Dr Oliver Mathematics
GCSE Mathematics
2022 November Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Write 500 as a product of powers of its prime factors. (3)

Solution

$$
\begin{array}{r|l}
 & 500 \\ \hline
2 & 250 \\ \hline
2 & 125 \\ \hline
5 & 25 \\ \hline
5 & 5 \\ \hline
5 & 1 \\ \hline
\end{array}
$$

So

$$500 = 2 \times 2 \times 5 \times 5 \times 5 = \underline{\underline{2^2 \times 5^3}}.$$

2. (a) Work out (2)

$$1\tfrac{3}{5} + 2\tfrac{1}{4}.$$

Give your answer as a mixed number.

Solution

$$
\begin{aligned}
1\tfrac{3}{5} + 2\tfrac{1}{4} &= 1 + \tfrac{3}{5} + 2 + \tfrac{1}{4} \\
&= 3 + \tfrac{12}{20} + \tfrac{5}{20} \\
&= \underline{\underline{3\tfrac{17}{20}}}.
\end{aligned}
$$

(b) Show that (2)

$$2\tfrac{2}{3} \div 6 = \tfrac{4}{9}.$$

Solution

$$
\begin{aligned}
2\tfrac{2}{3} \div 6 &= \tfrac{8}{3} \div 6 \\
&= \tfrac{8}{3} \times \tfrac{1}{6} \\
&= \tfrac{4}{3} \times \tfrac{1}{3} \\
&= \underline{\underline{\tfrac{4}{9}}},
\end{aligned}
$$

as required.

3. Simplify (2)

$$(2^{-5} \times 2^8)^2.$$

Give your answer as a power of 2

Solution

$$
\begin{aligned}
(2^{-5} \times 2^8)^2 &= (2^3)^2 \\
&= \underline{\underline{2^6}}.
\end{aligned}
$$

4. Work out (2)

$$0.004 \times 0.32.$$

Solution

Well,

$$4 \times 32 = 128$$

and count up the spaces to the right of decimal points: $3 + 2 = 5$. Hence,

$$0.004 \times 0.32 = \underline{\underline{0.001\,28}}.$$

5. A car factory is going to make four different car models **A**, **B**, **C**, and **D**. (2)

80 people are asked which of the four models they would be most likely to buy.

The table shows information about the results.

Car model	Number of people
A	23
B	15
C	30
D	12

The factory is going to make 40 000 cars next year.

Work out how many model **B** cars the factory should make next year.

Solution

They should make

$$\begin{aligned}\tfrac{15}{80} \times 40\,000 &= 15 \times 500 \\ &= \underline{\underline{7\,500 \text{ cars}}}\end{aligned}$$

of model **B**.

6. Rizwan writes down three numbers a, b, and c:

$$\begin{aligned}a : b &= 1 : 3 \\ b : c &= 6 : 5.\end{aligned}$$

(a) (i) Find (2)

$$a : b : c.$$

Solution

Well,

$$\begin{aligned}a : b &= 1 : 3 = 2 : 6 \\ b : c &= 6 : 5\end{aligned}$$

and

$$a : b : c = \underline{\underline{2 : 6 : 5}}.$$

(ii) Express a as a fraction of the total of the three numbers a, b, and c. (2)

Solution
Well,

$$2 + 6 + 5 = 13$$

and so, as a fraction, a equals $\underline{\underline{\tfrac{2}{13}}}$.

Emma writes down three numbers m, n, and p:

$$\begin{aligned} n &= 2m \\ p &= 5n. \end{aligned}$$

(b) Find (2)

$$m : p.$$

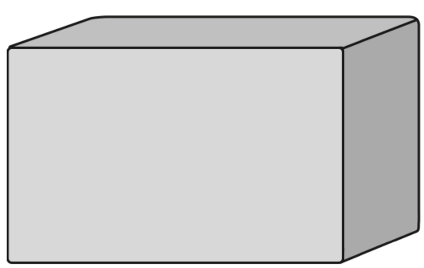
Solution
Well,

$$p = 5n \Rightarrow n = \tfrac{1}{5}p$$

and

$$\begin{aligned} \frac{n}{n} = \frac{1}{1} &\Rightarrow \frac{2m}{\frac{1}{5}p} = \frac{1}{1} \\ &\Rightarrow \frac{m}{p} = \frac{\frac{1}{2}}{5} \\ &\Rightarrow \frac{m}{p} = \frac{1}{10} \\ &\Rightarrow m : p = \underline{\underline{1 : 10}}. \end{aligned}$$

7. A storage tank exerts a force of 10 000 newtons on the ground. (2)



$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

The base of the tank in contact with the ground is a 4 m by 2 m rectangle.

