Dr Oliver Mathematics
GCSE Mathematics
2006 November Paper 5H: Non-Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. Mr Brown makes some compost.
   He mixes soil, manure, and leaf mould in the ratio $3 : 1 : 1$.
   Mr Brown makes 75 litres of compost.

   (a) How many litres of soil does he use? (3)

   Solution

   $$3 + 1 + 1 = 5$$

   and so he uses

   $$\frac{3}{5} \times 75 = 3 \times 15 = \underline{\underline{15 \text{ litres}}}.$$

   Mr Brown sows 200 flower seeds.
   For each flower seed the probability that it will produce a flower is 0.8.

   (b) Work out an estimate for the number of these flower seeds that will produce a flower. (2)

   Solution

   $$200 \times 0.8 = \underline{\underline{160}}.$$

2. Here are the first five terms of a number sequence:

   $$3 \quad 7 \quad 11 \quad 15 \quad 19$$

   (a) Write down an expression, in terms of n, for the nth term of this sequence. (2)

   Solution

   Let the

   $$n\text{th term} = an + b.$$

$$\begin{array}{ccccccccc} 3 & & 7 & & 11 & & 15 & & 19 \\ & 4 & & 4 & & 4 & & 4 & \end{array}$$

$$\begin{array}{ccccccccc} a+b & & 2a+b & & 3a+b & & 4a+b & & 5a+b \\ & a & & a & & a & & a & \end{array}$$

We compare terms:

$$a = 4$$

and

$$\begin{aligned} a + b = 4 &\Rightarrow 4 + b = 3 \\ &\Rightarrow b = -1. \end{aligned}$$

Hence,

$$n\text{th term} = \underline{\underline{4n - 1}}.$$

Adeel says that 319 is a term in the number sequence.

(b) Is Adeel correct? (2)
You must justify your answer.

Solution

$$\begin{aligned} 4n - 1 = 319 &\Rightarrow 4n = 320 \\ &\Rightarrow n = 80; \end{aligned}$$

<u>yes, it is: the 80th term of the sequence</u>.

3. The density of concrete is 2.3 grams per cm^3.

(a) Work out the mass of a piece of concrete with a volume of 20 cm^3. (2)

Solution

$$\begin{aligned} \text{Mass} &= \text{density} \times \text{volume} \\ &= 2.3 \times 20 \\ &= \underline{\underline{46 \text{ g}}}. \end{aligned}$$

480 grams of a cheese has a volume of 400 cm^3.

(b) Work out the density of the cheese. (2)

Solution

$$\begin{aligned}\text{Density} &= \frac{\text{mass}}{\text{volume}} \\ &= \frac{480}{400} \\ &= \underline{\underline{1.2 \text{ grams per cm}^3}}.\end{aligned}$$

4. Estimate the value of (2)

$$\frac{21 \times 3.86}{0.207}.$$

Solution

Round to 1 significant figure:

$$\begin{aligned}\frac{21 \times 3.86}{0.207} &\approx \frac{20 \times 4}{0.2} \\ &= \frac{80}{0.2} \\ &= \underline{\underline{400}}.\end{aligned}$$

5. (a) Solve (3)

$$3(x - 4) = x + 24.$$

Solution

$$\begin{aligned}3(x-4) = x + 24 &\Rightarrow 3x - 12 = x + 24 \\ &\Rightarrow 2x = 36 \\ &\Rightarrow \underline{\underline{x = 18}}.\end{aligned}$$

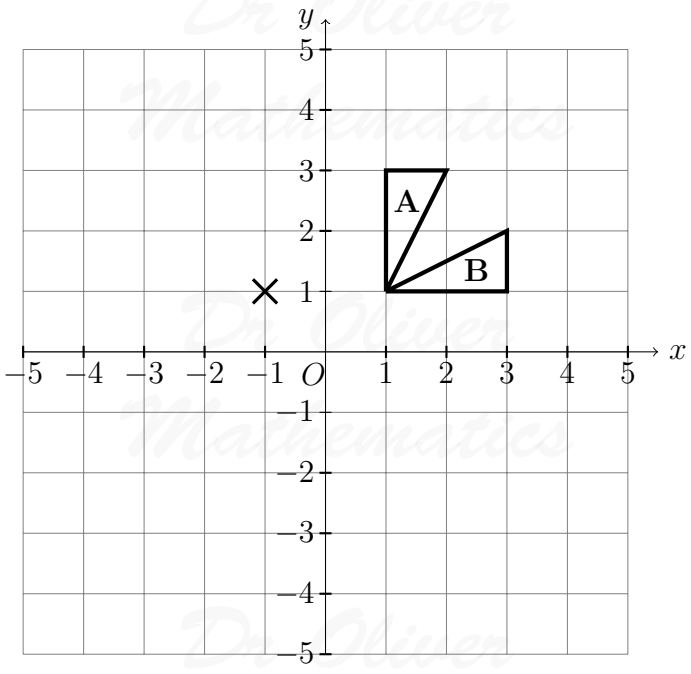
(b) Simplify fully (2)

