Dr Oliver Mathematics
Mathematics
Kinematics
Past Examination Questions

This booklet consists of 37 questions across a variety of examination topics.
The total number of marks available is 356.
Whenever a numerical value of g is required, take $g = 9.8$ ms^{-2}.
You should know the relevant formula:

$$v = u + at$$
$$s = ut + \tfrac{1}{2}at^2$$
$$s = vt - \tfrac{1}{2}at^2$$
$$s = \tfrac{1}{2}(u + v)t$$
$$v^2 = u^2 + 2as$$

Give any answer to 2 significant figures in order to be consistent with the degree of accuracy used of g.

1. A ball is projected vertically upwards with a speed of u ms^{-1} from a point A which is 1.5 m above the ground. The ball moves freely under gravity until it reaches the ground. The greatest height attained by the ball is 25.6 m above A.

   (a) Show that $u = 22.4$. (3)

   Solution

   $s = 25.6$, $u = ?\,(\uparrow)$, $v = 0$, $a = -9.8$, and $t = ?$:

   $$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = u^2 + 2 \times (-9.8) \times 25.6 \\ &\Rightarrow u^2 = 501.76 \\ &\Rightarrow \underline{\underline{u = 22.4}}. \end{aligned}$$

   The ball reaches the ground T seconds after it has been projected from A.

   (b) Find, to 2 decimal places, the value of T. (4)

   Solution

$s = -1.5$, $u = 22.4$, $v = ?$, $a = -9.8$, and $t = T$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow -1.5 = 22.4T - 4.9T^2 \\ &\Rightarrow 4.9T^2 - 22.4T - 1.5 = 0 \\ &\Rightarrow T = \frac{22.4 \pm \sqrt{22.4^2 - 4 \times (-4.9) \times (-1.5)}}{9.8} \\ &\Rightarrow T = \frac{22.4 \pm \sqrt{442.96}}{9.8} \\ &\Rightarrow T = 0.067\,975\,043\,39,\ 4.637\,439\,662 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{T = 4.64 \text{ (2 dp)}}}. \end{aligned}$$

The ground is soft and the ball sinks 2.5 cm into the the ground before coming to rest. The mass of the ball is 0.6 kg. The ground is assumed to exert a constant resistive force of magnitude F newtons.

(c) Find, to 3 significant figures, the value of F. (6)

Solution

Speed at the ground:

$$\begin{aligned} v = u + at &\Rightarrow v = 22.4 - 9.8 \times 4.637\,439\,662 \text{ (FCD)} \\ &\Rightarrow v = -23.046\,908\,69 \text{ (FCD)}. \end{aligned}$$

Now,

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = (-23.046\,908\,69)^2 + 2 \times a \times 0.025 \\ &\Rightarrow a = -10\,623.2. \end{aligned}$$

Finally,

$$\begin{aligned} F = ma &\Rightarrow F - 0.6g = 0.6a \\ &\Rightarrow F = 6379.8 \\ &\Rightarrow \underline{\underline{F = 6380 \text{ (3 sf)}}}. \end{aligned}$$

The printed solution has

$$T = 4.64 \Rightarrow v = -23.072 \Rightarrow a = -10\,646.343\,68 \Rightarrow F = 6393.686\,208.$$

(d) State one physical factor that could be taken into to account to make the model (1) used in this question more realistic.

Solution
e.g., air resistance, assume F is variable, etc

2. A sprinter runs a race of 200 m. Her total time for running the race is 25 s and Figure 1 is a sketch of the speed-time graph the the motion of the sprinter.

Figure 1: a sprinter

She starts from rest and accelerates uniformly to a speed of 9 ms^{-1} in 4 s. The speed of 9 ms^{-1} is maintained for 16 s and then she decelerates uniformly to a speed of u ms^{-1} at the end of the race.
Calculate

(a) the distance covered by the sprinter in the first 20 s of the race, (2)

Solution

$$\begin{aligned}\text{Distance} &= \text{area under the graph}\\ &= \tfrac{1}{2}\times 4\times 9+(20-4)\times 9\\ &= 18+144\\ &= \underline{\underline{162\text{ m}}}.\end{aligned}$$

(b) the value of of u, and (4)

Solution

The distance left is $200 - 162 = 38$ m and

$$\begin{aligned} \tfrac{1}{2} \times 5 \times (u + 9) = 38 &\Rightarrow u + 9 = 15.2 \\ &\Rightarrow \underline{\underline{u = 6.2}}. \end{aligned}$$

(c) the deceleration of the sprinter in the last 5 s of the race. (3)

Solution

$$\text{Deceleration} = \frac{9 - 6.2}{5} = \underline{\underline{0.56\ \text{ms}^{-2}}}.$$

3. In taking off, an aircraft moves on a straight runway AB of length 1.2 km. The aircraft moves from A with an initial speed 2 ms^{-1}. It moves with constant acceleration and 20 s later it leaves the runway at C with speed 74 ms^{-1}. Find

(a) the acceleration of the aircraft, and (2)

Solution

$s = ?$, $u = 2$, $v = 74$, $a = ?$, and $t = 20$:

$$\begin{aligned} v = u + at &\Rightarrow 74 = 2 + 20a \\ &\Rightarrow 20a = 72 \\ &\Rightarrow \underline{\underline{a = 3.6\ \text{ms}^{-2}}}. \end{aligned}$$

(b) the distance BC. (4)

Solution

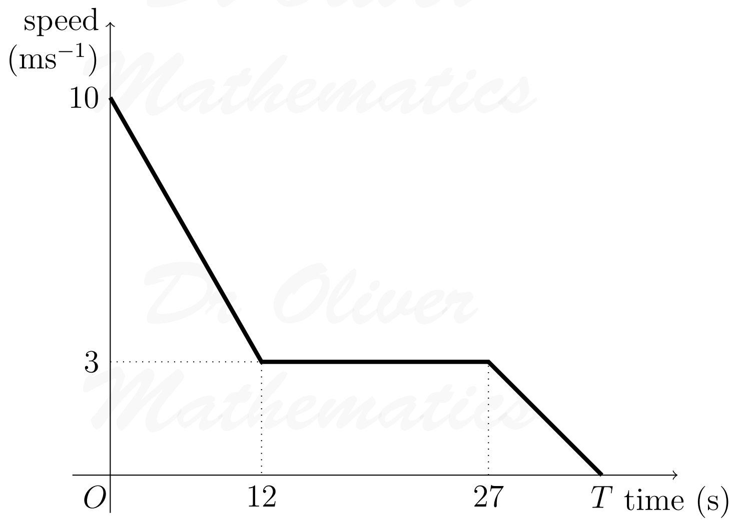
$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow AC = 2 \times 20 + \tfrac{1}{2} \times 3.6 \times 20^2 \\ &\Rightarrow AC = 760 \\ &\Rightarrow BC = 1200 - 760 \\ &\Rightarrow \underline{\underline{BC = 440\ \text{m}}}. \end{aligned}$$

4. A train is travelling at 10 ms^{-1} on a straight horizontal track. The driver sees a red signal 135 m ahead and immediately applies the brakes. The train decelerates with

constant deceleration for 12 s, reducing its speed to 3 ms^{-1}. The driver then releases the brakes and allows the train to travel at a constant speed of 3 ms^{-1} for a further 15 s. He then applies the brakes again and the train slows down with constant deceleration, coming to rest as it reaches the signal.

(a) Sketch a speed-time graph to show the motion of the train. (3)

Solution



(b) Find the distance travelled by the train from the moment when the brakes are first applied to the moment when its speed first reaches 3 ms^{-1}. (2)

Solution

$$\begin{aligned}\text{Distance} &= \text{area under the graph}\\ &= \tfrac{1}{2} \times (10+3) \times 12\\ &= \underline{\underline{78 \text{ m}}}.\end{aligned}$$

(c) Find the total time from the moment when the brakes are first applied to the moment when the train comes to rest. (5)

Solution

$$\begin{aligned}\text{Second stage of motion} &= (27-12) \times 3 = 45 \text{ m},\\ \text{first and second stages of motion} &= 78 + 45 = 123 \text{ m}\\ \text{third stage of motion} &= 135 - 123 = 12 \text{ m}.\end{aligned}$$

Now,

$$s = \tfrac{1}{2}(u+v)t \Rightarrow 12 = \tfrac{1}{2}(3+0)t$$
$$\Rightarrow t = 8 \text{ s}$$
$$\Rightarrow \text{total time} = 27 + 8$$
$$\Rightarrow \underline{\underline{\text{total time} = 35 \text{ s}}}$$

5. A stone is thrown vertically upwards with a speed of 16 ms^{-1} from a point h metres above the ground. The stone hits the ground 4 s later. Find

(a) the value of h, and (3)

Solution

$s =?$, $u = 16\,(\uparrow)$, $v =?$, $a = -9.8$, and $t = 4$:

$$s = ut + \tfrac{1}{2}at^2 \Rightarrow s = 16 \times 4 + \tfrac{1}{2} \times (-9.8) \times 4^2$$
$$\Rightarrow s = -14.4$$

and so

$$\text{distance} = \underline{\underline{14.4 \text{ m}}}.$$

(b) the speed of the stone as it hits the ground. (3)

Solution

$$v = u + at \Rightarrow v = 16 - 9.8 \times 4$$
$$\Rightarrow v = -23.2$$

and so

$$\text{speed} = \underline{\underline{23.2 \text{ ms}^{-1}}}.$$

6. Figure 2 shows the speed-time graph of a cyclist moving over a straight road over a 7 s period.

Figure 2: a cyclist

The sections of the graph from $t = 0$ to $t = 3$ and from $t = 3$ to $t = 7$ are straight lines. The section from $t = 3$ to $t = 7$ is parallel to the t-axis.
State what can can be deduced about the motion of the cyclist from the fact that

(a) the graph from $t = 0$ to $t = 3$ is a straight line, and (1)

Solution
constant acceleration

(b) the graph from $t = 3$ to $t = 7$ is parallel to the t-axis. (1)

Solution
constant speed or constant velocity

(c) Find the distance travelled by the cyclist during this 7 s period. (4)

