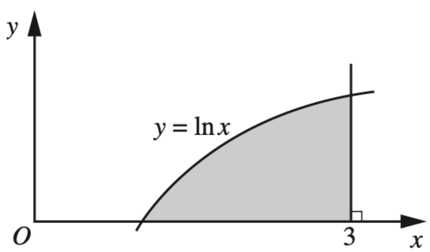
Product rule:

$$u = x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1$$
$$v = \ln x \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{x}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}x}(x \ln x - x) = \left[(x)\left(\frac{1}{x}\right) + (1)(\ln x)\right] - 1$$
$$= \underline{\underline{\ln x}}.$$

The diagram shows part of the graph of $y = \ln x$.



(b) Use your result from part (a) to evaluate the area of the shaded region bounded by the curve, the line $x = 3$, and the x-axis. (4)

Solution

Now,

$$\text{area} = \int_1^3 \ln x \, \mathrm{d}x$$
$$= [x \ln x - x]_{x=1}^3$$
$$= (3 \ln 3 - 3) - (0 - 1)$$
$$= \underline{\underline{3 \ln 3 - 2}}.$$

6. A curve has the equation

$$y = \frac{\mathrm{e}^{2x}}{\sin x}, \text{ for } 0 < x < \pi.$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and show that the x-coordinate of the stationary point satisfies (4)

$$2\sin x - \cos x = 0.$$

Solution

Quotient rule:

$$u = \mathrm{e}^{2x} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2\mathrm{e}^{2x}$$
$$v = \sin x \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x} = \cos x$$

and

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{(\sin x)(2\mathrm{e}^{2x}) - (\cos x)(\mathrm{e}^{2x})}{(\sin x)^2} \\ &= \frac{2\mathrm{e}^{2x}\sin x - \mathrm{e}^{2x}\cos x}{(\sin^2 x} \\ &= \underline{\underline{\frac{\mathrm{e}^{2x}(2\sin x - \cos x)}{\sin^2 x}}}\end{aligned}$$

and

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = 0 &\Rightarrow \frac{\mathrm{e}^{2x}(2\sin x - \cos x)}{\sin^2 x} = 0 \\ &\Rightarrow \underline{\underline{2\sin x - \cos x = 0}},\end{aligned}$$

as both $\mathrm{e}^{2x} > 0$ and $\sin^2 x > 0$.

(b) Find the x-coordinate of the stationary point. (2)

Solution

$$\begin{aligned}2\sin x - \cos x = 0 &\Rightarrow 2\sin x = \cos x \\ &\Rightarrow \tan x = \tfrac{1}{2} \\ &\Rightarrow x = 0.463\,647\,609 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 0.464 \text{ (3 sf)}}}.\end{aligned}$$

7. Solve, for x and y, the simultaneous equations (6)

$$
\begin{aligned}
125^x &= 25(5^y), \\
7^x \div 49^y &= 1.
\end{aligned}
$$

Solution

Now,

$$
\begin{aligned}
25^x = 25(5^y) &\Rightarrow (5^3)^x = (5^2)(5^y) \\
&\Rightarrow 5^{3x} = 5^{2+y} \\
&\Rightarrow 3x = 2 + y \quad (1)
\end{aligned}
$$

and

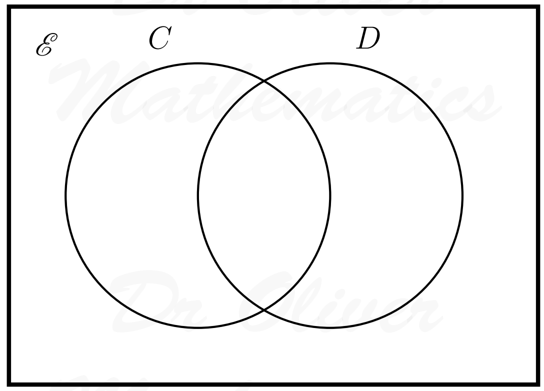
$$
\begin{aligned}
7^x \div 49^y = 1 &\Rightarrow 7^x \div (7^2)^y = 1 \\
&\Rightarrow 7^x = 7^{2y} \\
&\Rightarrow x = 2y \quad (2).
\end{aligned}
$$

Next,

$$
\begin{aligned}
3x = 2 + y &\Rightarrow 3(2y) = 2 + y \\
&\Rightarrow 5y = 2 \\
&\Rightarrow \underline{\underline{y = \tfrac{2}{5}}} \\
&\Rightarrow \underline{\underline{x = \tfrac{4}{5}}}.
\end{aligned}
$$

8. The Venn diagram below represents the sets

- $\mathscr{E}$ = {homes in a certain town},
- C = {homes with a computer}, and
- D = {homes with a dishwasher}.