$$(2x^3y)^4.$$

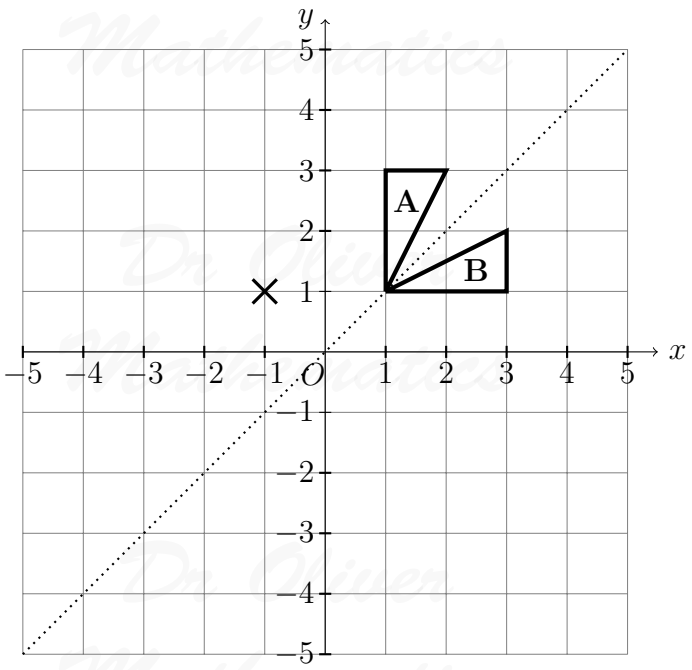
Solution

$$(2x^3y)^4 = \underline{\underline{16x^{12}y^4}}.$$

6. (a) Describe fully the single transformation that maps triangle **A** onto triangle **B**. (2)



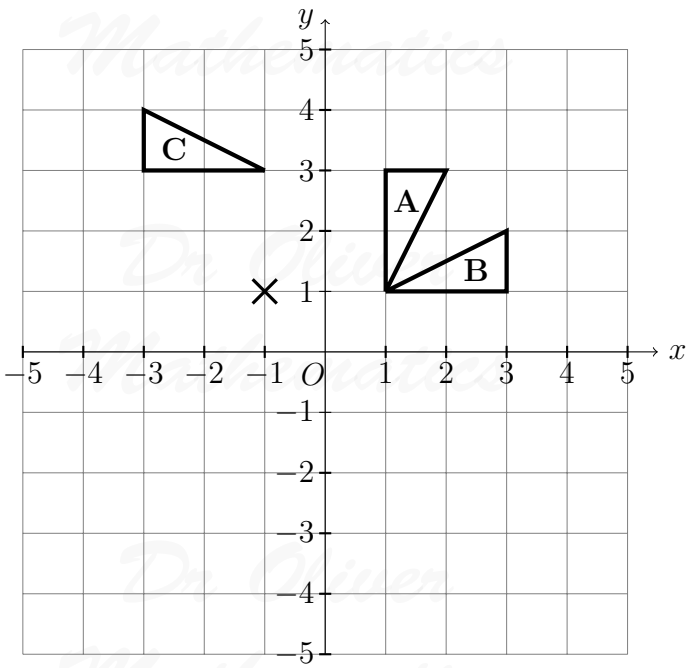
Solution



Reflection, in the line $y = x$.

(b) On the grid, rotate triangle **A** 90° anti-clockwise about the point $(-1, 1)$. (2)
Label your new triangle **C**.

Solution



7. (a) $-3 \leqslant n < 2$. (2)

n is an integer.

Write down all the possible values of n.

Solution

-3, -2, -1, 0, 1.

(b) Solve the inequality (2)

$$5x < 2x - 6.$$

Solution

$$5x < 2x - 6 \Rightarrow 3x < -6$$
$$\Rightarrow x < -2.$$

8. Work out (3)

$$3\tfrac{2}{5} - 1\tfrac{3}{4}.$$

Solution

$$\begin{aligned} 3\tfrac{2}{5} - 1\tfrac{3}{4} &= 2 + \tfrac{2}{5} - \tfrac{3}{4} \\ &= 2 + \tfrac{8}{20} - \tfrac{15}{20} \\ &= 2 - \tfrac{7}{20} \\ &= \underline{\underline{1\tfrac{13}{20}}}. \end{aligned}$$

9. The table shows some expressions. (3)

Expression	Length	Area	Volume	None of these
πab				
$a + 2b$				
$\pi a^2 + b$				

The letters a and b represent lengths.
π and 2 are numbers that have no dimensions.
Place a tick (✓) in the correct column to show whether the expression can be used to represent a length, an area, a volume, or none of these.