Solution

$$\begin{aligned}\text{Distance} &= \text{area under the graph}\\ &= \tfrac{1}{2} \times (2 + 5) \times 3 + 4 \times 5\\ &= \underline{\underline{30.5 \text{ m}}}.\end{aligned}$$

7. A train moves along a straight track with constant acceleration. Three telegraph poles are set at equal intervals beside the track at points A, B, and C, where $AB = 50$ m and

$BC = 50$ m. The front of the train passes A with speed 22.5 ms^{-1}and, 2 s, it passes B. Find

(a) the acceleration of the train, (3)

Solution

$s = 50$, $u = 22.5$, $v =?$, $a =?$, and $t = 2$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow 50 = 22.5 \times 2 + \tfrac{1}{2} \times a \times 2^2 \\ &\Rightarrow 5 = 2a \\ &\Rightarrow \underline{\underline{a = 2.5 \text{ ms}^{-2}}}. \end{aligned}$$

(b) the speed of the front of the train when it passes C, and (3)

Solution

$s = 100$, $u = 22.5$, $v =?$, $a = 2.5$, and $t =?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow v^2 = 22.5^2 + 2 \times 2.5 \times 100 \\ &\Rightarrow v^2 = 1006.25 \\ &\Rightarrow v = \frac{5\sqrt{161}}{2} \\ &\Rightarrow \underline{\underline{v = 32 \text{ (2 sf)}}}. \end{aligned}$$

(c) the time that elapses from the instant the front of the train passes B to instant the front of the train passes C. (4)

Solution

$s =?$, $u = 22.5$, $v = \frac{5\sqrt{161}}{2}$, $a = 2.5$, and $t =?$:

$$\begin{aligned} v = u + at &\Rightarrow \frac{5\sqrt{161}}{2} = 22.5 + 2.5t \\ &\Rightarrow 2.5t = \frac{-45 + 5\sqrt{161}}{2} \\ &\Rightarrow t = \sqrt{161} - 9 \end{aligned}$$

and

$$\text{time passed} = (\sqrt{161} - 9) - 2 = \sqrt{161} - 11 = \underline{\underline{1.7 \text{ (2 sf)}}}.$$

8. A ball is projected vertically upwards with speed 21 ms^{-1} from a point A, which is 1.5 m above the ground. After projection, the ball moves freely under gravity until it reaches the ground. Modelling the ball as a particle, find

(a) the greatest height above A reached by the ball, (3)

Solution

$s = ?$, $u = 21\,(\uparrow)$, $v = 0$, $a = -9.8$, and $t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = 21^2 + 2 \times (-9.8) \times s \\ &\Rightarrow 19.6s = 441 \\ &\Rightarrow \underline{\underline{s = 22.5 \text{ m}}}. \end{aligned}$$

(b) the speed of the ball as it reaches the ground, and (3)

Solution

$s = 24$, $u = 0$, $v = ?$, $a = 9.8$, and $t = ?$ (why?):

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow v^2 = 0 + 2 \times 9.8 \times 24 \\ &\Rightarrow v^2 = 470.4 \\ &\Rightarrow v = \frac{28\sqrt{15}}{5} \\ &\Rightarrow \underline{\underline{v = 22 \text{ ms}^{-1} \text{ (2 sf)}}}. \end{aligned}$$

(c) the time between the instant when ball is projected from A and the instant when the ball reaches the ground. (4)

Solution

$s = ?$, $u = 21$, $v = -\frac{28\sqrt{15}}{5}$, $a = -9.8$, and $t = ?$:

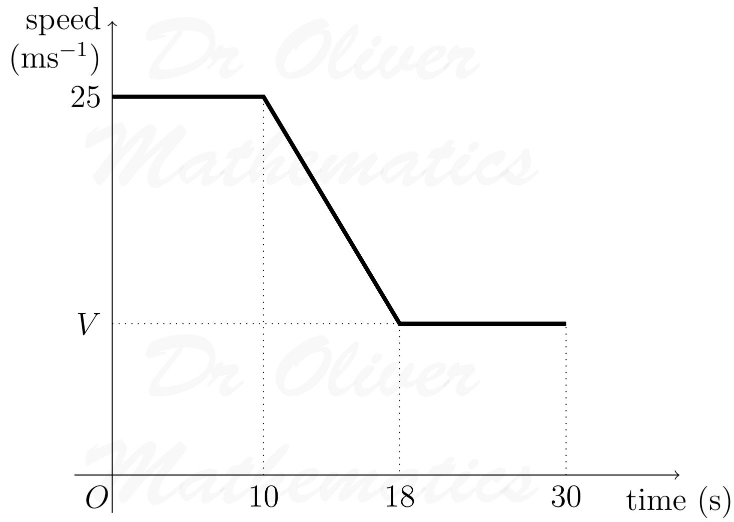
$$\begin{aligned} v = u + at &\Rightarrow -\frac{28\sqrt{15}}{5} = 21 - 9.8t \\ &\Rightarrow t = \frac{15 + 4\sqrt{15}}{7} \\ &\Rightarrow \underline{\underline{t = 4.4 \text{ s (2 sf)}}}. \end{aligned}$$

9. A car is moving along a straight road. At time $t = 0$, the car passes a point A with speed 25 ms^{-1}. The car moves with constant speed 25 ms^{-1} until $t = 10$ s. The car then

decelerates uniformly for 8 s. At time $t = 18$ s, the speed of the car is $2V$ ms^{-1} and this speed is maintained until the car reaches the point B at time $t = 30$ s.

(a) Sketch a speed-graph to show the motion of the car from A to B. (3)

Solution



Given that $AB = 526$ m, find

(b) the value of V, and (5)

Solution

$$\begin{aligned} & 25 \times 10 + \tfrac{1}{2} \times (25 + V) \times 8 + 12 \times V = 526 \\ \Rightarrow\ & 4(25 + V) + 12V = 276 \\ \Rightarrow\ & 100 + 4V + 12V = 276 \\ \Rightarrow\ & 16V = 176 \\ \Rightarrow\ & \underline{\underline{V = 11}}. \end{aligned}$$

(c) the deceleration of the car between $t = 10$ s and $t = 18$ s. (3)

Solution

$$\text{Deceleration} = \frac{25 - 11}{8} = \underline{\underline{1.75 \text{ ms}^{-1}}}.$$

10. A firework rocket starts from rest at ground level and moves vertically. In the first 3 s

of its motion, the rocket rises 27 m. The rocket is modelled as a particle moving with constant acceleration a ms^{-2}. Find

(a) the value of a, and (2)

Solution

$s = 27$, $u = ?\,(\uparrow)$, $v = ?$, $a = ?$, and $t = 3$:

$$s = ut + \tfrac{1}{2}at^2 \Rightarrow 27 = 0 + \tfrac{1}{2} \times a \times 3^2$$
$$\Rightarrow \underline{\underline{a = 6}}.$$

(b) the speed of the rocket 3 s after it has left the ground. (2)

Solution

$s = 27$, $u = ?$, $v = ?$, $a = 6$, and $t = 3$:

$$s = vt - \tfrac{1}{2}at^2 \Rightarrow 27 = 3v - \tfrac{1}{2} \times 6 \times 3^2$$
$$\Rightarrow \underline{\underline{v = 18 \text{ ms}^{-1}}}.$$

After 3 s, the rocket burns out. The motion of the rocket is now modelled as that of a particle moving freely under gravity.

(c) Find the height of the rocket above the ground 5 s after it has left the ground. (4)

Solution

$s = ?$, $u = 18\,(\uparrow)$, $v = ?$, $a = -9.8$, and $t = 2$:

$$s = ut + \tfrac{1}{2}at^2 \Rightarrow s = 18 \times 2 - \tfrac{1}{2} \times 9.8 \times 2^2$$
$$\Rightarrow s = 16.4$$

and thus it has gone up

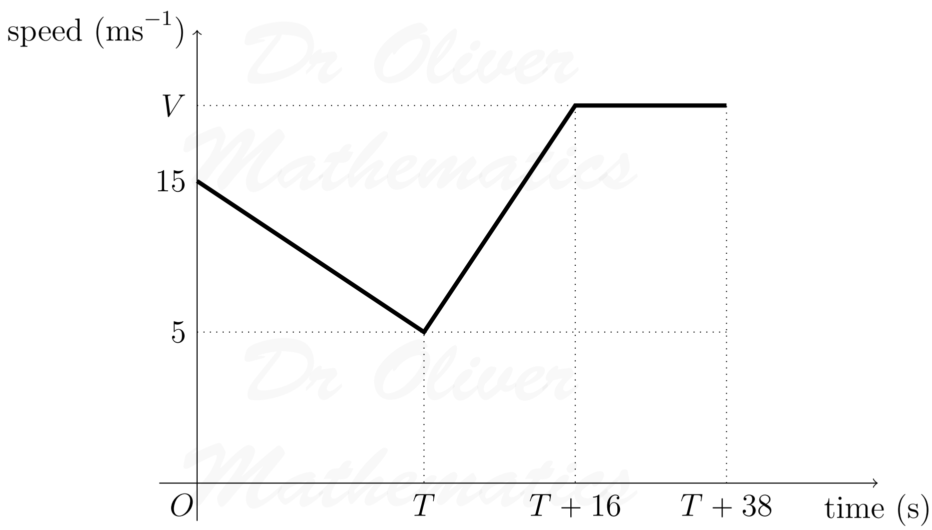
$$\text{distance} = 27 + 16.4 = \underline{\underline{43.4 \text{ m}}}.$$

11. A car moves along a horizontal straight road, passing two points A to B. At A the speed of the car is 15 ms^{-1}. When the driver passes A, he sees a warning sign W ahead of him, 120 m away. He immediately applies the brake and the car decelerates with uniform deceleration, reaching W with a speed 5 ms^{-1}. At W, the driver sees that the road is clear. He then immediately accelerates the car with uniform acceleration for 16 s

to reach a speed V ms^{-1} ($V > 15$). He then maintains the car at a constant speed of V ms^{-1}. Moving at this speed, the car passes B after a further 22 s.

