Dr Oliver Mathematics
AQA GCSE Mathematics
2020 June Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Circle the fraction that is equivalent to 4.75. (1)

$$\frac{15}{4} \quad \frac{19}{4} \quad \frac{21}{4} \quad \frac{23}{4}$$

Solution

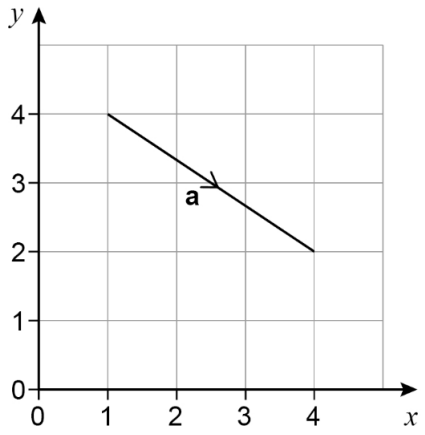
Well,

$$4.75 = \frac{16}{4} + \frac{3}{4}$$
$$= \frac{19}{4}$$

so

$$\frac{15}{4} \quad \underline{\underline{\frac{19}{4}}} \quad \frac{21}{4} \quad \frac{23}{4}$$

2. Here is vector **a**. (1)



Circle the column vector that represents **a**.

$$\begin{pmatrix}3\\2\end{pmatrix} \quad \begin{pmatrix}-3\\2\end{pmatrix} \quad \begin{pmatrix}3\\-2\end{pmatrix} \quad \begin{pmatrix}-3\\-2\end{pmatrix}$$

Solution

$$\begin{pmatrix}3\\2\end{pmatrix} \quad \begin{pmatrix}-3\\2\end{pmatrix} \quad \underline{\underline{\begin{pmatrix}3\\-2\end{pmatrix}}} \quad \begin{pmatrix}-3\\-2\end{pmatrix}$$

3. Which one of these is a square number **and** a cube number? (1)
Circle your answer.

$$100 \quad 1\,000 \quad 10\,000 \quad 1\,000\,000$$

Solution

Well,

$$\begin{aligned} 100 &= 10^2 \text{ but it is not a cube number,} \\ 1\,000 &= 10^3 \text{ but it is not a square number,} \\ 10\,000 &= 100^2 \text{ but it is not a cube number, and} \\ 1\,000\,000 &= 1\,000^2 \textbf{ and } 100^3 \end{aligned}$$

so

$$100 \quad 1\,000 \quad 10\,000 \quad \underline{\underline{1\,000\,000}}$$

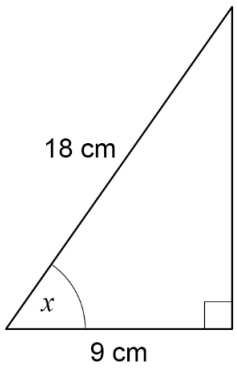
4. Circle the reciprocal of $\frac{5}{6}$. (1)

$$\frac{6}{5} \quad \frac{1}{6} \quad -\frac{1}{6} \quad -\frac{6}{5}$$

Solution

$$\underline{\underline{\frac{6}{5}}} \quad \frac{1}{6} \quad -\frac{1}{6} \quad -\frac{6}{5}$$

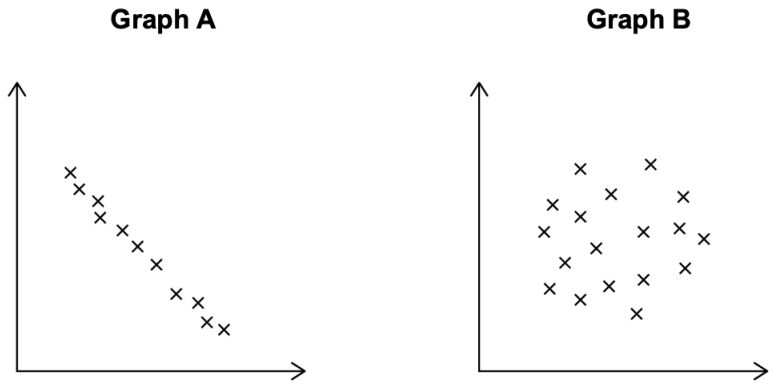
5. Use trigonometry to work out the size of angle x. (2)