Solution

Expression	Length	Area	Volume	None of these
πab		✓		
$a + 2b$	✓			
$\pi a^2 + b$				✓

10. (a) Write (1)

$$5.7 \times 10^{-4}$$

as an ordinary number.

Solution

$$5.7 \times 10^{-4} = \underline{\underline{0.000\,57}}.$$

(b) Work out the value of (2)

$$(7 \times 10^4) \times (3 \times 10^5).$$

Give your answer in standard form.

Solution

$$\begin{aligned}(7 \times 10^4) \times (3 \times 10^5) &= 21 \times 10^9 \\ &= \underline{\underline{2.1 \times 10^{10}}}.\end{aligned}$$

11. Solve the simultaneous equations (4)

$$\begin{aligned}3x - 4y &= 13 \\ 2x + 3y &= 3.\end{aligned}$$

Solution

$$\begin{aligned}3x - 4y &= 13 \quad (1) \\ 2x + 3y &= 3 \quad (2)\end{aligned}$$

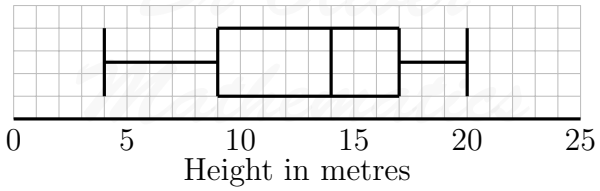
E.g.,

$$\begin{aligned}3 \times (1): &\quad 9x - 12y = 39 \quad (3) \\ 4 \times (2): &\quad 8x + 12y = 12 \quad (4)\end{aligned}$$

Now, add (3) and (4):

$$\begin{aligned}17x = 51 &\Rightarrow \underline{\underline{x = 3}} \\ &\Rightarrow 6 + 3y = 3 \\ &\Rightarrow 3y = -3 \\ &\Rightarrow \underline{\underline{y = -1}}.\end{aligned}$$

12. The box plot gives information about the distribution of the heights of all the trees in a wood.



(a) Write down the median height of the trees. (1)

Solution

14 m.

(b) Work out the interquartile range of the heights of the trees. (1)

Solution

$$\begin{aligned} \text{IQR} &= \text{UQ} - \text{LQ} \\ &= 17 - 9 \\ &= 8 \text{ m}. \end{aligned}$$

There are 300 trees in the wood.

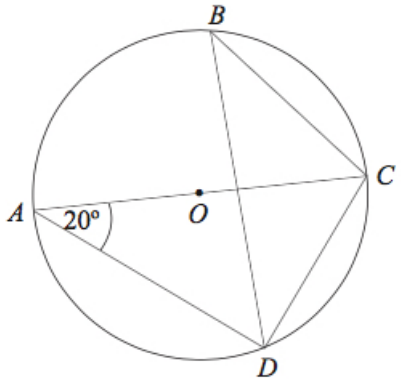
(c) Work out the number of trees in the wood with a height of 17 m or more. (2)

Solution

There are 25% with a height of 17 m or more and so the number of trees is

$$\frac{300}{4} = 75 \text{ trees}.$$

13. A, B, C, and D are points on the circumference of a circle, centre O



AC is a diameter of the circle.
Angle $DAC = 20°$.

(a) Find the size of angle ACD. (2)

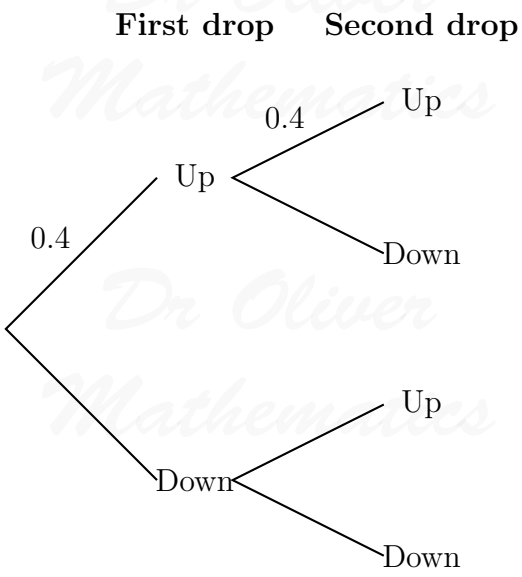
Solution
$90 - 20 = \underline{\underline{70°}}$ because ADC is a right-angle.