(a) Sketch a speed-time to illustrate the motion of the car from A to B. (3)

Solution



(b) Find the time taken for the car to move A to B. (3)

Solution

$s = 120$, $u = 15$, $v = 5$, $a = ?$, and $t = ?$:

$$\begin{aligned} s = \tfrac{1}{2}(u+v)t \Rightarrow 120 &= \tfrac{1}{2}(15+5)t \\ \Rightarrow 10t &= 120 \\ \Rightarrow t &= 12 \end{aligned}$$

and

$$\text{distance} = 12 + 16 + 22 = \underline{\underline{50 \text{ s}}}.$$

The distance from A to B is 1 km.

(c) Find the value of B. (5)

Solution

$$
\begin{aligned}
& 1000 = \tfrac{1}{2} \times (15+5) \times 12 + \tfrac{1}{2} \times (5+V) \times 16 + 22V \\
\Rightarrow\ & 1000 = 120 + 8(5+V) + 22V \\
\Rightarrow\ & 880 = 40 + 8V + 22V \\
\Rightarrow\ & 30V = 840 \\
\Rightarrow\ & \underline{\underline{V = 28}}.
\end{aligned}
$$

12. At time $t = 0$, a particle is projected vertically upwards with a speed of u ms^{-1} from a point 10 m above the ground. After time T seconds, the particle hits the ground with speed 17.5 ms^{-1}. Find

(a) the value of u, and (3)

Solution

$s = -10$, $u = ?\,(\uparrow)$, $v = -17.5$, $a = -9.8$, and $t = T$:

$$
\begin{aligned}
v^2 = u^2 + 2as \Rightarrow (-17.5)^2 &= u^2 + 2 \times (-9.8) \times (-10) \\
\Rightarrow u^2 &= 110.25 \\
\Rightarrow \underline{\underline{u = 10.5}}&.
\end{aligned}
$$

(b) the value of T. (4)

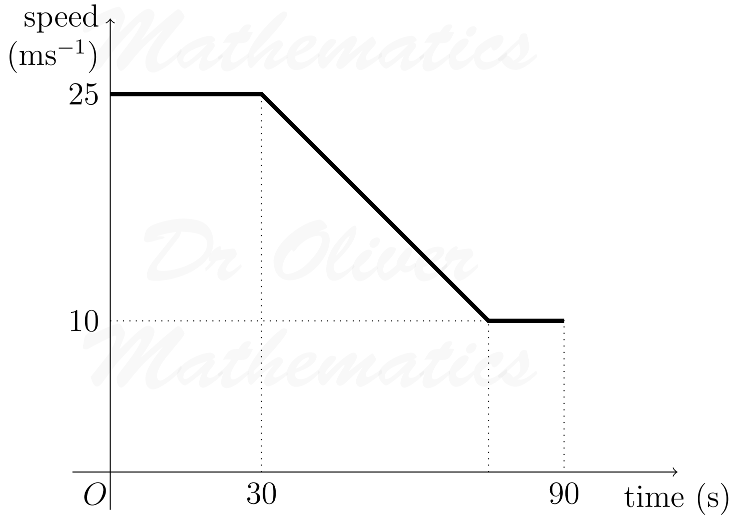
Solution

$$
\begin{aligned}
v = u + at \Rightarrow -17.5 &= 10.5 - 9.8T \\
\Rightarrow 9.8T &= 28 \\
\Rightarrow \underline{\underline{T = \tfrac{20}{7}}}&.
\end{aligned}
$$

13. A car is moving along a straight horizontal road. The speed of the car as it passes the point A is 25 ms^{-1} and the car maintains this speed for 30 s. The car then decelerates uniformly to a speed of 10 ms^{-1}. The speed of 10 ms^{-1} is then maintained until the car passes the point B. The time taken from A to B is 90 s and $AB = 1410$ m.

(a) Sketch a speed-time graph to show the motion of the car from A to B. (2)

Solution



(b) Calculate the deceleration of the car as it decelerates from 25 ms^{-1} to 10 ms^{-1}. (7)

Solution

$$
\begin{aligned}
& 25 \times 30 + \tfrac{1}{2} \times (25+10) \times (T-30) + (90-T) \times 10 = 1410 \\
\Rightarrow\ & 750 + 17.5(T-30) + 10(90-T) = 1410 \\
\Rightarrow\ & 17.5T - 525 + 900 - 10T = 660 \\
\Rightarrow\ & 7.5T = 285 \\
\Rightarrow\ & T = 38.
\end{aligned}
$$

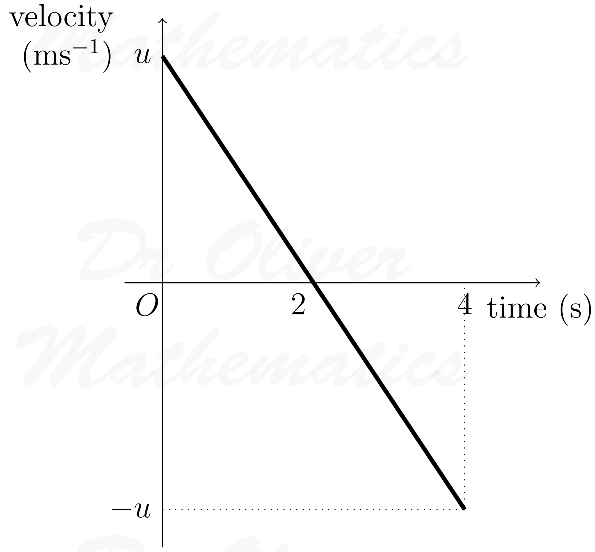
Finally,

$$\text{deceleration} = \frac{25-10}{38-30} = \underline{\underline{1.875 \text{ ms}^{-2}}}.$$

14. A small ball is projected vertically upwards from ground level with speed u ms^{-1}. The ball takes 4 s to return to ground level.

(a) Draw a velocity-time graph to represent the motion of the ball during the first 4 s. (2)

Solution



The maximum height of the ball above the ground during the first 4 s is 19.6 m.

(b) Find the value of u. (3)

Solution

$s = 0$, $u = u\,(\uparrow)$, $v = -u$, $a = -9.8$, and $t = 4$:

$$\begin{aligned} v = u + at &\Rightarrow -u = u - 9.8 \times 4 \\ &\Rightarrow 2u = 39.2 \\ &\Rightarrow \underline{\underline{u = 19.6}} \end{aligned}$$

15. Three posts P, Q, and R are fixed in that order at the side of a straight horizontal road. The distance from P to Q is 45 m and distance from Q to R is 120 m. A car is moving along the road with constant acceleration a ms^{-2}. The speed of the car, as it passes P, is u ms^{-1}. The car passes Q two seconds after passing P, and the car passes R four secons after passing Q. Find

(a) the value of u, and

(b) the value of a.

Solution

$s = 45$, $u =?$, $v =?$, $a =?$, and $t = 2$ which gives

$$s = ut + \tfrac{1}{2}at^2 \Rightarrow 45 = 2u + 2a \Rightarrow \boxed{135 = 6u + 6a}.$$

$s = 165$, $u = ?$, $v = ?$, $a = ?$, and $t = 6$ which gives

$$s = ut + \tfrac{1}{2}at^2 \Rightarrow \boxed{165 = 6u + 18a}.$$

Solve:

$$12a = 30 \Rightarrow \underline{\underline{a = 2.5}}$$

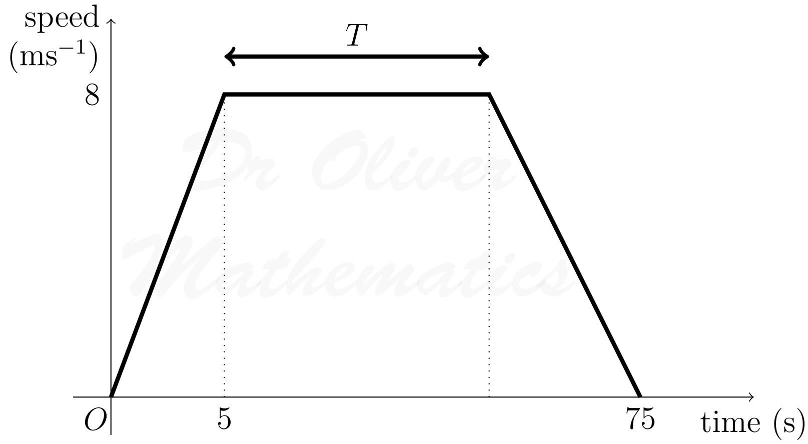
and

$$45 = 2u + 5 \Rightarrow 2u = 40 \Rightarrow \underline{\underline{u = 20}}.$$

16. An athlete runs along a straight road. She starts from rest and moves with constant acceleration for 5 seconds, reaching a speed of 8 ms^{-1}. This speed is then maintained for T seconds. She then decelerates at a constant rate until she stops. She has run a total distance of 500 m in 75 s.

(a) Sketch a speed-time graph to illustrate the motion of the athlete. (3)

Solution



(b) Calculate the value of T. (5)

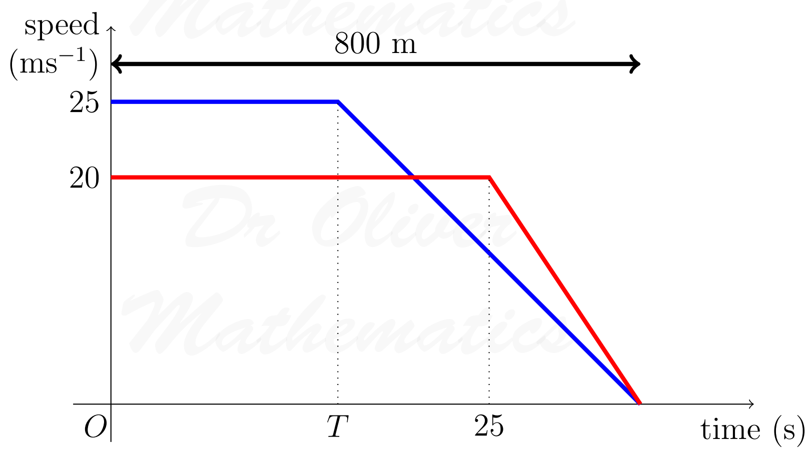
Solution

$$\begin{aligned}\tfrac{1}{2} \times (T + 75) \times 8 = 500 &\Rightarrow 4(T + 75) = 500\\ &\Rightarrow T + 75 = 125\\ &\Rightarrow \underline{\underline{T = 50}}.\end{aligned}$$

17. Two cars P and Q are moving in the direction along the same straight horizontal road. Car P is moving with constant speed 25 ms^{-1}. At time $t = 0$, P overtakes Q, which is moving with constant speed 20 ms^{-1}. From $t = T$ seconds, P decelerates uniformly, coming to rest at a point X, which is 800 m from the where P overtook Q. From $t = 25$ s, Q decelerates uniformly, coming to rest at same point X at the same instant as P.