Solution

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos x = \tfrac{9}{18}$$
$$\Rightarrow \cos x = \tfrac{1}{2}$$
$$\Rightarrow \underline{\underline{x = 60^\circ}}.$$

6. A and B are scatter graphs. (2)



What type of correlation is shown by each graph?
Choose from

Weak positive
Strong positive
Weak negative
Strong negative
No correlation

Solution

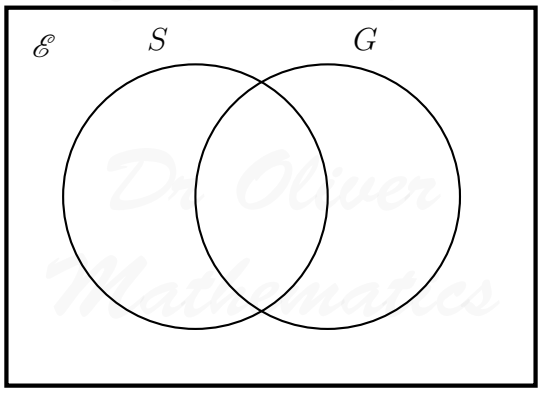
Graph A is Strong negative whereas graph B is No correlation.

7. Here is some information about 80 people who play in bands. (4)

- 12 are singers but not guitar players.
- 30% are neither a singer nor a guitar player.
- 41 of the guitar players are also singers.

Complete this Venn diagram to represent the information.

- $\mathscr{E}$ = 80 people who play in bands.
- S = singers.
- G = guitar players.



Solution

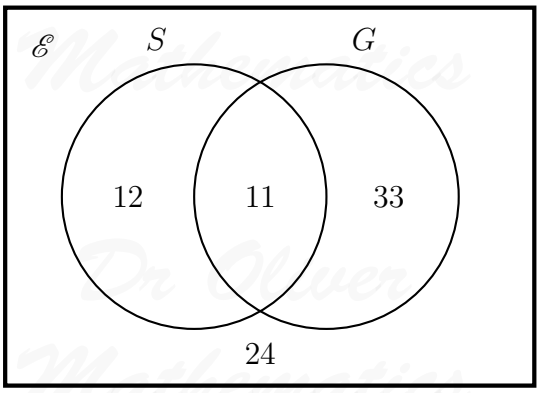
Well,

$$80 - 12 - 24 = 44$$

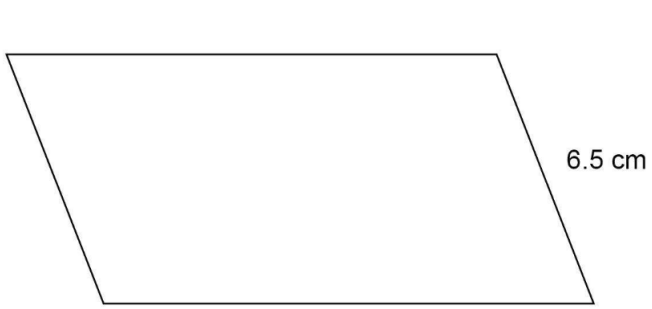
and

$$\tfrac{1}{4} \times 44 = 11.$$

Hence,



8. The shorter side of a parallelogram has length 6.5 cm. (3)



The length of the shorter side is $\frac{1}{9}$ of the perimeter.

Work out the length of the longer side.

Solution

Let the length of the longer side of x cm. Well,

$$\begin{aligned}\text{perimeter} &= 6.5 \times 9 \\ &= (6.5 \times 10) - 6.5 \\ &= 65 - 6.5 \\ &= 58.5\end{aligned}$$

and

$$\begin{aligned}2(x + 6.5) = 58.5 &\Rightarrow 2x + 13 = 58.5 \\ &\Rightarrow 2x = 45.5 \\ &\Rightarrow \underline{\underline{x = 22.75}}.\end{aligned}$$

9. (a) All the terms of a **geometric** progression are positive. (2)
The second and fourth terms are shown.

$$\ldots \; 4 \; \ldots \; 16$$

Work out the first and third terms.

Solution

Well, let the first term be a and the common ratio be r. Then

$$\begin{aligned}ar &= 4 \quad (1) \\ ar^3 &= 16 \quad (2).\end{aligned}$$

Divide (2) $\div$ (1):

$$\begin{aligned}r^2 = 4 &\Rightarrow r = 2 \\ r^2 = 4 &\Rightarrow a = 2.\end{aligned}$$

Hence, the first and third terms are $\underline{\underline{2}}$ and $2 \times 2^2 = \underline{\underline{8}}$ respectively.