Work out the pressure on the ground due to the tank.

Solution

$$\begin{aligned}\text{Pressure} &= \frac{\text{force}}{\text{area}} \\ &= \frac{10\,000}{4 \times 2} \\ &= \frac{10\,000}{8} \\ &= \underline{\underline{1\,250 \text{ newtons/m}^2}}.\end{aligned}$$

8. Two numbers m and n are such that (2)

- m is a multiple of 5,
- n is an even number, and
- the highest common factor (HCF) of m and n is 7.

Write down a possible value for m and a possible value for n.

Solution

E.g., the number are $\underline{\underline{m = 35}}$ and $\underline{\underline{n = 14}}$: $35 = 5 \times 7$, 28 is a even number, and

$$m = 5 \times 7 \text{ and } n = 2 \times 7.$$

9. (a) Complete the table of values for (2)

$$y = 6x - x^3.$$

x	-3	-2	-1	0	1	2	3
y	9					4	-9

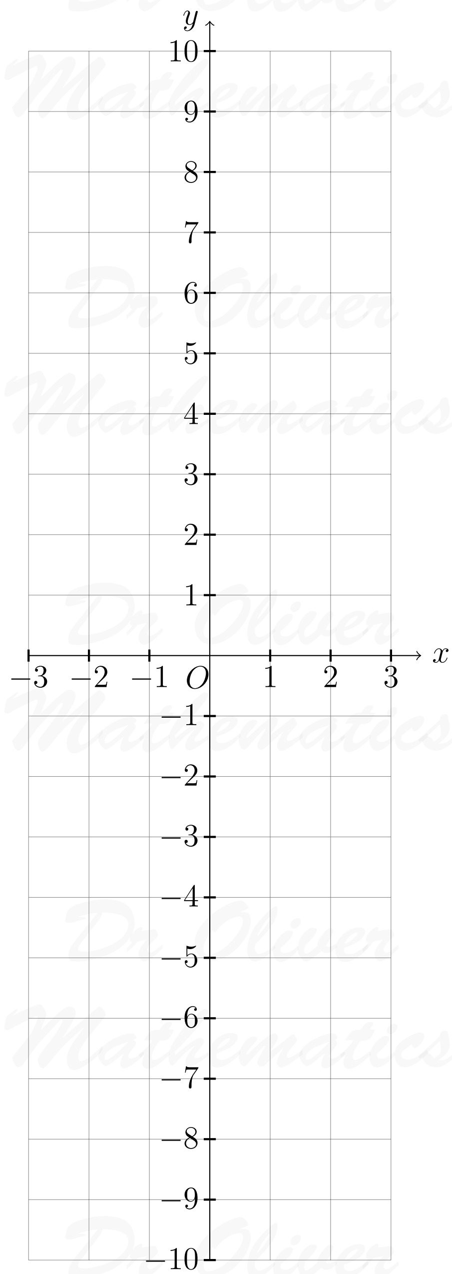
Solution

x	-3	-2	-1	0	1	2	3
y	9	$\underline{\underline{-4}}$	$\underline{\underline{-5}}$	$\underline{\underline{0}}$	$\underline{\underline{5}}$	4	-9