(a) Sketch, on the same axis, the speed-time graphs of the two cars for the period from $t = 0$ to the time when they both come to rest at point point X. (4)

Solution



(b) Find the value of T. (8)

Solution

For Q,

$$\begin{aligned}\tfrac{1}{2} \times (t + 25) \times 20 = 800 &\Rightarrow 10(t + 25) = 800 \\ &\Rightarrow t + 25 = 80 \\ &\Rightarrow t = 55,\end{aligned}$$

and, for P,

$$\begin{aligned}\tfrac{1}{2} \times (T + 55) \times 25 = 800 &\Rightarrow 12.5(T + 55) = 800 \\ &\Rightarrow T + 55 = 64 \\ &\Rightarrow \underline{\underline{T = 9}}.\end{aligned}$$

18. A ball is projected vertically upwards with a speed of 14.7 ms^{-1} from a point which is 49 m above horizontal ground. Modelling the ball as a particle moving freely under

gravity, find

(a) the greatest height, above the ground, reached by the ball, (4)

Solution

$s = ?$, $u = 14.7\,(\uparrow)$, $v = 0$, $a = -9.8$, and $t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = 14.7^2 - 2 \times 9.8 \times s \\ &\Rightarrow 19.6s = 216.09 \\ &\Rightarrow s = 11.025; \end{aligned}$$

thus, the greatest height reached is

$$\text{distance} = 49 + 11.025 = 60.025 = \underline{\underline{60 \text{ m (2 sf)}}}.$$

(b) the speed with which the ball first strikes the ground, and (3)

Solution

$s = 60.025$, $u = 0$, $v = ?$, $a = 9.8$, and $t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow v^2 = 0 + 2 \times 9.8 \times 60.025 \\ &\Rightarrow v^2 = 1176.49 \\ &\Rightarrow \underline{\underline{v = 34.3}}. \end{aligned}$$

(c) the total time from when the ball is projected to when it first strikes the ground. (3)

Solution

$s = -49$, $u = 14.7$, $v = -34.3$, $a = -9.8$, and $t = ?$:

$$\begin{aligned} v = u + at &\Rightarrow -34.3 = 14.7 - 9.8t \\ &\Rightarrow 9.8t = 49 \\ &\Rightarrow \underline{\underline{t = 5}}. \end{aligned}$$

19. A ball is thrown vertically upwards with a speed u ms^{-1} from a point P at height h metres above the ground. The ball hits the ground 0.75 s later. The speed of the ball immediately before it hits the ground is 6.45 ms^{-1}. The ball is modelled as a particle.

(a) Show that $u = 0.9$. (3)

Solution

$s = h$, $u = ?\,(\uparrow)$, $v = -6.45$, $a = -9.8$, and $t = 0.75$:

$$\begin{aligned} v = u + at &\Rightarrow -6.45 = u - 9.8 \times 0.75 \\ &\Rightarrow \underline{\underline{u = 0.9}}. \end{aligned}$$

(b) Find the height above P to which the ball rises before it starts to fall towards the ground again. (2)

Solution

$s = ?$, $u = 0.9\,(\uparrow)$, $v = 0$, $a = -9.8$, and $t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = 0.9^2 - 2 \times 9.8 \times s \\ &\Rightarrow 19.6s = 0.81 \\ &\Rightarrow s = \tfrac{81}{1960} \\ &\Rightarrow \underline{\underline{s = 0.041 \text{ (2 sf)}}}. \end{aligned}$$

(c) Find the value of h. (3)

Solution

$s = ?$, $u = 0.9\,(\uparrow)$, $v = -6.45$, $a = -9.8$, and $t = 0.75$:

$$\begin{aligned} s = \tfrac{1}{2}(u + v)t &\Rightarrow s = \tfrac{1}{2} \times (0.9 - 6.45) \times 0.75 \\ &\Rightarrow s = -2\tfrac{13}{160}; \end{aligned}$$

thus,

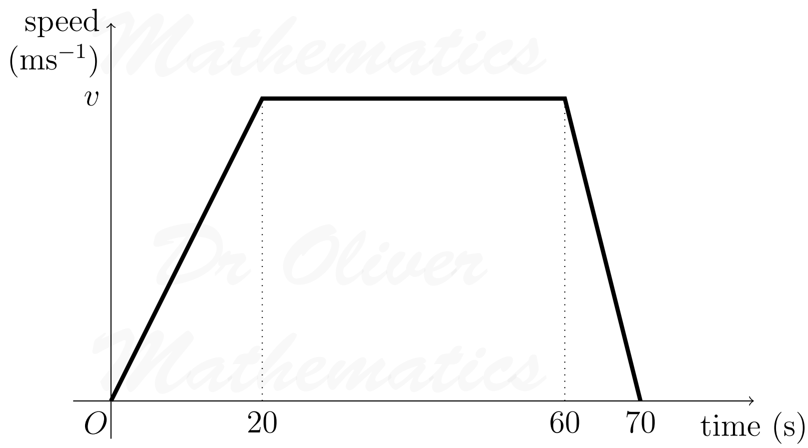
$$\text{distance} = \underline{\underline{2.1 \text{ (2 sf)}}}.$$

20. A car accelerates uniformly from rest for 20 seconds. It moves at constant speed v ms^{-1} for the next 40 seconds and then decelerates uniformly for 10 seconds until it comes to rest.

(a) For the motion of the car, sketch

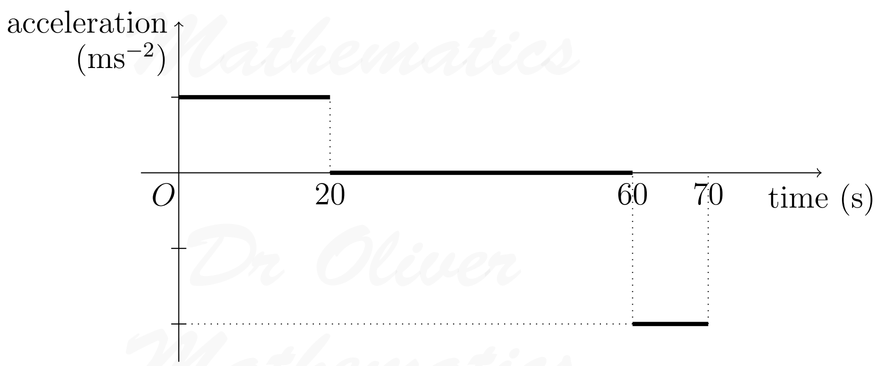
(i) a speed-time graph, and (3)

Solution



(ii) an acceleration-time graph. (3)

Solution



Given that the total distance moved by the car is 880 m,

(b) find the value of v. (4)

Solution

$$\tfrac{1}{2} \times (40 + 70) \times v = 880 \Rightarrow 55v = 880$$
$$\Rightarrow \underline{\underline{v = 16}}.$$

21. At time $t = 0$ a ball is projected vertically upwards from a point O and rises to a maximum height of 40 m above O. The ball is modelled as a particle moving freely under gravity.

(a) Show that the speed of projection is 28 ms^{-1}. (3)

Solution

$s = 40$, $u = ?\,(\uparrow)$, $v = 0$, $a = -9.8$, and $t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = u^2 - 2 \times 9.8 \times 40 \\ &\Rightarrow u^2 = 784 \\ &\Rightarrow u = \pm\, 28; \end{aligned}$$

as we are not interested in the direction, $\underline{\underline{u = 28}}$.

(b) Find the times, in seconds, when the ball is 33.6 m above O. (5)

Solution

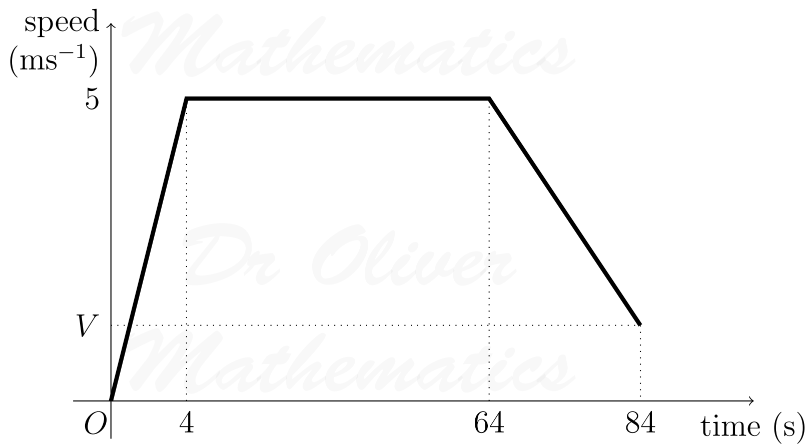
$s = 33.6$, $u = 28$, $v = ?$, $a = -9.8$, and $t = ?$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow 33.6 = 28t - 4.9t^2 \\ &\Rightarrow 4.9t^2 - 28t + 33.6 = 0 \\ &\Rightarrow t = \frac{28 \pm \sqrt{28^2 - 4 \times 4.9 \times 33.6}}{9.8} \\ &\Rightarrow t = \frac{28 \pm \sqrt{125.44}}{9.8} \\ &\Rightarrow t = \frac{28 \pm 11.2}{9.8} \\ &\Rightarrow t = \tfrac{12}{7} \text{ or } t = 4 \\ &\Rightarrow \underline{\underline{t = 1.7 \text{ (2 sf)}}} \text{ or } \underline{\underline{t = 4}}. \end{aligned}$$

22. A girl runs a 400 m race in a time of 84 s. In a model of this race, starting from rest, she moves with constant acceleration for 4 s, reaching a speed of 5 ms^{-1}. She maintains this speed for 60 s and then moves with constant deceleration for 20 s, crossing the finish line with a speed of V ms^{-1}.