(b) The first two terms of an arithmetic progression are shown. (3)

$$p \; 5p \; \ldots$$

The sum of the first three terms is 90.

Work out the value of p.

Solution

Well,

$$\begin{aligned}\text{third term} &= (5p - p) + 5p \\ &= 9p.\end{aligned}$$

Finally,

$$\begin{aligned}p + 5p + 9p = 90 &\Rightarrow 15p = 90 \\ &\Rightarrow \underline{\underline{p = 6}}.\end{aligned}$$

10. The cost of a holiday is £2 400. (4)
Rana pays a deposit followed by monthly payments, in the ratio

deposit : total of the monthly payments = 3 : 5

She makes 6 equal monthly payments.

Work out her monthly payment.

Solution

Well,

$$3 + 5 = 8$$

and so she pays

$$\tfrac{5}{8} \times 2\,400 = 5 \times 300 = 1\,500$$

in her monthly payments.

Finally, each monthly payment is

$$\frac{1\,500}{6} = \underline{\underline{£250}}.$$

11. As a decimal, (2)

$$\tfrac{11}{40} = 0.275.$$

Work out

$$\frac{33}{400}$$

as a decimal.

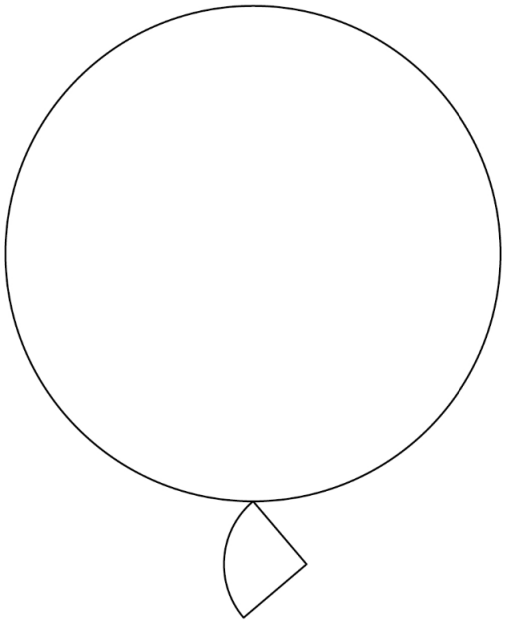
Solution

We have multiply by 3 and the divide by 10 (or vice versa):

$$\frac{11}{40} = 0.275 \Rightarrow \frac{33}{40} = 0.825$$
$$\Rightarrow \frac{33}{400} = 0.082\,5.$$

12. Two wire shapes make an earring.
The shapes are

- a circle with radius 21 mm and
- a quarter circle.



Not drawn accurately

radius of circle : radius of quarter circle = 7 : 2

12

(a) Show that the radius of the quarter circle is 6 mm. (1)

Solution

Well, the radius of the quarter circle is

$$\begin{aligned} \tfrac{2}{7} \times 21 &= 2 \times 3 \\ &= \underline{\underline{6 \text{ mm}}}, \end{aligned}$$

as required.

(b) Work out the **total** length of the wire in the earring. (4)
Give your answer in the form

$$a\pi + b,$$

where a and b are integers.

Solution

$$\begin{aligned} \text{Total length} &= \text{circle} + \text{quarter circle} \\ &= (2 \times \pi \times 21) + (\tfrac{1}{4} \times 2 \times \pi \times 6 + 6 + 6) \\ &= 42\pi + (3\pi + 12) \\ &= \underline{\underline{(45\pi + 12) \text{ cm}}}; \end{aligned}$$

hence, $\underline{\underline{a = 45}}$ and $\underline{\underline{b = 12}}$.

13. (a) s and t are **positive** integers. (2)

- $(x + s)(x - t)$ is expanded and simplified.
- The answer is

$$x^2 + kx - 40,$$

where k is a positive integer.

Work out the smallest possible value of k.