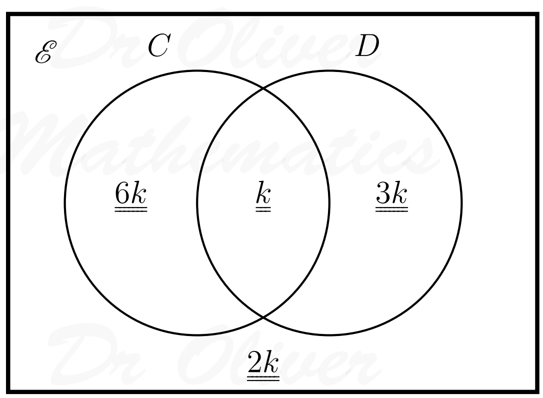


It is given that

- $\text{n}(C \cap D) = k$,
- $\text{n}(C) = 7 \times \text{n}(C \cap D)$,
- $\text{n}(D) = 4 \times \text{n}(C \cap D)$, and
- $\text{n}(\mathscr{E}) = 6 \times \text{n}(C' \cap D')$.

(a) Copy the Venn diagram above and insert, in each of its four regions, the number, in terms of k, of homes represented by that region. (5)

Solution



There are

$$\begin{aligned} \text{n}(C \cup D) = \tfrac{5}{6}\,\text{n}(\mathscr{E}) &\Rightarrow \text{n}(\mathscr{E}) = \tfrac{6}{5}\,\text{n}(C \cup D) \\ &\Rightarrow \text{n}(\mathscr{E}) = \tfrac{6}{5} \times 10k \\ &\Rightarrow \text{n}(\mathscr{E}) = 12k. \end{aligned}$$

(b) Given that there are 165 000 homes which do not have both a computer and a dishwasher, calculate the number of homes in the town. (2)

Solution

Well,

$$11k = 165\,000 \Rightarrow k = 15\,000$$

and

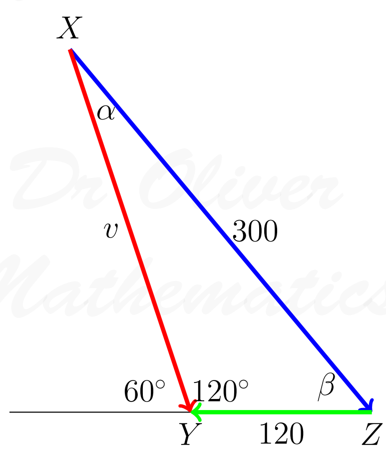
$$\mathrm{n}(\mathscr{E}) = \underline{\underline{180\,000}}.$$

9. A plane, whose speed in still air is 300 km h^{-1}, flies directly from X to Y. (7)

Given that Y is 720 km from X on a bearing of 150° and that there is a constant wind of 120 km h^{-1} blowing towards the west, find the time taken for the flight.

Solution

Let v ms^{-1} be the speed of the aircraft.



Sine rule:

$$\frac{\sin YXZ}{YZ} = \frac{\sin XYZ}{XZ} \Rightarrow \frac{\sin \alpha^\circ}{120} = \frac{\sin 120^\circ}{300}$$
$$\Rightarrow \sin \alpha^\circ = \frac{120 \sin 120^\circ}{300}$$
$$\Rightarrow \alpha = 20.267\,901\,06 \text{ (FCD)}.$$

Now,

$$\beta = 180 - (120 + \alpha) = 39.732\,090\,94 \text{ (FCD)}$$

and we apply the sine rule once again:

$$\frac{XY}{\sin XZY} = \frac{XZ}{\sin XYZ} \Rightarrow \frac{v}{\sin 39.732\ldots^\circ} = \frac{300}{\sin 120^\circ}$$
$$\Rightarrow v = \frac{300 \sin 39.732\ldots^\circ}{\sin 120^\circ}$$
$$\Rightarrow v = 221.424\,945\,6 \text{ (FCD)}.$$