(b) Find the size of angle DBC. (2)
Give a reason for your answer.

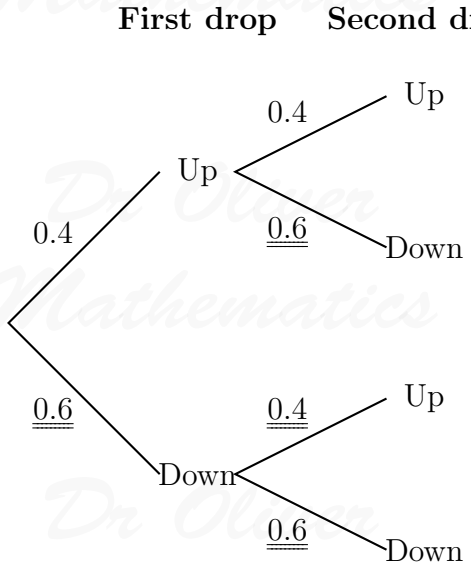
Solution
Angle $DBC = \underline{\underline{20°}}$ (angles in the same segment).

14. Mary has a drawing pin.
When the drawing pin is dropped it can land either 'point up' or 'point down'.
The probability of it landing 'point up' is 0.4.
Mary drops the drawing pin twice.

(a) Complete the probability tree diagram. (2)



Solution



(b) Work out the probability that the drawing pin will land 'point up' both times. (2)

Solution

$$0.4 \times 0.4 = \underline{\underline{0.16}}.$$

15. The table shows some rows of a number pattern.

Row 1	$1^2 - (0 \times 2)$
Row 2	$2^2 - (1 \times 3)$
Row 3	$3^2 - (2 \times 4)$
Row 4	$4^2 - (3 \times 5)$
	.....
Row n	.....

(a) In the table, write down an expression, in terms of n, for Row n. (1)

Solution

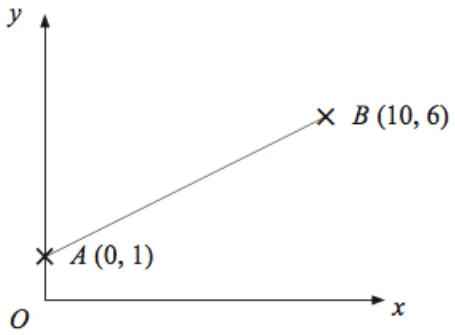
$n^2 - (n-1)(n+1)$.

(b) Simplify fully your expression for Row n. (2)
You must show your working.

Solution

$$\begin{aligned} n^2 - (n-1)(n+1) &= n^2 - (n^2 - 1) \\ &= 1. \end{aligned}$$

16. A is the point $(0, 1)$.
B is the point $(10, 6)$.



The equation of the straight line through A and B is $y = \frac{1}{2}x + 1$.

(a) Write down the equation of another straight line that is parallel to $y = \frac{1}{2}x + 1$. (1)

Solution

E.g., $\underline{\underline{y = \frac{1}{2}x + 2}}$, $\underline{\underline{y = \frac{1}{2}x - 7}}$, etc.

(b) Write down the equation of another straight line that passes through the point $(0, 1)$. (1)

Solution

E.g., $\underline{\underline{y = -7x + 1}}$, $\underline{\underline{y = 1}}$, etc.

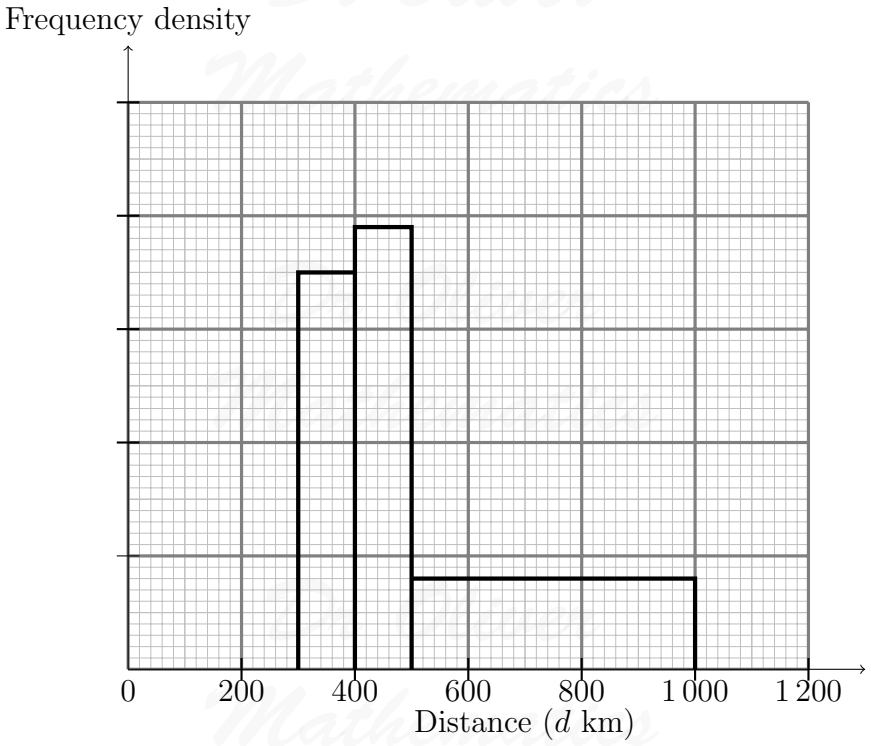
(c) Find the equation of the line perpendicular to AB passing through B. (3)