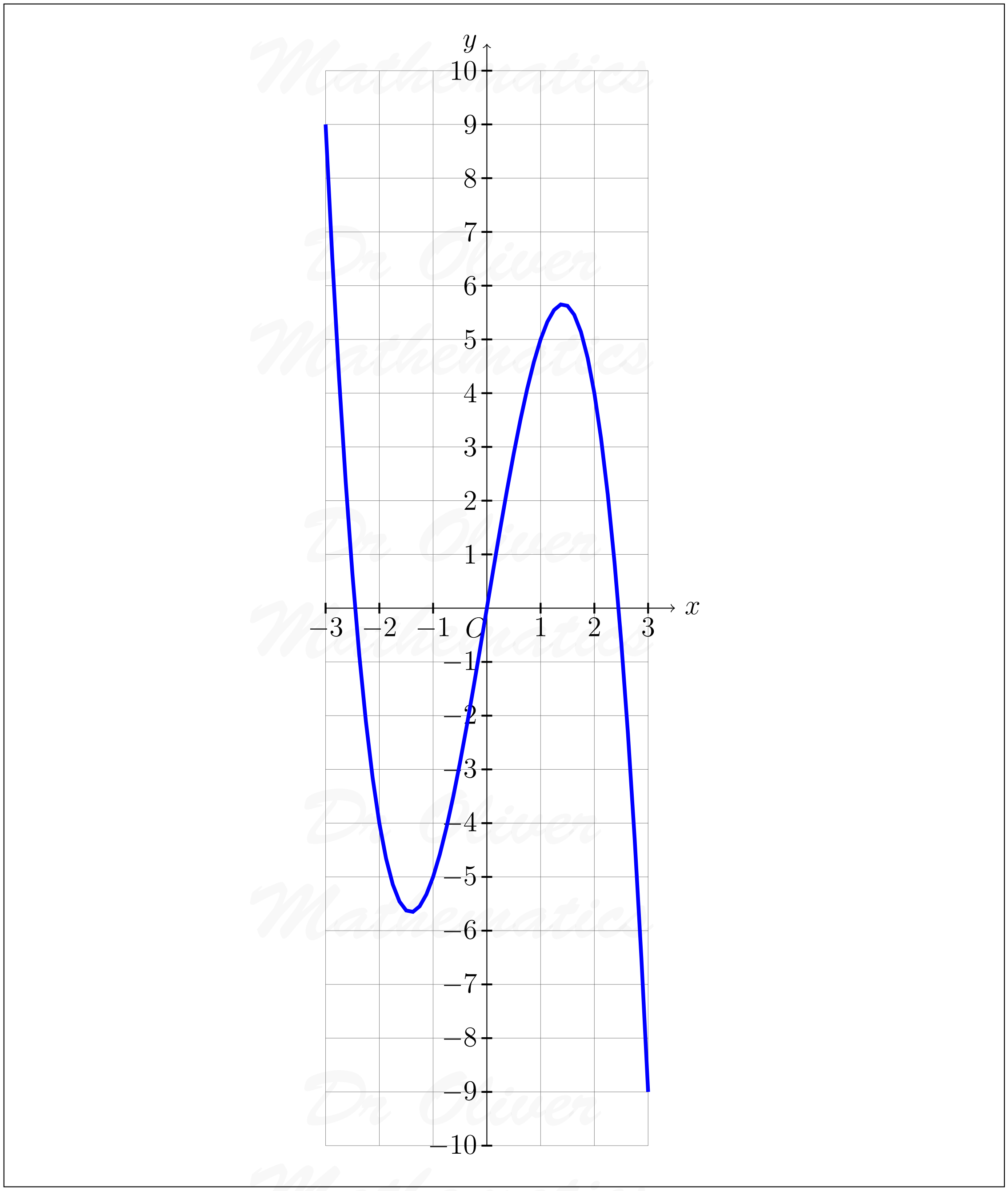
(b) On the grid, draw the graph of (2)

$$y = 6x - x^3$$

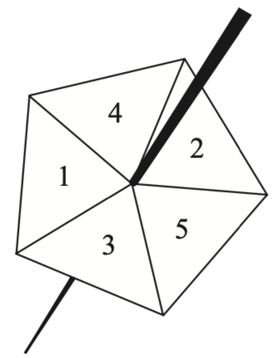
for values of x from -3 to 3.



Solution



10. Lina spins a biased 5-sided spinner 40 times.



Here are her results.

Score	1	2	3	4	5
Frequency	6	8	9	7	10

Lina is now going to spin the spinner another two times.

(a) Work out an estimate for the probability that she gets a score of 5 both times. (2)

Solution

$$\begin{aligned} \mathrm{P}(2,2) &= \tfrac{10}{40} \times \tfrac{10}{40} \\ &= \tfrac{1}{4} \times \tfrac{1}{4} \\ &= \underline{\underline{\tfrac{1}{16}}}. \end{aligned}$$