Finally,

$$\begin{aligned}\text{time taken} &= \frac{\text{distance}}{\text{speed}} \\ &= \frac{720}{221.424\ldots} \\ &= 3.251\,666\,148 \text{ (FCD)} \\ &= \underline{\underline{3.25 \text{ hours (3 sf)}}}.\end{aligned}$$

10. (a) Solve, for $0^\circ < x < 360^\circ$, (4)

$$4\tan^2 x + 15\sec x = 0.$$

Solution

$$\begin{aligned}4\tan 2x + 15\sec x = 0 &\Rightarrow 4(\sec^2 x - 1) + 15\sec x = 0 \\ &\Rightarrow 4\sec^2 x + 15\sec x - 4 = 0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & +15 \\ \text{multiply to:} & (+4)\times(-4) = -16\end{array}\right\} -1,\ +16$$

e.g.,

$$\begin{aligned}&\Rightarrow 4\sec^2 x + 16\sec x - \sec x - 4 = 0 \\ &\Rightarrow 4\sec x(\sec x + 4) - 1(\sec x + 4) = 0 \\ &\Rightarrow (4\sec x - 1)(\sec x + 4) = 0 \\ &\Rightarrow \sec x = \tfrac{1}{4} \text{ or } \sec x = -4 \\ &\Rightarrow \cos x = 4 \text{ (impossible) or } \cos x = -\tfrac{1}{4} \\ &\Rightarrow x = 104.477\,512\,2,\ 255.522\,487\,8 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 105,\ 256 \text{ (3 sf)}}}.\end{aligned}$$

(b) Given that $y > 3$, find the smallest value of y such that (4)

$$\tan(3y - 2) = -5.$$

Solution

$$\begin{aligned} & \tan(3y-2) = -5 \\ \Rightarrow\ & 3y - 2 = -1.373\ldots,\ 1.768\ldots,\ 4.909\ldots,\ 8.051\,377\,194 \text{ (FCD)} \\ \Rightarrow\ & 3y = 10.051\,377\,194 \text{ (FCD)} \\ \Rightarrow\ & y = 3.350\,459\,065 \text{ (FCD)} \\ \Rightarrow\ & \underline{\underline{y = 3.35 \text{ (3 sf)}}}. \end{aligned}$$

11. (a) (i) Expand (3)

$$(2 + x)^5.$$

Solution

$$\begin{aligned} (2+x)^5 &= 2^5 + \binom{5}{1}(2^4)(x) + \binom{5}{2}(2^3)(x^2) \\ &\quad + \binom{5}{3}(2^2)(x^3) + \binom{5}{4}(2)(x^4) + x^5 \\ &= \underline{\underline{32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5}}. \end{aligned}$$

(ii) Use your answer to part (a)(i) to find the integers a and b for which (3)

$$(2 + \sqrt{3})^5$$

can be expressed in the form

$$a + b\sqrt{3}.$$

Solution

$$\begin{aligned} (2+\sqrt{3})^5 &= 32 + 80(\sqrt{3}) + 80(\sqrt{3})^2 + 40(\sqrt{3})^3 + 10(\sqrt{3})^4 + (\sqrt{3})^5 \\ &= 32 + 80\sqrt{3} + 240 + 120\sqrt{3} + 90 + 9\sqrt{3} \\ &= \underline{\underline{362 + 209\sqrt{3}}}; \end{aligned}$$

so, $\underline{\underline{a = 362}}$ and $\underline{\underline{b = 209}}$.

(b) Find the coefficient of x in the expansion of (3)

$$\left(x - \frac{4}{x}\right)^7.$$

Solution

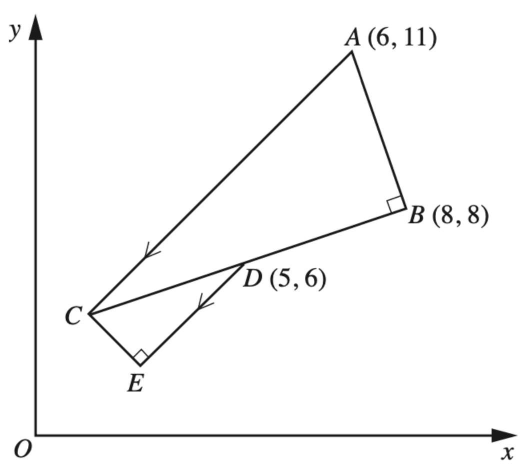
$$\begin{aligned}\left(x - \frac{4}{x}\right)^7 &= \ldots + \binom{7}{4}(x^4)\left(-\frac{4}{x}\right)^3 + \ldots \\ &= \ldots - 2\,240x + \ldots;\end{aligned}$$

hence, the coefficient of x is $\underline{\underline{-2\,240}}$.