(a) Sketch a speed-time graph for the motion of the girl during the whole race. (2)

Solution



(b) Find the distance run by the girl in the first 64 s of the race. (3)

Solution

$$\text{Distance} = \tfrac{1}{2} \times 5 \times 4 + 60 \times 5 = \underline{\underline{310 \text{ m}}}.$$

(c) Find the value of V. (5)

Solution

There is only $400 - 310 = 90$ m to go. Now,

$$\tfrac{1}{2} \times (V + 5) \times 20 = 90 \Rightarrow 10(V + 5) = 90 \Rightarrow V + 5 = 9 \Rightarrow \underline{\underline{V = 4}}.$$

(d) Find the deceleration of the girl in the final 20 s of her race. (2)

Solution

$$\text{Deceleration} = \frac{5 - 4}{20} = \underline{\underline{0.05 \text{ ms}^{-2}}}.$$

23. A stone is projected vertically upwards from a point A with speed u ms^{-1}. After projection, the stone moves freely under gravity until it returns to A. The time between the

instant that the stone is projected and the instant that it returns to A is $3\frac{4}{7}$ seconds. Modelling the stone as a particle,

(a) show that $u = 17\frac{1}{2}$, (3)

Solution

$s = ?$, $u = ?\,(\uparrow)$, $v = -u$, $a = -9.8$, and $t = \frac{25}{7}$:

$$\begin{aligned} v = u + at &\Rightarrow -u = u - 35 \\ &\Rightarrow 2u = 35 \\ &\Rightarrow \underline{\underline{u = 17\tfrac{1}{2}}}. \end{aligned}$$

(b) find the greatest height above A reached by the stone, and (2)

Solution

$s = ?$, $u = 17.5\,(\uparrow)$, $v = 0$, $a = -9.8$, and $t = \frac{25}{14}$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = 17.5^2 - 2 \times 9.8 \times s \\ &\Rightarrow 19.6s = 306.25 \\ &\Rightarrow s = 15\tfrac{5}{6} \\ &\Rightarrow \underline{\underline{s = 16 \text{ m}}}. \end{aligned}$$

(c) find the length of time for which the stone is at least $6\frac{3}{5}$ m above A. (6)

Solution

$s = 6.6$, $u = 17.5\,(\uparrow)$, $v = ?$, $a = -9.8$, and $t = ?$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow 6.6 = 17.5t - \tfrac{1}{2} \times 9.8 \times t^2 \\ &\Rightarrow 4.9t^2 - 17.5t + 6.6 = 0 \\ &\Rightarrow t = \frac{17.5 \pm \sqrt{(-17.5)^2 - 4 \times 4.9 \times 6.6}}{9.8} \\ &\Rightarrow t = \frac{17.5 \pm \sqrt{176.89}}{9.8} \\ &\Rightarrow t = \frac{17.5 \pm 13.3}{9.8} \\ &\Rightarrow t = \tfrac{3}{7} \text{ or } \tfrac{22}{7}; \end{aligned}$$

hence, the time is

$$\text{time taken} = \tfrac{22}{7} - \tfrac{3}{7} = \tfrac{19}{7} = \underline{\underline{2.7 \text{ (2 sf)}}}.$$

24. A car moves along a straight horizontal road from a point A to a point B, where $AB = 885$ m. The car accelerates from rest at A to a speed of 15 ms^{-1} at a constant rate a ms^{-2}. The time for which the car accelerates is $\frac{1}{3}T$ seconds. The car maintains the speed of 15 ms^{-1} for T seconds. The car the decelerates at a constant rate of 2.5 ms^{-2} stopping at B.

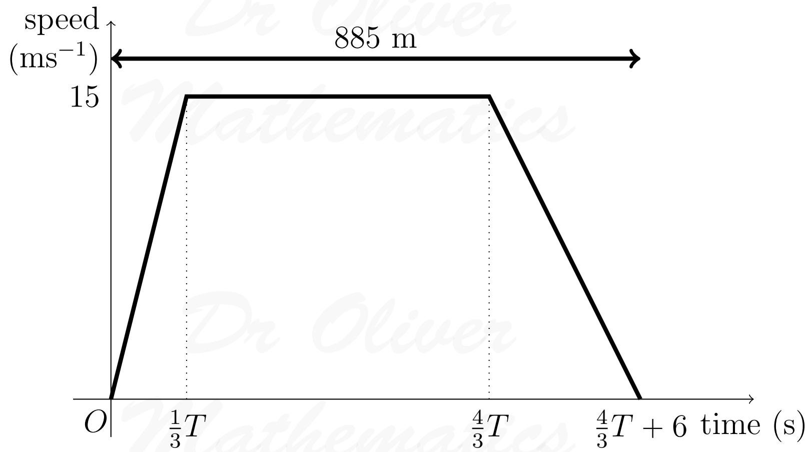
(a) Find the time for which the car decelerates. (2)

Solution

$$\text{time} = \frac{15-0}{2.5} = \underline{\underline{6 \text{ ms}^{-2}}}.$$

(b) Sketch a speed-time graph for the motion of the car. (2)

Solution



(c) Find the value of T. (4)

Solution

$$\begin{aligned} \tfrac{1}{2} \times (\tfrac{4}{3}T + 6 + T) \times 15 = 885 &\Rightarrow \tfrac{7}{3}T + 6 = 118 \\ &\Rightarrow \tfrac{7}{3}T + 6 = 118 \\ &\Rightarrow \tfrac{7}{3}T = 112 \\ &\Rightarrow \underline{\underline{T = 48}}. \end{aligned}$$

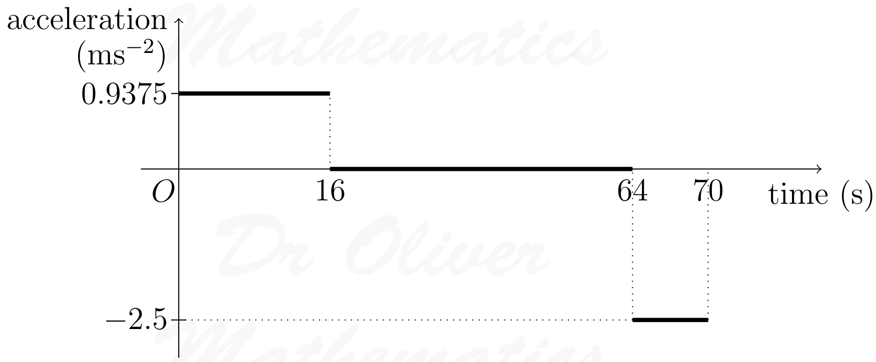
(d) Find the value of a. (2)

Solution

$$\text{Acceleration} = \frac{15 - 0}{\frac{1}{3} \times 48} = \frac{15}{16} = \underline{\underline{0.94 \text{ (2 sf)}}}.$$

(e) Sketch a acceleration-time graph for the motion of the car. (3)

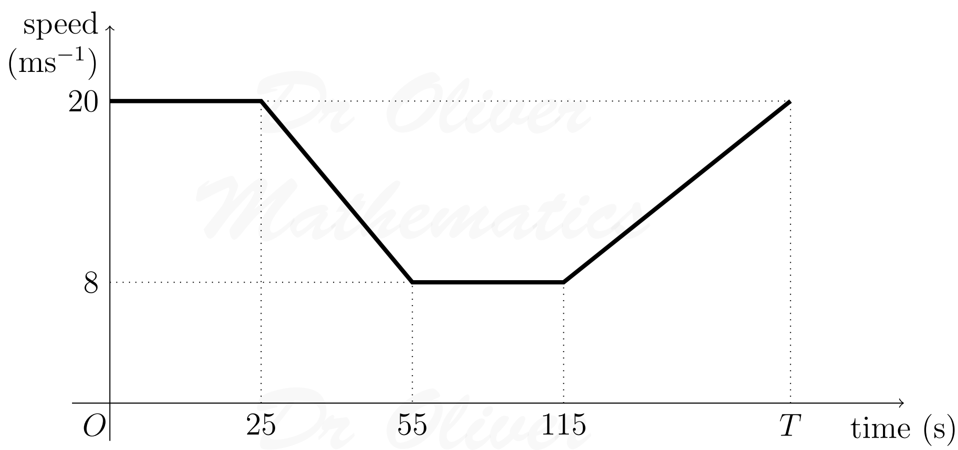
Solution



25. A car is moving on a straight horizontal road. At time $t = 0$, the car is moving with speed 20 ms^{-1} and is at the point A. The car maintains the speed of 20 ms^{-1} for 25 s. The car then moves with constant deceleration 0.4 ms^{-2}, reducing its speed from 20 ms^{-1} to 8 ms^{-1}. The car then move with constant speed 8 ms^{-1} for 60 s. The car then moves with constant acceleration until it is moving with speed 20 ms^{-1} at the point B.

(a) Sketch a speed-time graph for the motion of the car from A to B. (3)

Solution



(b) Find the time for which the car is decelerating. (2)

Solution

$$\text{Decelerating} = \frac{20-8}{0.4} = \underline{\underline{30 \text{ s}}}.$$

Given that the distance from A to B is 1960 m,

(c) find the time taken from the car to move from A to B. (8)

Solution

$$20 \times 25 + \tfrac{1}{2} \times (8+20) \times 30 + 8 \times 60 + \tfrac{1}{2} \times (8+20) \times (T-115) = 1960$$
$$\Rightarrow 500 + 420 + 480 + 14(T-115) = 1960$$
$$\Rightarrow 14(T-115) = 560$$
$$\Rightarrow T - 115 = 40$$
$$\Rightarrow \underline{\underline{T = 155}}.$$

26. A particle P is projected vertically upwards from a point A with speed u ms^{-1}. The point A is 17.5 m above horizontal ground. The particle P moves freely under gravity until it reaches the ground with speed 28 ms^{-1}.