Solution

The gradient of the line perpendicular to AB is -2 and the equation is

$$y - 6 = -2(x - 10) \Rightarrow y - 6 = -2x + 20$$
$$\Rightarrow \underline{\underline{y = -2x + 26}}.$$

17. The incomplete table and histogram give some information about the distances walked by some students in a school in one year.



(a) Use the information in the histogram to complete the frequency table. (3)

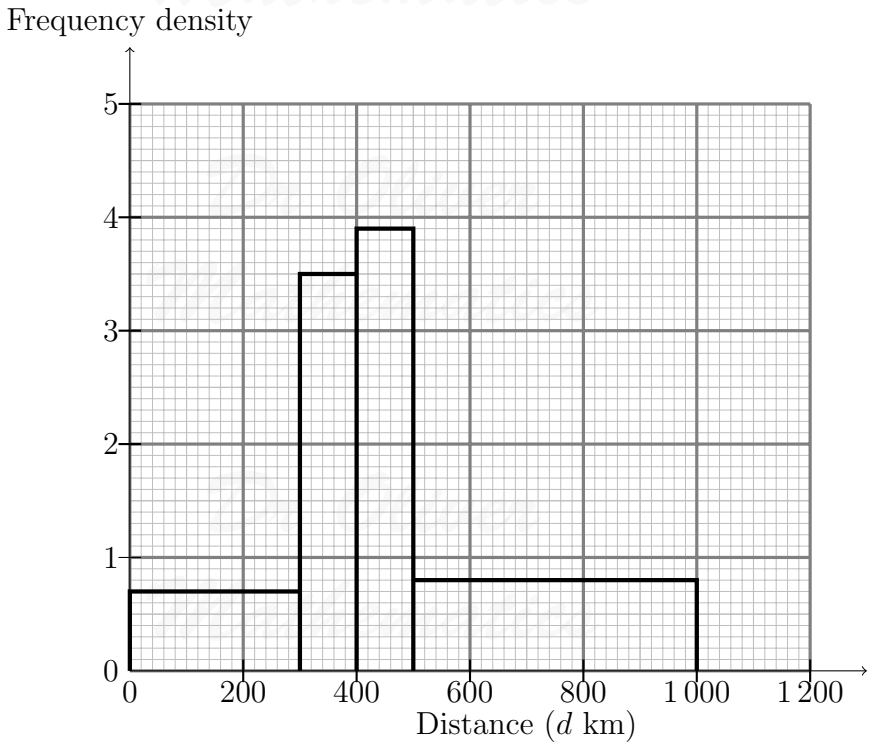
Distance (d) in km	Frequency
$0 < d \leqslant 300$	210
$300 < d \leqslant 400$	350
$400 < d \leqslant 500$	
$500 < d \leqslant 1\,000$	

Solution

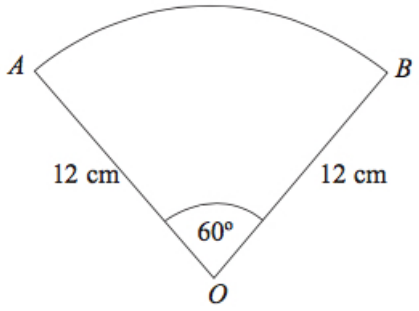
Distance (d) in km	Frequency	Width	Frequency density
$0 < d \leqslant 300$	210	300	$\frac{210}{300} = 0.7$
$300 < d \leqslant 400$	350	100	$\frac{350}{100} = 3.5$
$400 < d \leqslant 500$	$\underline{\underline{390}}$	100	$\frac{390}{100} = 3.9$
$500 < d \leqslant 1\,000$	$\underline{\underline{400}}$	500	$\frac{400}{500} = 0.8$