Answer only **one** of the following two alternatives.

EITHER

12. **Solutions to this question by accurate drawing will not be accepted**. (11)
The diagram, which is not drawn to scale, shows a right-angled triangle ABC, where A is the point $(6, 11)$ and B is the point $(8, 8)$.



The point $D(5, 6)$ is the mid-point of BC.

The line DE is parallel to AC and angle DEC is a right-angle.

Find the area of the entire figure $ABDECA$.

Solution

Now,

$$\begin{aligned} m_{AB} &= \frac{11-8}{6-8} \\ &= -\tfrac{3}{2} \end{aligned}$$

and

$$m_{BC} = -\frac{1}{-\frac{3}{2}} = \tfrac{2}{3}.$$

Next,

$$\begin{aligned} \overrightarrow{OC} &= \overrightarrow{OB} + 2\overrightarrow{BD} \\ &= \begin{pmatrix} 8 \\ 8 \end{pmatrix} + 2\begin{pmatrix} 5-8 \\ 6-8 \end{pmatrix} \\ &= \begin{pmatrix} 8 \\ 8 \end{pmatrix} + 2\begin{pmatrix} -3 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 4 \end{pmatrix}; \end{aligned}$$

so, $C(2, 4)$. Now,

$$\begin{aligned} m_{AC} &= \frac{11-4}{6-2} \\ &= \tfrac{7}{4} \end{aligned}$$

so

$$m_{DE} = m_{AC} = \tfrac{7}{4}$$

and

$$m_{CE} = -\frac{1}{\frac{7}{4}} = -\tfrac{4}{7}.$$

Next, the equation of DE is

$$\begin{aligned} y - 6 = \tfrac{7}{4}(x-5) &\Rightarrow y - 6 = \tfrac{7}{4}x - \tfrac{35}{4} \\ &\Rightarrow y = \tfrac{7}{4}x - \tfrac{11}{4} \quad (1) \end{aligned}$$

and the equation of CE is

$$y - 4 = -\tfrac{4}{7}(x-2) \Rightarrow y - 4 = -\tfrac{4}{7}x + \tfrac{8}{7}$$
$$\Rightarrow y = -\tfrac{4}{7}x + \tfrac{36}{7} \quad (2).$$

Equate (1) = (2):

$$\tfrac{7}{4}x - \tfrac{11}{4} = -\tfrac{4}{7}x + \tfrac{36}{7} \Rightarrow \tfrac{65}{28}x = \tfrac{221}{28}$$
$$\Rightarrow x = 3\tfrac{2}{5}$$
$$\Rightarrow y = 3\tfrac{1}{5};$$

so, $E(3\frac{2}{5}, 3\frac{1}{5})$.

$\underline{\triangle ABC}$:

$$\begin{aligned}\text{area of } \triangle ABC &= \tfrac{1}{2} \times AB \times BC \\ &= \tfrac{1}{2} \times \sqrt{(6-8)^2 + (11-8)^2} \times \sqrt{(2-8)^2 + (4-8)^2} \\ &= \tfrac{1}{2} \times \sqrt{13} \times \sqrt{52} \\ &= 13.\end{aligned}$$

$\underline{\triangle CDE}$:

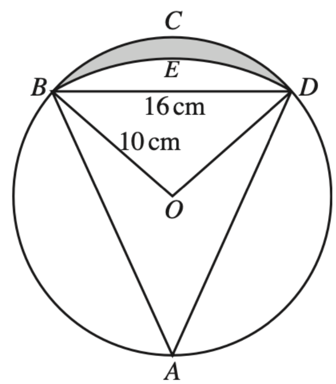
$$\begin{aligned}\text{area of } \triangle CDE &= \tfrac{1}{2} \times CE \times ED \\ &= \tfrac{1}{2} \times \sqrt{(2-3\tfrac{2}{5})^2 + (4-3\tfrac{1}{5})^2} \times \sqrt{(5-3\tfrac{2}{5})^2 + (6-3\tfrac{1}{5})^2} \\ &= \tfrac{1}{2} \times \tfrac{1}{5}\sqrt{65} \times \tfrac{2}{5}\sqrt{65} \\ &= 2\tfrac{3}{5}.\end{aligned}$$

Finally,

$$\begin{aligned}\text{area of } ABDECA &= \text{area of } \triangle ABC + \text{area of } \triangle CDE \\ &= 13 + 2\tfrac{3}{5} \\ &= \underline{\underline{15\tfrac{3}{5} \text{ cm}}}.\end{aligned}$$

OR

13. The diagram, which is not drawn to scale, shows a circle $ABCDA$, centre O and radius 10 cm.



The chord BD is 16 cm long.