(a) Show that $u = 21$. (3)

Solution

$s = -17.5$, $u = ?\,(\uparrow)$, $v = -28$, $a = -9.8$, and $t = ?$:

$$v^2 = u^2 + 2as \Rightarrow (-28)^2 = u^2 - 2 \times 9.8 \times 17.5$$
$$\Rightarrow u^2 = 441$$
$$\Rightarrow \underline{\underline{u = 21}}.$$

At t seconds after projection, P is 19 m above A.

(b) Find the possible values of t. (5)

Solution

$s = 19$, $u = 21\,(\uparrow)$, $v = ?$, $a = -9.8$, and $t = ?$:

$$
\begin{aligned}
s = ut + \tfrac{1}{2}at^2 &\Rightarrow 19 = 21t - 4.9t^2 \\
&\Rightarrow 4.9t^2 - 21t + 19 = 0 \\
&\Rightarrow t = \frac{21 \pm \sqrt{(-21)^2 - 4 \times 4.9 \times 19}}{9.8} \\
&\Rightarrow t = \frac{21 \pm \sqrt{68.6}}{9.8} \\
&\Rightarrow t = \frac{15 \pm \sqrt{35}}{7} \\
&\Rightarrow \underline{\underline{t = 1.3,\ 3.0 \text{ (2 sf)}}}.
\end{aligned}
$$

The ground is soft and, after P reaches the ground, P sinks vertically downwards into the ground before coming to rest. The mass of P is 4 kg and the ground is assumed to exert a constant resistive force of magnitude 5000 N on P.

(c) Find the vertically distance that P sinks into the ground before coming to rest. (4)

Solution

Newton's 2nd Law:

$$4g - 5000 = 4a \Rightarrow a = -1240.2 \text{ ms}^{-2}.$$

$s = ?$, $u = 28\,(\downarrow)$, $v = 0$, $a = -1240.2$, and $t = ?$:

$$
\begin{aligned}
v^2 = u^2 + 2as &\Rightarrow 0 = 28^2 + 2 \times (-1240.2) \times s \\
&\Rightarrow 2480.4s = 784 \\
&\Rightarrow s = \tfrac{1960}{6201} \\
&\Rightarrow \underline{\underline{s = 0.32 \text{ m (2 sf)}}}.
\end{aligned}
$$

27. The velocity-time graph in Figure 3 represents the journey of a train P travelling along a straight horizontal track between two stations, which are 1.5 km apart.

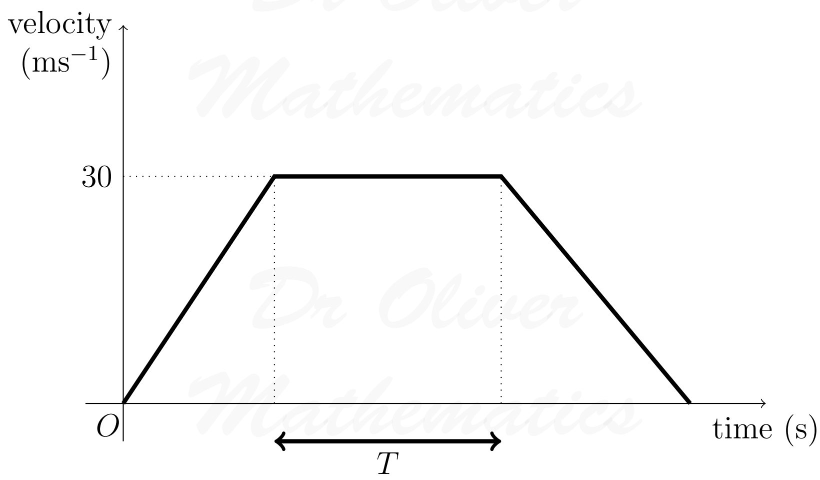


Figure 3: a train journey

The train P leaves the first station, accelerating uniformly from rest the 300 m until it reaches a speed of 30 ms^{-1}. The train then maintains this speed for T seconds before decelerating uniformly at 1.25 ms^{-2}, coming to rest at the next station.

(a) Find the acceleration of P during the first 300 m of its journey. (2)

Solution

$s = 300$, $u = 0$, $v = 30$, $a =?$, and $t =?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 30^2 = 0 + 2 \times a \times 300 \\ &\Rightarrow 600a = 900 \\ &\Rightarrow \underline{\underline{a = 1.5 \text{ ms}^{-2}}}. \end{aligned}$$

(b) Find the value of T. (5)

Solution

$s =?$, $u = 30$, $v = 0$, $a = -1.25$, and $t =?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = 30^2 - 2 \times 1.25 \times s \\ &\Rightarrow 2.5s = 900 \\ &\Rightarrow s = 360 \text{ m} \end{aligned}$$

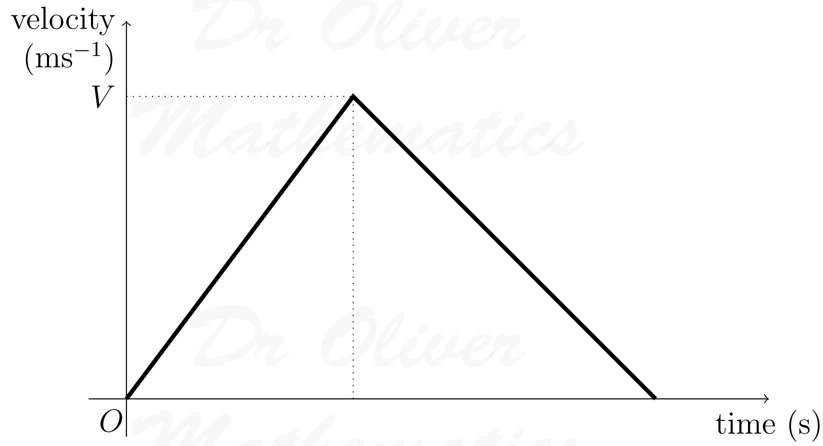
and

$$\begin{aligned} 300 + 30 \times T + 360 = 1500 &\Rightarrow 30T = 840 \\ &\Rightarrow \underline{\underline{T = 28}}. \end{aligned}$$

A second train Q completes the same journey in the same total time. The train leaves the first station, accelerating uniformly from rest until it reaches a speed of V ms^{-1} and then immediately decelerates uniformly until it comes to rest at the next station.

(c) Sketch a velocity-time graph which represents the journey of train Q. (2)

Solution



(d) Find the value of V. (6)

Solution

$$30 = 0 + 1.5t_1 \Rightarrow t_1 = 20$$

and

$$30 = 0 + 1.25t_1 \Rightarrow t_2 = 24.$$

Now,

$$\begin{aligned} \tfrac{1}{2} \times (20 + 28 + 24) \times V = 1500 &\Rightarrow 72V = 3000 \\ &\Rightarrow V = 41\tfrac{2}{3} \\ &\Rightarrow \underline{\underline{V = 42 \text{ (2 sf)}}}. \end{aligned}$$

28. A lorry is moving along a straight horizontal road with constant acceleration. The lorry passes a point A with speed u ms^{-1}, $u < 34$, and, 10 s later, passes a point B with speed 34 ms^{-1}. Given that $AB = 240$ m, find

(a) the value of u, and (3)

Solution

$s = 240$, $u = ?$, $v = 34$, $a = ?$, and $t = 10$:

$$\begin{aligned} s = \tfrac{1}{2}(u+v)t &\Rightarrow 240 = \tfrac{1}{2} \times (u+34) \times 10 \\ &\Rightarrow u + 34 = 48 \\ &\Rightarrow \underline{\underline{u = 14}}. \end{aligned}$$

(b) the time taken for the lorry to move from A to the mid-point of AB. (6)

Solution

$s = 240$, $u = 14$, $v = 34$, $a = ?$, and $t = 10$:

$$v = u + at \Rightarrow 34 = 14 + 10a \Rightarrow 10a = 20 \Rightarrow a = 2.$$

$s = 120$, $u = 14$, $v = ?$, $a = 2$, and $t = ?$:

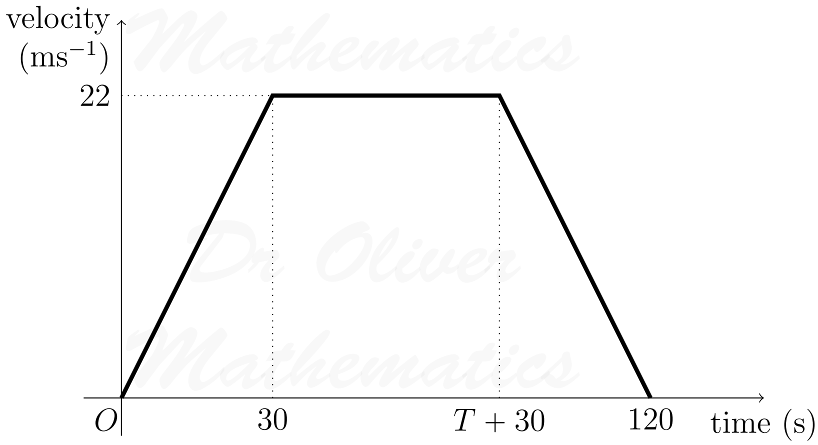
$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow 120 = 14t + t^2 \\ &\Rightarrow t^2 + 14t - 120 = 0 \\ &\Rightarrow (t-6)(t+20) = 0 \\ &\Rightarrow t = 6 \text{ or } t = -20; \end{aligned}$$

hence, $\underline{\underline{t = 6}}$.

29. A car is moving along a straight horizontal road. The car takes 120 s to travel between two sets of traffic lights, which are 2145 m apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for 30 s until its speed is 22 ms^{-1}. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

(a) Sketch a speed-time graph for the motion of the car between the two sets of traffic lights. (2)

Solution



(b) Find the value of T. (3)

Solution

$$\begin{aligned} \tfrac{1}{2} \times (T + 120) \times 22 = 2145 &\Rightarrow T + 120 = 195 \\ &\Rightarrow \underline{\underline{T = 75}}. \end{aligned}$$

A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, a ms^{-2}, and passes the car at the point A, which is 990 m from the first set of traffic lights. When motorcycle passes the car, the car is moving with speed 22 ms^{-1}.

