

Dr Oliver Mathematics

Arithmetic Sequences

In this note, we investigate arithmetic sequences. The easiest way to do this is to do a few examples.

1. Here are the first five terms of an arithmetic sequence:

$$2 \quad 5 \quad 8 \quad 11 \quad 14.$$

Find, in terms of n , an expression for the n th term of this sequence.

Solution

Let the

$$n\text{th term} = an + b.$$

$$\begin{array}{l} \text{Write down the sequence:} \quad 2 \quad 5 \quad 8 \quad 11 \quad 14 \\ (n+1)\text{st} - n\text{th term:} \quad 3 \quad 3 \quad 3 \quad 3 \end{array}$$

Compare this with the first five $(an + b)$ s:

$$\begin{array}{l} n = 1: \quad a \times 1 + b = a + b, \\ n = 2: \quad a \times 2 + b = 2a + b, \\ n = 3: \quad a \times 3 + b = 3a + b, \\ n = 4: \quad a \times 4 + b = 4a + b, \text{ and} \\ n = 5: \quad a \times 5 + b = 5a + b. \end{array}$$

Now,

$$\begin{array}{l} \text{write down the sequence:} \quad a + b \quad 2a + b \quad 3a + b \quad 4a + b \quad 5a + b \\ (n+1)\text{st} - n\text{th term:} \quad a \quad a \quad a \quad a \end{array}$$

We compare terms:

$$a = 3$$

and

$$\begin{array}{l} a + b = 2 \Rightarrow 3 + b = 2 \\ \Rightarrow b = -1. \end{array}$$

Hence,

$$n\text{th term} = \underline{\underline{3n - 1}}.$$

2. Here are the first five terms of an arithmetic sequence:

$$9 \quad 16 \quad 23 \quad 30 \quad 37.$$

Find, in terms of n , an expression for the n th term of this sequence.

Solution

Let the

$$n\text{th term} = an + b.$$

Write down the sequence:	9	16	23	30	37
$(n + 1)$ st - n th term:	7	7	7	7	

Compare this with the first five $(an + b)$ s:

$$\begin{aligned}n = 1: & \quad a \times 1 + b = a + b, \\n = 2: & \quad a \times 2 + b = 2a + b, \\n = 3: & \quad a \times 3 + b = 3a + b, \\n = 4: & \quad a \times 4 + b = 4a + b, \text{ and} \\n = 5: & \quad a \times 5 + b = 5a + b.\end{aligned}$$

Now,

write down the sequence:	$a + b$	$2a + b$	$3a + b$	$4a + b$	$5a + b$
$(n + 1)$ st - n th term:	a	a	a	a	

We compare terms:

$$a = 7$$

and

$$\begin{aligned}a + b = 9 & \Rightarrow 7 + b = 9 \\ & \Rightarrow b = 2.\end{aligned}$$

Hence,

$$n\text{th term} = \underline{\underline{7n + 2}}.$$

3. Here are the first four terms of an arithmetic sequence:

$$10 \quad 6 \quad 2 \quad -2.$$

Find, in terms of n , an expression for the n th term of this sequence.

Solution

Let the

$$nth \text{ term} = an + b.$$

10	6	2	-2
-4	-4	-4	
$a + b$	$2a + b$	$3a + b$	$4a + b$
a	a	a	

We compare terms:

$$a = -4$$

and

$$\begin{aligned} a + b = 10 &\Rightarrow -4 + b = 10 \\ &\Rightarrow b = 14. \end{aligned}$$

Hence,

$$nth \text{ term} = \underline{\underline{14 - 4n}}.$$

4. Here are the first five terms of an arithmetic sequence:

$$x \quad 11 \quad y \quad 19 \quad 23.$$

Find x and y .

Solution

Let the

$$nth \text{ term} = an + b.$$

x	11	y	19	23
$11 - x$	$y - 11$	$y - 19$	4	
$a + b$	$2a + b$	$3a + b$	$4a + b$	$5a + b$
a	a	a	a	

Dr Oliver
Mathematics

So

$$a = 4.$$

Next,

$$11 - x = 4 \Rightarrow \underline{\underline{x = 7}}$$

and

$$19 - y = 4 \Rightarrow \underline{\underline{y = 15.}}$$

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