BED is an arc of a circle, centre A.

(a) Show that the length of AB is approximately 17.9 cm. (6)

Solution

So, $OD = 10$ cm (isosceles triangle) and applying the cosine rule:

$$\begin{aligned} \cos BOD &= \frac{OB^2 + OD^2 - BD^2}{2 \times OB \times OD} \\ \Rightarrow \quad \cos BOD &= \frac{10^2 + 10^2 - 16^2}{2 \times 10 \times 10} \\ \Rightarrow \quad \cos BOD &= -\tfrac{7}{25} \\ \Rightarrow \quad \angle BOD &= 106.260\,204\,7 \text{ (FCD)} \\ \Rightarrow \quad \angle BAD &= 53.130\,102\,235 \text{ (FCD)} \\ \Rightarrow \quad \angle BAO &= 26.565\,051\,18 \text{ (FCD)}. \end{aligned}$$

Finally, apply the sine rule:

$$\begin{aligned} \frac{AB}{\sin AOB} = \frac{OB}{\sin BAO} \Rightarrow \frac{AB}{\sin(180 - 2 \times 26.565\ldots)} &= \frac{10}{\sin 26.565\ldots} \\ \Rightarrow AB &= \frac{10 \sin 126.869\ldots}{\sin 26.565\ldots} \\ \Rightarrow AB &= 17.888\,543\,82 \text{ (FCD)} \\ \Rightarrow \underline{\underline{AB = 17.9 \text{ cm (3 sf)}}}&, \end{aligned}$$

as required.

For the shaded region enclosed by the arcs BCD and BED, find

(b) its perimeter, (2)

Solution

$$\begin{aligned}\text{Perimeter} &= \text{perimeter } BCD + \text{perimeter } BED\\ &= \left(10 \times \frac{106.260\ldots}{180} \times \pi\right) + \left(17.888\ldots \times \frac{53.130\ldots}{180} \times \pi\right)\\ &= 18.545\ldots + 16.587\ldots\\ &= 35.133\,865\,5 \text{ (FCD)}\\ &= \underline{\underline{35.1 \text{ cm (3 sf)}}}.\end{aligned}$$

(c) its area. (3)

Solution

Well,

$$\text{area} = \text{area of segment } BCDAB - \text{area of segment } BEDAB.$$

What do we need for that?

- Twice the area of triangle OAB,
- AB,
- area of segment $BCDAB$, and
- area of segment $BEDAB$.

Okay? Deep breath ...

- Twice the area of triangle OAB:

$$\begin{aligned}\text{area} &= 2 \times \tfrac{1}{2} \times 10^2 \times \sin[180 - (2 \times 26.565\ldots)]\\ &= 80.\end{aligned}$$

- AB:

$$\begin{aligned}& AB^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 126.869\ldots\\ \Rightarrow\quad & AB^2 = 320\\ \Rightarrow\quad & AB = 8\sqrt{5}.\end{aligned}$$

- area of segment $BCDAB$:

$$\begin{aligned} \text{area} &= \tfrac{1}{2} \times 10^2 \times \left(\frac{106.260\ldots}{180} \times \pi \right) + 80 \\ &= 92.729\,521\,8 + 80 \\ &= 172.729\,521\,8 \text{ (FCD)}. \end{aligned}$$

- area of segment $BEDAB$:

$$\begin{aligned} \text{area} &= \tfrac{1}{2} \times (8\sqrt{5})^2 \times \left(\frac{53.130\ldots}{180} \times \pi \right) \\ &= 148.367\,234\,9 \text{ (FCD)}. \end{aligned}$$

- Subtract:

$$\begin{aligned} \text{area} &= \text{area of segment } BCDAB - \text{area of segment } BEDAB \\ &= 172.729\ldots - 148.367\ldots \\ &= 24.362\,286\,92 \text{ (FCD)} \\ &= \underline{\underline{24.4 \text{ cm}^2 \text{ (3 sf)}}}. \end{aligned}$$