(b) Use the information in the table to complete the histogram. (1)

Solution



18. OAB is a sector of a circle, centre O. (3)

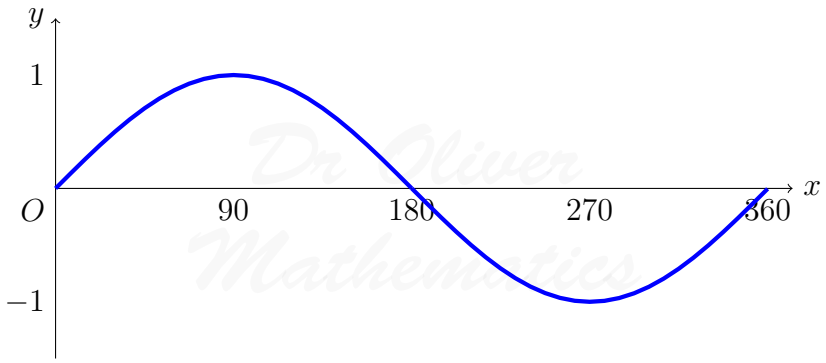


Angle $AOB = 60°$.
$OA = OB = 12$ cm.
Work out the length of the arc AB.
Give your answer in terms of π.

Solution

$$\begin{aligned}\text{Arc length} &= \frac{60}{360} \times 2 \times \pi \times 12 \\ &= \tfrac{1}{6} \times \pi \times 24 \\ &= \underline{\underline{4\pi}}.\end{aligned}$$

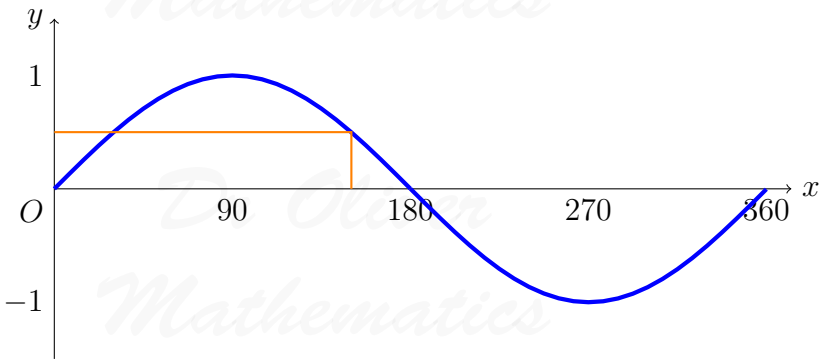
19. Here is a sketch of the curve $y = \sin x^\circ$ for $0 \leqslant x \leqslant 360$.



(a) Given that $\sin 30^\circ = \frac{1}{2}$, write down the value of (2)

(i) $\sin 150^\circ$,

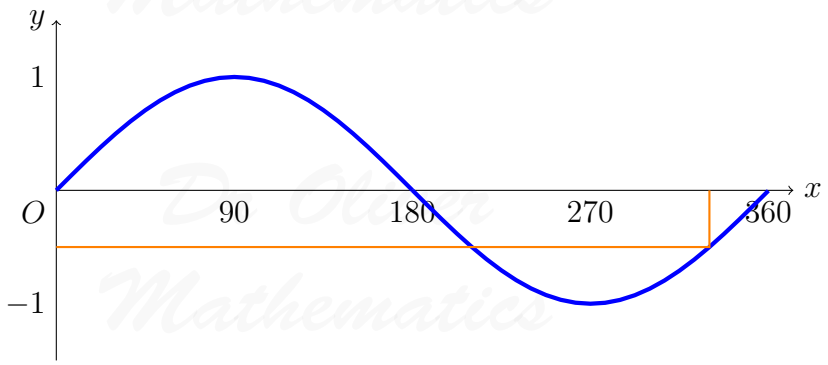
Solution



$$\sin 150^\circ = \sin 30^\circ = \underline{\underline{\tfrac{1}{2}}}.$$

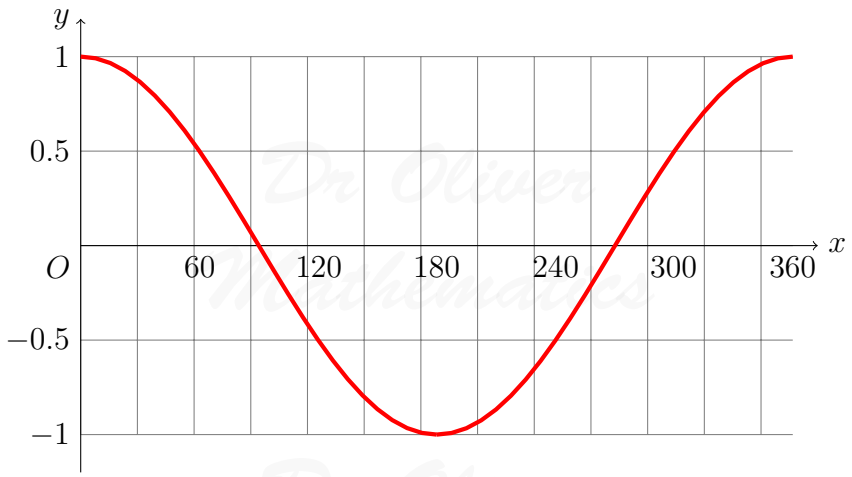
(ii) $\sin 330^\circ$.