(c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A. (4)

Solution

$$\begin{aligned} \tfrac{1}{2} \times [t + (t - 30)] \times 22 = 990 &\Rightarrow 2t - 30 = 90 \\ &\Rightarrow 2t = 120 \\ &\Rightarrow t = 60 \end{aligned}$$

and the

$$\text{time taken} = 60 - 50 = \underline{\underline{10 \text{ s}}}.$$

(d) Find the value of a. (2)

Solution

$s = 990$, $u = 0$, $v = ?$, $a = ?$, and $t = 50$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow 990 = 0 + \tfrac{1}{2} \times a \times 50^2 \\ &\Rightarrow 1250a = 990 \\ &\Rightarrow a = \tfrac{99}{125} \\ &\Rightarrow \underline{\underline{a = 0.79 \text{ (2 sf)}}}. \end{aligned}$$

30. At time $t = 0$, two balls A and B are projected vertically upwards. The ball A is projected vertically upwards with a speed 2 ms^{-1} from a point 50 m above horizontal ground. The ball B is projected vertically upwards from the ground with a speed 20 ms^{-1}. At the time $t = T$, the two balls are at the same vertical height, h metres, above the ground. The balls are modelled as particles moving freely under gravity. Find

(a) the value of T, and (5)

Solution

For A, $s = 50$, $u = 2\,(\uparrow)$, $v = 0$, $a = -9.8$, and $t = ?$:

$$s = ut + \tfrac{1}{2}at^2 \Rightarrow 0 = 2t - 4.9t^2 + 50$$

For B, $s = 0$, $u = 20\,(\uparrow)$, $v = ?$, $a = -9.8$, and $t = ?$:

$$s = ut + \tfrac{1}{2}at^2 \Rightarrow 0 = 20t - 4.9t^2.$$

Subtract:

$$0 = (2t + 50) - 20t \Rightarrow 18t = 50 \Rightarrow t = \tfrac{25}{9} = \underline{\underline{2.8 \text{ (2 sf)}}}.$$

(b) the value of h. (2)

Solution

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow h = 20 \times \tfrac{25}{9} - 4.9 \times (\tfrac{25}{9})^2 \\ &\Rightarrow h = 17\tfrac{121}{162} \\ &\Rightarrow \underline{\underline{h = 18 \text{ (2 sf)}}}. \end{aligned}$$

31. A ball of mass 0.3 kg is released from rest at a point which is 2 m above horizontal ground. The ball moves freely under gravity. After striking the ground, the ball rebounds vertically and rises to a maximum height of 1.5 m above the ground, before falling to the ground again. The ball is modelled as a particle.

(a) Find the speed of the ball at the instant before it strikes the ground for the first time. (2)

Solution

$s = 2$, $u = 0\,(\downarrow)$, $v = ?$, $a = 9.8$, and $t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow v^2 = 0 + 2 \times 9.8 \times 2 \\ &\Rightarrow v^2 = 39.2 \\ &\Rightarrow v = \frac{14\sqrt{5}}{5} \\ &\Rightarrow \underline{\underline{v = 6.3 \text{ (2 sf)}}}. \end{aligned}$$

(b) Find the speed of the ball at the instant after it rebounds from the ground for the first time. (2)

Solution

$s = 1.5$, $u = ?\,(\uparrow)$, $v = 0$, $a = -9.8$, and $t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = u^2 - 2 \times 9.8 \times 1.5 \\ &\Rightarrow u^2 = 29.4 \\ &\Rightarrow u = \frac{7\sqrt{15}}{5} \\ &\Rightarrow \underline{\underline{u = 5.4 \text{ (2 sf)}}}. \end{aligned}$$

(c) Find the magnitude of the impulse on the ball in the first impact with the ground. (2)

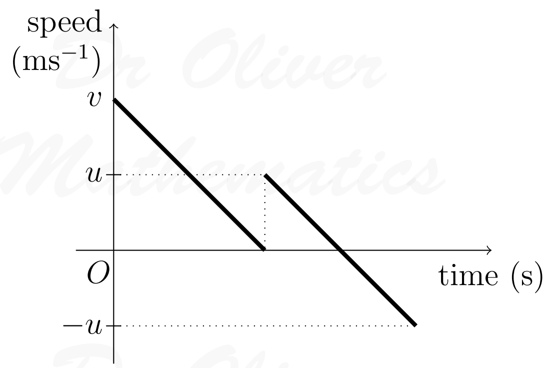
Solution

$$\begin{aligned} \text{Impulse} &= 0.3\left[\frac{14\sqrt{5}}{5} - \left(\frac{7\sqrt{15}}{5}\right)\right] \\ &= \frac{21\sqrt{15} + 42\sqrt{5}}{50} \\ &= \underline{\underline{3.5 \text{ Ns (2 sf)}}}. \end{aligned}$$

(d) Sketch a velocity-time graph for the motion of the ball from the instant when it is released until the instant when it strikes the ground for the second time. (3)

Solution

Going up is chosen to be positive.



(e) Find the time between the instant when the ball is released and the instant when the ball strikes the ground for the second time. (4)

Solution

First impact — $s = 2$, $u = 0\,(\downarrow)$, $v = \frac{14\sqrt{5}}{5}$, $a = 9.8$, and $t = ?$:

$$\text{time} = \frac{\frac{14\sqrt{5}}{5} - 0}{9.8} = \frac{2\sqrt{5}}{7}.$$

Second impact — $s = 0$, $u = \frac{7\sqrt{15}}{5}\,(\uparrow)$, $v = -\frac{7\sqrt{15}}{5}$, $a = -9.8$, and $t = ?$:

$$\text{time} = \frac{\frac{-7\sqrt{15}}{5} - \left(\frac{-7\sqrt{15}}{5}\right)}{-9.8} = \frac{2\sqrt{15}}{7}.$$

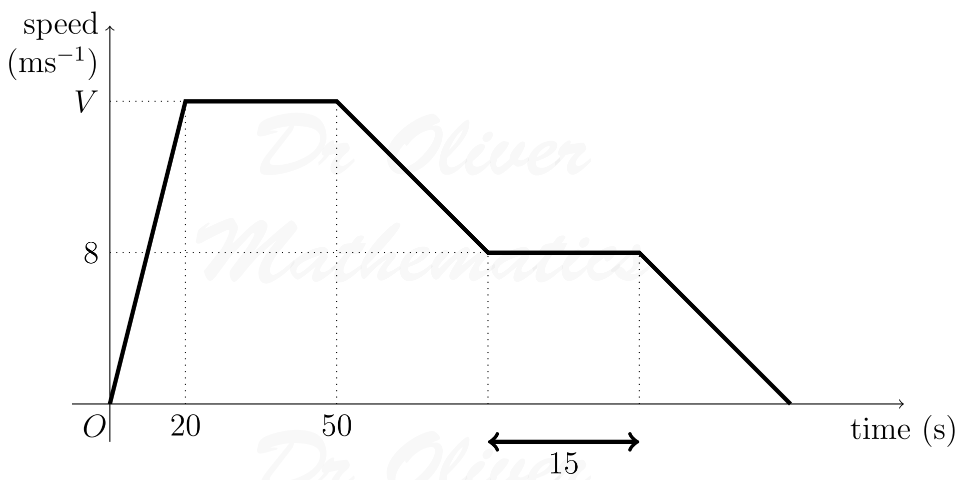
Finally,

$$\text{total time} = \frac{2\sqrt{5}}{7} + \frac{2\sqrt{15}}{7} = \frac{2\sqrt{15} + 2\sqrt{5}}{7} = \underline{\underline{1.7 \text{ (2 sf)}}}.$$

32. A car starts from rest and moves with constant acceleration along a straight horizontal road. The car reaches a speed of V ms^{-1} in 20 seconds. It moves at constant speed V ms^{-1} for the next 30 seconds, then moves with constant deceleration $\frac{1}{2}$ ms^{-2} until it has speed 8 ms^{-1}. It moves at constant speed 8 ms^{-1} for the next 15 seconds and then moves with constant deceleration $\frac{1}{3}$ ms^{-2} until it comes to rest.

(a) Sketch a speed-time graph for this journey. (3)

Solution



In the first 20 seconds of this journey, the car travels 140 m. Find

(b) the value of V, (2)

Solution

We use the area under the graph:

$$\tfrac{1}{2} \times V \times 20 = 140 \Rightarrow 10V = 140$$
$$\Rightarrow \underline{\underline{V = 14}}.$$

(c) the total time for this journey, and (4)

Solution

$\underline{\tfrac{1}{2}\text{ ms}^{-2}}$:

$$t_1 = \frac{14 - 6}{\frac{1}{2}} = 12.$$

$\underline{\tfrac{1}{3}\text{ ms}^{-2}}$:

$$t_2 = \frac{8 - 0}{\frac{1}{3}} = 24.$$

Thus,

$$\text{total time} = 20 + 30 + 12 + 15 + 24 = \underline{\underline{101}}.$$

(d) the total distance travelled by the car. (4)

Solution

$$\begin{aligned}\text{Distance} &= 140 + (30 \times 14) + \left[\tfrac{1}{2} \times (14+8) \times 12\right] + (8 \times 15) + (\tfrac{1}{2} \times 8 \times 24) \\ &= 140 + 420 + 132 + 120 + 96 \\ &= \underline{\underline{908}}.\end{aligned}$$

33. At time $t = 0$, a particle is projected vertically upwards with speed u ms^{-1} from a point A. The particle moves freely under gravity. At time T, the particle is at its maximum height H above A.

(a) Find T in terms of u and g. (2)

Solution

$s = H$, $u =?$, $v = 0$, $a = -g$, and $t = T$:

$$v = u + at \Rightarrow 0 = u - gT \Rightarrow \underline{\underline{T = \frac{u}{g}}}.$$

(b) Show that $H = \dfrac{u^2}{2g}$. (2)

Solution

$$\begin{aligned}s = vt - \tfrac{1}{2}at^2 &\Rightarrow H = 0 + \tfrac{1}{2}gT^2 \\ &\Rightarrow H = \frac{g}{2}\left(\frac{u}{g}\right)^2 \\ &\Rightarrow \underline{\underline{H = \frac{u^2}{2g}}}.\end{aligned}$$

The point A is at a height $3H$ above the ground.