Solution

$\times$	x	$+s$
x	x^2	$+sx$
$-t$	$-tx$	$-st$

so

$$(x+s)(x-t) = x^2 + (s-t)x - st.$$

We want $s - t > 0$:

$$\begin{aligned}(x+40)(x-1) &= x^2 + 39x - 40\\ (x+20)(x-2) &= x^2 + 18x - 40\\ (x+10)(x-4) &= x^2 + 6x - 40\\ (x+8)(x-5) &= x^2 + 3x - 40\end{aligned}$$

and we run out of pairs. Hence,

$$\underline{\underline{k = 3}}.$$

(b) Faisal tries to solve (1)

$$(x+2)(x-7) = 0.$$

Here is his working.

	$(x+2)=0$	or	$(x-7)=0$
Answer	$x=2$	or	$x=7$

Give a reason why his answer is wrong.

Solution

E.g., $\underline{\underline{x = -2}}$.

14. (a) (2)

$$c = 2^{10} \times 3 \times 5^6.$$

Work out $18c$.
Give your answer as a product of prime factors in index form.

Solution

$$\begin{aligned}18c &= (2 \times 3^2) \times (2^{10} \times 3 \times 5^6)\\ &= \underline{\underline{2^{11} \times 3^3 \times 5^6}}.\end{aligned}$$

(b) Work out (2)

$$\sqrt[3]{\frac{2^7 \times 11^3}{2}}.$$

Give your answer as an integer.

Solution

$$\begin{aligned}\sqrt[3]{\frac{2^7 \times 11^3}{2}} &= \sqrt[3]{2^6 \times 11^3} \\ &= \sqrt[3]{(2^2 \times 11)^3} \\ &= 2^2 \times 11 \\ &= 4 \times 11 \\ &= \underline{\underline{44}}.\end{aligned}$$

15. (1)

$$3x = \tfrac{1}{2}y.$$

Circle the ratio $x : y$

$$6 : 1 \quad 1 : 6 \quad 3 : 2 \quad 2 : 3$$

Solution

Well,

$$3x = \tfrac{1}{2}y \Rightarrow 6x = y$$

and so

$$6 : 1 \quad \underline{\underline{1 : 6}} \quad 3 : 2 \quad 2 : 3.$$

16. A sequence of numbers is formed by the iterative process (2)

$$u_{n+1} = \frac{4}{u_n - 1} \text{ and } u_1 = 9.$$

Work out the values of u_2 and u_3.

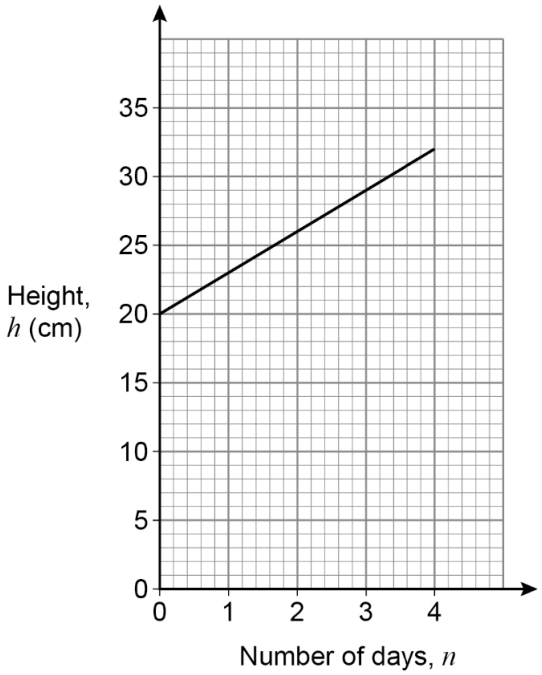
Solution

Well,

$$\begin{aligned} u_2 &= \frac{4}{u_1 - 1} \\ &= \frac{4}{9-1} \\ &= \underline{\underline{\tfrac{1}{2}}} \end{aligned}$$

$$\begin{aligned} u_3 &= \frac{4}{u_2 - 1} \\ &= \frac{4}{\frac{1}{2} - 1} \\ &= \frac{4}{-\frac{1}{2}} \\ &= \underline{\underline{-8}}. \end{aligned}$$

17. Jim buys a plant of height 20 cm. (3)
The graph shows how the height of the plant changes during the next 4 days.



Work out a formula for h in terms of n.