Solution



$\sin 330^\circ = -\sin 30^\circ = \underline{\underline{-\frac{1}{2}}}$.

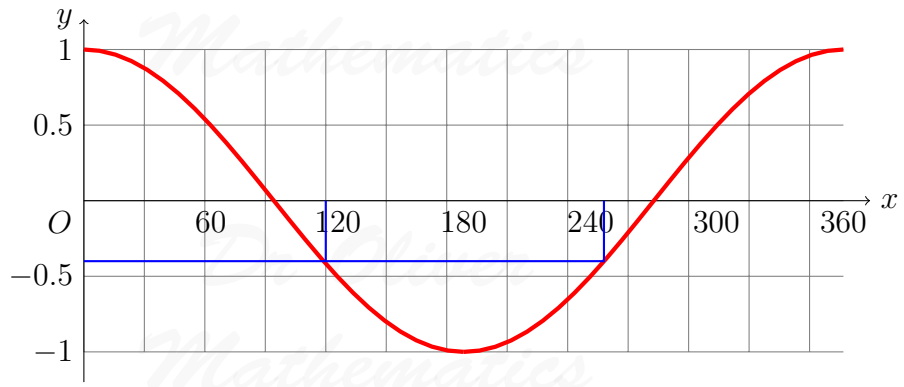
The graph of $y = \cos x^\circ$ for $0 \leqslant x \leqslant 360$ is drawn below.



(b) Use the graph to find estimates of the solutions, in the interval $0 \leqslant x \leqslant 360$, of the equation (4)

(i) $\cos x^\circ = -0.4$,

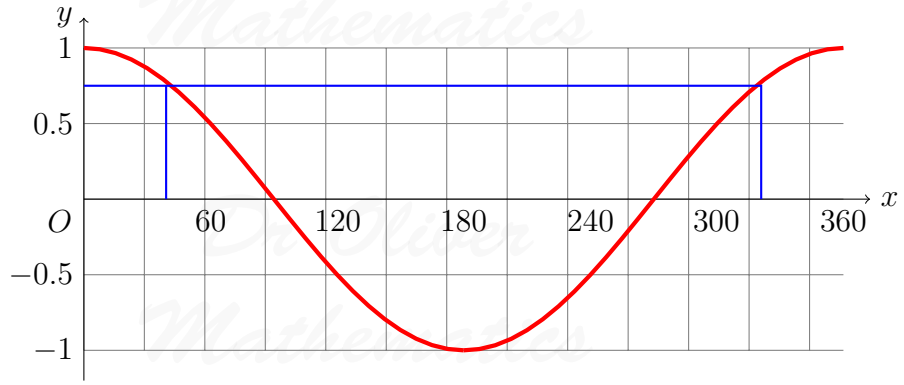
Solution



Correct read-off: approximately 114° and 246°.

(ii) $4\cos x^\circ = 3$.

Solution



$$4\cos x^\circ = 3 \Rightarrow \cos x^\circ = \tfrac{3}{4}.$$

Correct read-off: approximately 41° and 319°.

20. (a) Expand and simplify (3)

$$(2x + 5)(3x - 2).$$

Solution

$\times$	$2x$	$+5$
$3x$	$6x^2$	$+15x$
-2	$-4x$	-10

$$(2x+5)(3x-2) = \underline{\underline{6x^2+11x-10}}.$$

(b) Given that (3)

$$x^2+6x-5 = (x+p)^2+q$$

for all values of x, find the value of

(i) p,

Solution

$$\begin{aligned} x^2+6x-5 &= (x^2+6x+9)-14 \\ &= (x+3)^2-14; \end{aligned}$$

hence, $\underline{\underline{p=3}}$ ...

(ii) q.

Solution

... and $\underline{\underline{q=-14}}$.