(c) Find, in terms of T, the total time from the instant of projection to the instant when the particle hits the ground. (4)

Solution

$s = -3H = -\frac{3u^2}{2g}$, $u = gT$, $v =?$, $a = -g$, and $t =?$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow -\frac{3u^2}{2g} = g\left(\frac{u}{g}\right)t - \tfrac{1}{2}gt^2 \\ &\Rightarrow -3u^2 = 2gut - g^2t^2 \\ &\Rightarrow g^2t^2 - 2gut - 3u^2 = 0 \\ &\Rightarrow (gt+u)(gt-3u) \\ &\Rightarrow t = -\frac{u}{g} \text{ or } t = \frac{3u}{g}; \end{aligned}$$

hence,

$$t = \frac{3u}{g} = \underline{\underline{3T}}.$$

34. A small stone is projected vertically upwards from a point O with a speed of 19.6 ms^{-1}. Modelling the stone as a particle moving freely under gravity,

(a) find the greatest height above O reached by the stone, and (2)

Solution

$s =?$, $u = 19.6$, $v = 0$, $a = -9.8$, and $t =?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = 19.6^2 - 2 \times 9.8 \times s \\ &\Rightarrow 19.6s = 19.6^2 \\ &\Rightarrow \underline{\underline{s = 19.6}}. \end{aligned}$$

(b) find the length of time for which the stone is more than 14.7 m above O. (5)

Solution

$s = 14.7$, $u = 19.6$, $v =?$, $a = -9.8$, and $t =?$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow 14.7 = 19.6t - 4.9t^2 \\ &\Rightarrow 4.9t^2 - 19.6t + 14.7 = 0 \\ &\Rightarrow 49t^2 - 196t + 147 = 0 \\ &\Rightarrow t^2 - 4t + 3 = 0 \\ &\Rightarrow (t-1)(t-3) = 0 \\ &\Rightarrow t = 1 \text{ or } t = 3; \end{aligned}$$

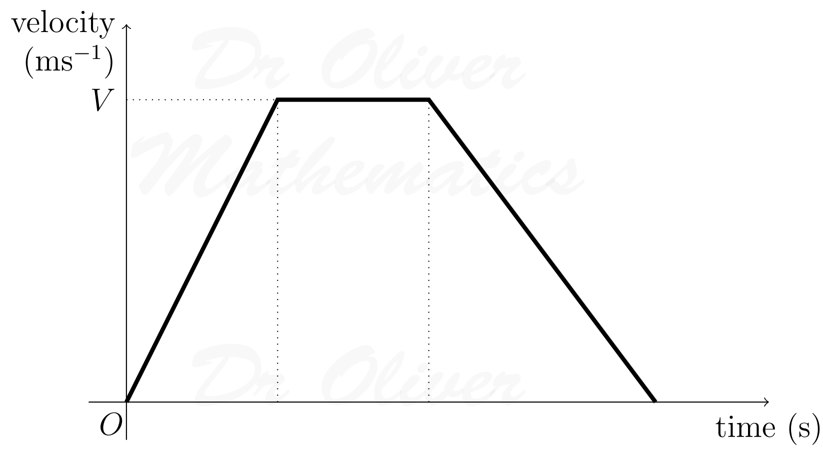
hence,

$$\text{length of time} = 3 - 1 = \underline{\underline{2}}.$$

35. A train travels along a straight horizontal track between two stations, A and B. The train starts from rest at A and moves with constant acceleration 0.5 ms^{-2} until it reaches a speed of V ms^{-1}, $V < 50$. The train then travels at this constant speed before it moves with constant deceleration 0.25 ms^{-2} until it comes to rest at B.

(a) Sketch a speed-time graph for the motion of the train between the two stations A andf B. (2)

Solution



The total time for the journey from A to B is 5 minutes.

(b) Find, in terms of V, the length of time, in seconds, for which the train is

(i) accelerating, (2)

Solution

$$\text{time spent accelerating} = \frac{V - 0}{0.5} = \underline{\underline{2V}}.$$

(ii) decelerating, and (1)

Solution

$$\text{time spent decelerating} = \frac{V - 0}{0.25} = \underline{\underline{4V}}.$$

(iii) moving with constant speed. (2)

Solution

time spent at constant speed $= 300 - 2V - 4V - 300 = \underline{\underline{300 - 5V}}$.

Given that the total distance between the two stations A and B is 6.3 km,

(c) find the value of V. (6)

Solution

$$\begin{aligned}\tfrac{1}{2} \times [(300 - 6V) + 300] \times V = 6300 &\Rightarrow V(800 - 5V) = 12\,600 \\ &\Rightarrow 600V - 6V^2 = 12\,600 \\ &\Rightarrow 6V^2 - 600V + 12\,600 = 0 \\ &\Rightarrow V^2 - 100V + 2\,100 = 0 \\ &\Rightarrow (V - 30)(V - 70) = 0 \\ &\Rightarrow V = 30 \text{ or } V = 70;\end{aligned}$$

hence, because $V < 50$, $\underline{\underline{V = 30}}$.

36. Two trains M and N are moving in the same direction along parallel straight horizontal tracks. At time $t = 0$, M overtakes N whilst they are travelling with speeds of 40 ms^{-1} and 30 ms^{-1} respectively. Train M overtakes train N as they pass a point X at the side of the tracks.

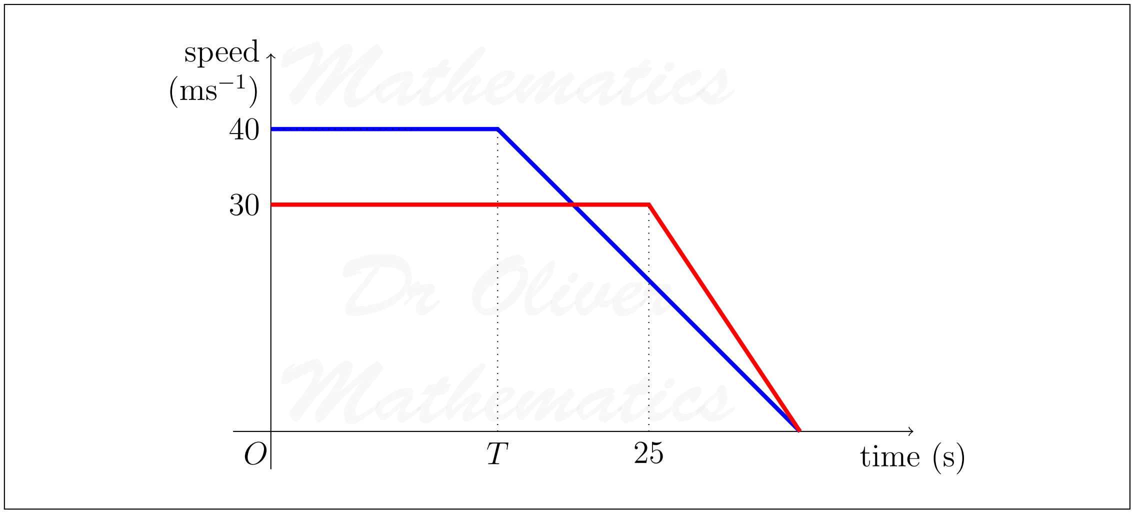
After overtaking N, train M maintains its speed of 40 ms^{-1} for T seconds and then decelerates uniformly, coming to rest next to a point Y at the side of the tracks.

After being overtaking, train N maintains its speed of 30 ms^{-1} for 25 seconds and then decelerates uniformly, also coming to rest next to the point Y.

The times taken by the trains to travel between X and Y are the same.

(a) Sketch the speed-time graphs for the motion of the two trains between X and Y. (4)

Solution



Given that $XY = 975$ m,

(b) find the value of T. (8)

Solution

For Y,

$$
\begin{aligned}
30 \times 25 + \tfrac{1}{2} \times (T_2 - 25) \times 30 = 975 &\Rightarrow 750 + 15(T_2 - 25) = 975 \\
&\Rightarrow 15(T_2 - 25) = 225 \\
&\Rightarrow T_2 - 25 = 15 \\
&\Rightarrow T_2 = 40.
\end{aligned}
$$

Finally,

$$
\begin{aligned}
\tfrac{1}{2} \times (T + 40) \times 40 = 975 &\Rightarrow 20(T + 40) = 975 \\
&\Rightarrow T + 40 = 48.75 \\
&\Rightarrow \underline{\underline{T = 8.75}}.
\end{aligned}
$$

37. A cyclist is moving along a straight horizontal road and passes a point A. Five seconds later, at the instant when she is moving with speed 10 ms^{-1}, she passes the point B. She moves with constant acceleration from A to B. Given that $AB = 40$ m, find

(a) the acceleration of the cyclist as she moves from A to B, (4)

Solution

$s = 40$, $u =?$, $v = 10$, $a =?$, and $t = 5$:

$$\begin{aligned} s = vt - \tfrac{1}{2}at^2 &\Rightarrow 40 = 10 \times 5 - \tfrac{1}{2} \times a \times 5^2 \\ &\Rightarrow 40 = 50 - 12.5a \\ &\Rightarrow \underline{\underline{a = 0.8}}. \end{aligned}$$

(b) the time it takes her to travel from A to the midpoint of AB. (5)

Solution

$$v = u + at \Rightarrow 10 = u + 0.8 \times 5 \Rightarrow u = 6.$$

$s = 20$, $u = 6$, $v =?$, $a = 0.8$, and $t =?$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow 20 = 6t + 0.4t^2 \\ &\Rightarrow 0.4t^2 + 6t - 20 \\ &\Rightarrow t = \frac{-6 \pm \sqrt{6^2 - 4 \times 0.8 \times (-20)}}{0.8} \\ &\Rightarrow t = \frac{-6 \pm \sqrt{68}}{0.8} \\ &\Rightarrow t = -17.807\,764\,06 \text{ or } 2.807\,764\,064 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{t = 2.8 \text{ (2 sf)}}}. \end{aligned}$$