Solution

Well,

$$\begin{aligned} m &= \frac{32-20}{4-0} \\ &= \frac{12}{4} \\ &= 3 \end{aligned}$$

and the equation is

$$\underline{\underline{h = 3n + 12}}.$$

18. Solve the simultaneous equations (4)

$$\begin{aligned} 2x + 4y &= -9 \\ 2y &= 4x - 7. \end{aligned}$$

Solution

Well,

$$\begin{aligned} 2x + 4y = -9 &\Rightarrow 2x + 2(2y) = -9 \\ &\Rightarrow 2x + 2(4x - 7) = -9 \\ &\Rightarrow 2x + 8x - 14 = -9 \\ &\Rightarrow 10x = 5 \\ &\Rightarrow \underline{\underline{x = \tfrac{1}{2}}} \\ &\Rightarrow 2y = 4(\tfrac{1}{2}) - 7 \\ &\Rightarrow 2y = -5 \\ &\Rightarrow \underline{\underline{y = -2\tfrac{1}{2}}}. \end{aligned}$$

19. Circle the expression that is equivalent to (1)

$$\tfrac{1}{5}x + \tfrac{1}{10}x.$$

$$\tfrac{3}{10}x \quad \tfrac{2}{15}x \quad \tfrac{1}{25}x \quad \tfrac{1}{50}x^2$$

Solution

$$\begin{aligned}\tfrac{1}{5}x + \tfrac{1}{10}x &= \tfrac{2}{10}x + \tfrac{1}{10}x \\ &= \tfrac{3}{10}x\end{aligned}$$

so

$$\underline{\underline{\tfrac{3}{10}x}} \quad \tfrac{2}{15}x \quad \tfrac{1}{25}x \quad \tfrac{1}{50}x^2$$

20. (a) Write down the value of 7^0. (1)

Solution

$$7^0 = \underline{\underline{1}}.$$

(b) Work out the value of $32^{-\frac{3}{5}}$. (2)

Solution

$$\begin{aligned}32^{-\frac{3}{5}} &= \frac{1}{(32^{\frac{1}{5}})^3} \\ &= \frac{1}{2^3} \\ &= \underline{\underline{\tfrac{1}{8}}}.\end{aligned}$$

21. Write these numbers in order of size. (2)

$$15.6 \quad 3\sqrt{23} \quad 2.1^4 \quad \tfrac{47}{3}$$

Start with the smallest.

Solution

Well,

$$\begin{aligned} 3\sqrt{23} &\approx 3\sqrt{25} \\ &= 3 \times 5 \\ &= 15, \\ 2.1^4 &\approx 2^4 \\ &= 16, \text{ and} \\ \tfrac{47}{3} &= 15\tfrac{2}{3}. \end{aligned}$$

So, starting with the smallest,

$$\underline{\underline{3\sqrt{23} \quad 15.6 \quad \tfrac{47}{3} \quad 2.1^4}}.$$

22. (a) y is directly proportional to x^3. (3)

$y = 17$ when $x = 4$.

Work out an equation connecting y and x.

Solution

Well,

$$y \propto x^3 \Rightarrow y = kx^3,$$

where k is some constant. Now,

$$\begin{aligned} x = 4, y = 17 &\Rightarrow 17 = k \times 4^3 \\ &\Rightarrow k = \tfrac{17}{64}; \end{aligned}$$

hence,

$$\underline{\underline{y = \tfrac{17}{64}x^3}}.$$

(b) m is inversely proportional to $\sqrt{r}$. (1)

The value of r is multiplied by 4.

Circle what happens to the value of m.

$$\times 2 \quad \times 16 \quad \div 2 \quad \div 16$$

Solution

Well,

$$m \propto \frac{1}{\sqrt{r}} \Rightarrow m = \frac{k}{\sqrt{r}},$$

where k is some constant. Now,

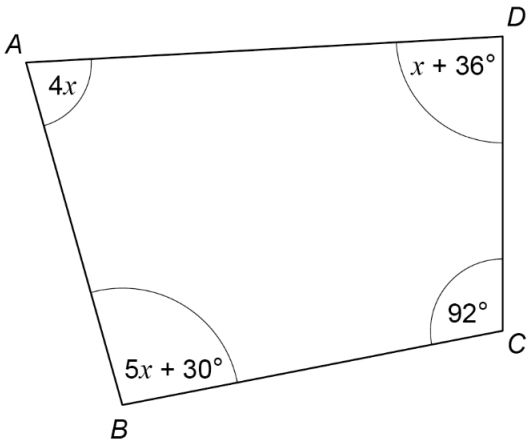
$$\begin{aligned} m &= \frac{k}{\sqrt{4r}} \\ &= \frac{k}{2\sqrt{r}} \end{aligned}$$

so

$$\times 2 \quad \times 16 \quad \underline{\underline{\div 2}} \quad \div 16$$

23. $ABCD$ is a quadrilateral. (4)

Not drawn accurately



Prove that $ABCD$ is **not** a cyclic quadrilateral.