21. (a) Write down the value of $4^{\frac{3}{2}}$. (1)

Solution

$$\begin{aligned} 4^{\frac{3}{2}} &= (4^{\frac{1}{2}})^3 \\ &= 2^3 \\ &= \underline{\underline{8}}. \end{aligned}$$

(b) Write $\sqrt{8}$ in the form $m\sqrt{2}$, where m is an integer. (2)

Solution

$$\begin{aligned}\sqrt{8} &= \sqrt{4 \times 2} \\ &= \sqrt{4} \times \sqrt{2} \\ &= \underline{\underline{2\sqrt{2}}}.\end{aligned}$$

(c) Write $\sqrt{50}$ in the form $k\sqrt{2}$, where k is an integer. (2)

Solution

$$\begin{aligned}\sqrt{50} &= \sqrt{25 \times 2} \\ &= \sqrt{25} \times \sqrt{2} \\ &= \underline{\underline{5\sqrt{2}}}.\end{aligned}$$

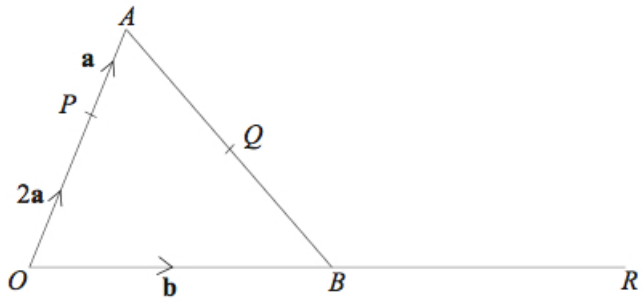
(d) Rationalise (2)

$$\frac{1+\sqrt{2}}{\sqrt{2}}.$$

Solution

$$\begin{aligned}\frac{1+\sqrt{2}}{\sqrt{2}} &= \frac{1+\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}(1+\sqrt{2})}{2} \\ &= \underline{\underline{\frac{\sqrt{2}+2}{2}}}.\end{aligned}$$

22. OAB is a triangle.
B is the midpoint of OR.
Q is the midpoint of AB.



$\overrightarrow{OP} = 2\mathbf{a}$.
$\overrightarrow{PA} = \mathbf{a}$.
$\overrightarrow{OB} = \mathbf{b}$.

(a) Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the vectors (4)

(i) $\overrightarrow{AB}$,

Solution

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\ &= \overrightarrow{AP} + \overrightarrow{PO} + \overrightarrow{OB} \\ &= -\mathbf{a} - 2\mathbf{a} + \mathbf{b} \\ &= \underline{\underline{-3\mathbf{a} + \mathbf{b}}}.\end{aligned}$$

(ii) $\overrightarrow{PR}$,

Solution

$$\begin{aligned}\overrightarrow{PR} &= \overrightarrow{PO} + \overrightarrow{OR} \\ &= \underline{\underline{-2\mathbf{a} + 2\mathbf{b}}}.\end{aligned}$$

(iii) $\overrightarrow{PQ}$.

Solution

$$\begin{aligned}\overrightarrow{PQ} &= \overrightarrow{PA} + \overrightarrow{AQ} \\ &= \overrightarrow{PA} + \tfrac{1}{2}\overrightarrow{AB} \\ &= \mathbf{a} + \tfrac{1}{2}(-3\mathbf{a} + \mathbf{b}) \\ &= \underline{\underline{-\tfrac{1}{2}\mathbf{a} + \tfrac{1}{2}\mathbf{b}}}.\end{aligned}$$

(b) Hence explain why PQR is a straight line. (2)

Solution
Well, $\overrightarrow{PR} = 4\overrightarrow{PQ}$ and, hence, they are parallel. So, $\underline{\underline{PQR \text{ is a straight line}}}$.

The length of PQ is 3 cm.

(c) Find the length of PR. (1)

Solution

$$PR = 3 \times 4 = \underline{\underline{12 \text{ cm}}}.$$

23. By eliminating y, find the solutions to the simultaneous equations: (6)

$$x^2 + y^2 = 25$$
$$y = x - 7.$$

Solution

$$\begin{aligned} x^2 + y^2 = 25 &\Rightarrow x^2 + (x-7)^2 = 25 \\ &\Rightarrow x^2 + (x^2 - 14x + 49) = 25 \\ &\Rightarrow 2x^2 - 14x + 24 = 0 \\ &\Rightarrow x^2 - 7x + 12 = 0 \\ &\Rightarrow (x-3)(x-4) = 0 \\ &\Rightarrow x - 3 = 0 \text{ or } x - 4 = 0 \\ &\Rightarrow x = 3 \text{ or } x = 4 \\ &\Rightarrow y = -4 \text{ or } y = -3; \end{aligned}$$

hence,

$$\underline{\underline{x = 3, y = -4}} \text{ or } \underline{\underline{x = 4, y = -3}}.$$