Solution

Well,

$$\begin{aligned} 4x + (x + 36) + 92 + (5x + 30) = 360 &\Rightarrow 10x + 158 = 360 \\ &\Rightarrow 10x = 202 \\ &\Rightarrow x = 20.2 \\ &\Rightarrow 4x = 80.8. \end{aligned}$$

Now,

$$\begin{aligned} \angle BAD + \angle BCD &= 80.8 + 92 \\ &= 172.8 \\ &\neq 180, \end{aligned}$$

so $ABCD$ is not a cyclic quadrilateral.

24. y is an obtuse angle. (1)

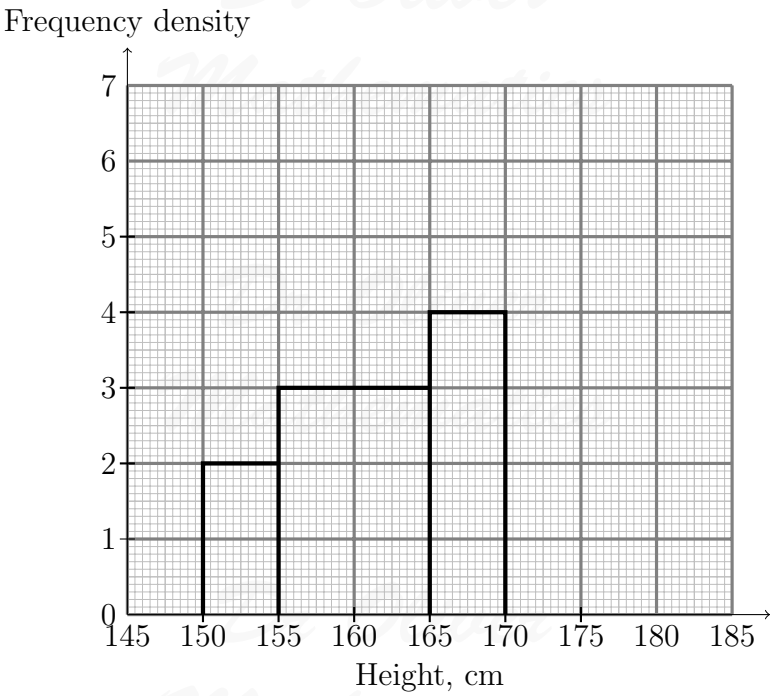
Which statement is true?
Tick **one** box.

- ☐ $\sin y > 0$ and $\cos y > 0$
- ☐ $\sin y > 0$ and $\cos y < 0$
- ☐ $\sin y < 0$ and $\cos y > 0$
- ☐ $\sin y < 0$ and $\cos y < 0$

Solution

Tick $\sin y > 0$ and $\cos y < 0$.

25. A histogram is drawn to represent the heights of a sample of women. (4)
Three of the four bars are shown.
The bar for 170 cm $\leqslant$ height $<$ 180 cm is missing.



There are 74 women in the sample.

Complete the histogram.

Solution

We make up a table:

Interval	Frequency	Width	Frequency Density
$150 \leqslant h < 155$	$5 \times 2 = 10$	5	2
$155 \leqslant h < 165$	$10 \times 3 = 30$	10	3
$165 \leqslant h < 170$	$5 \times 4 = 20$	5	4
$170 \leqslant h < 180$			

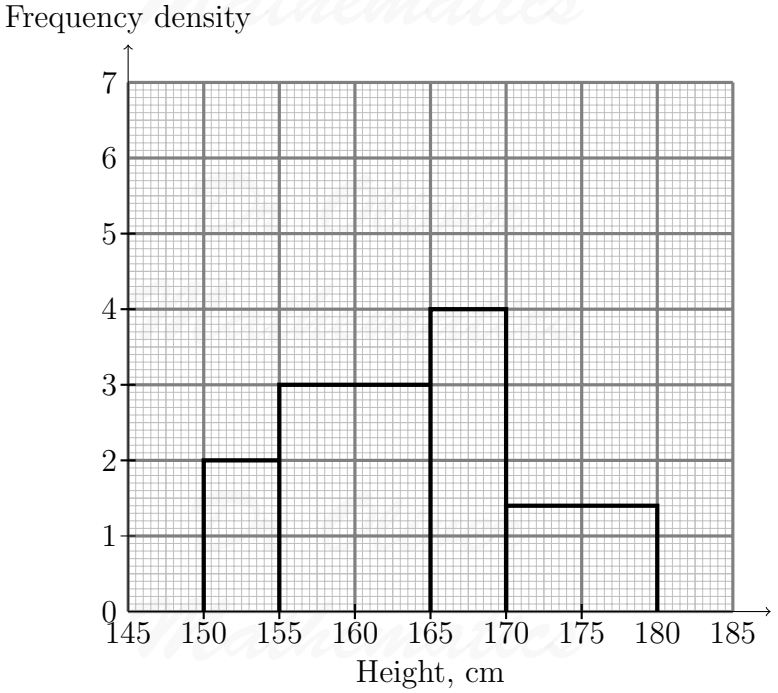
We know that they are 74 women:

$$74 - (10 + 30 + 20) = 14.$$

So

Interval	Frequency	Width	Frequency Density
$150 \leqslant h < 155$	$5 \times 2 = 10$	5	2
$155 \leqslant h < 165$	$10 \times 3 = 30$	10	3
$165 \leqslant h < 170$	$5 \times 4 = 20$	5	4
$170 \leqslant h < 180$	14	10	$\frac{14}{10} = 1.4$

and we complete the histogram:



26. (a) Show that (2)

$$\frac{14}{\sqrt{7}}$$

can be written in the form

$$a\sqrt{b},$$

where a and b are integers.

Solution

$$\frac{14}{\sqrt{7}} = \frac{14}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$$
$$= \frac{14\sqrt{7}}{7}$$
$$= \underline{\underline{2\sqrt{7}}};$$

hence, $\underline{\underline{a = 2}}$ and $\underline{\underline{b = 7}}$.

(b) Work out (3)

$$2\sqrt{10} \times \sqrt{80} \times \sqrt{18}.$$

Give your answer as an integer.

Solution

$$2\sqrt{10} \times \sqrt{80} \times \sqrt{18} = (2\sqrt{2 \times 5}) \times (\sqrt{2^4 \times 5}) \times (\sqrt{2 \times 3^2})$$
$$= (2\sqrt{2}\sqrt{5}) \times (2^2\sqrt{5}) \times (3\sqrt{2})$$
$$= 24 \times 2 \times 5$$
$$= \underline{\underline{240}}.$$

27. A and B are similar solid cylinders. (2)

Base area of A : base area of $B = 9 : 25$.

Complete these ratios.

Curved surface area of A : curved surface area of B = ... : .....

Height of A : height of B = ... : .....

Solution

Curved surface area of A : curved surface area of $B = \underline{\underline{9 : 25}}$

Height of A : height of $B = \sqrt{9} : \sqrt{25} = \underline{\underline{3 : 5}}$.

28. Factorise fully (2)

$$144 - 4x^2.$$

Solution

$$144 - 4x^2 = 4(36 - x^2)$$

difference of two squares:

$$= \underline{\underline{4(6 - x)(6 + x)}}.$$

29. The graph of (4)

$$y = x^3 + 6$$

is translated 4 units to the right.
The translated graph has equation $y = \mathrm{f}(x)$.

Work out $\mathrm{f}(x)$.
Give your answer in the form

$$x^3 + ax^2 + bx + c,$$

where a, b, and c are integers.

Solution

Well,

$$\mathrm{f}(x) = (x - 4)^3 + 6$$

$\times$	x	-4
x	x^2	$-4x$
-4	$-4x$	$+16$

$$= (x - 4)(x^2 - 8x + 16) + 6$$

$\times$	x^2	$-8x$	$+16$
x	x^3	$-8x^2$	$+16x$
-4	$-4x^2$	$+32x$	-64

$$= (x^3 - 12x^2 + 48x - 64) + 6$$
$$= \underline{\underline{x^3 - 12x^2 + 48x - 58}};$$

hence, $\underline{\underline{a = -12}}$, $\underline{\underline{b = 48}}$, and $\underline{\underline{c = -58}}$.