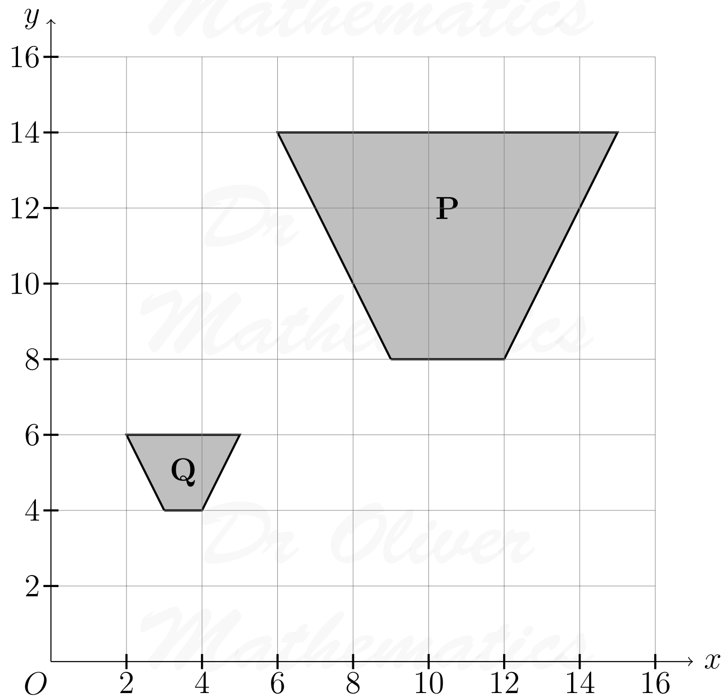
Derek is going to spin the spinner a large number of times.

(b) Work out an estimate for the percentage of times Derek can expect to get a score of 1. (2)

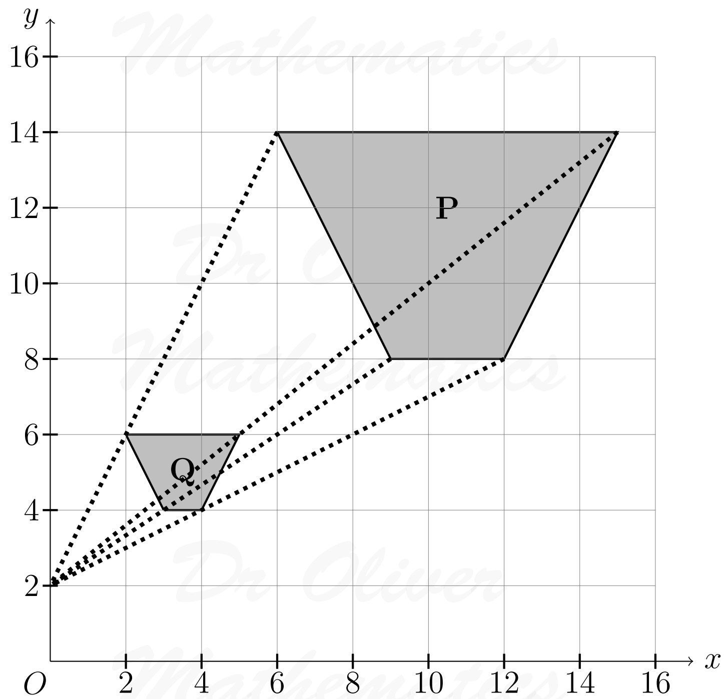
Solution

$$\begin{aligned} \text{Estimate} &= \tfrac{6}{40} \times 100\% \\ &= \tfrac{3}{20} \times 100\% \\ &= 3 \times 5\% \\ &= \underline{\underline{15\%}}. \end{aligned}$$

11. Describe fully the single transformation that maps shape **P** onto shape **Q**. (2)



Solution



Enlargement, centre $(0, 1)$, scale factor $\frac{1}{3}$.

12. Solve the simultaneous equations (4)

$$
\begin{aligned}
5x + 2y &= 11 \\
4x + 3y &= 6.
\end{aligned}
$$

Solution

Well,

$$
\begin{aligned}
5x + 2y &= 11 \quad (1) \\
4x + 3y &= 6 \quad (2)
\end{aligned}
$$

E.g., do $3 \times (1)$ and $2 \times (2)$:

$$
\begin{aligned}
15x + 6y &= 33 \quad (3) \\
8x + 6y &= 12 \quad (4)
\end{aligned}
$$

Do (3) − (4):

$$7x = 21 \Rightarrow \underline{\underline{x = 3}}$$

insert into (1):

$$\begin{aligned}
&\Rightarrow 5(3) + 2y = 11\\
&\Rightarrow 15 + 2y = 11\\
&\Rightarrow 2y = -4\\
&\Rightarrow \underline{\underline{y = -2}}.
\end{aligned}$$

13. p is inversely proportional to t. (3)

Complete the table of values.

t	100	25		2
p	1		5	

Solution

Well,

$$p \propto \frac{1}{t} \Rightarrow pt = k,$$

for some constant k. Now,

$$t = 100, p = 1 \Rightarrow pt = 100$$

and so

$$pt = 100.$$

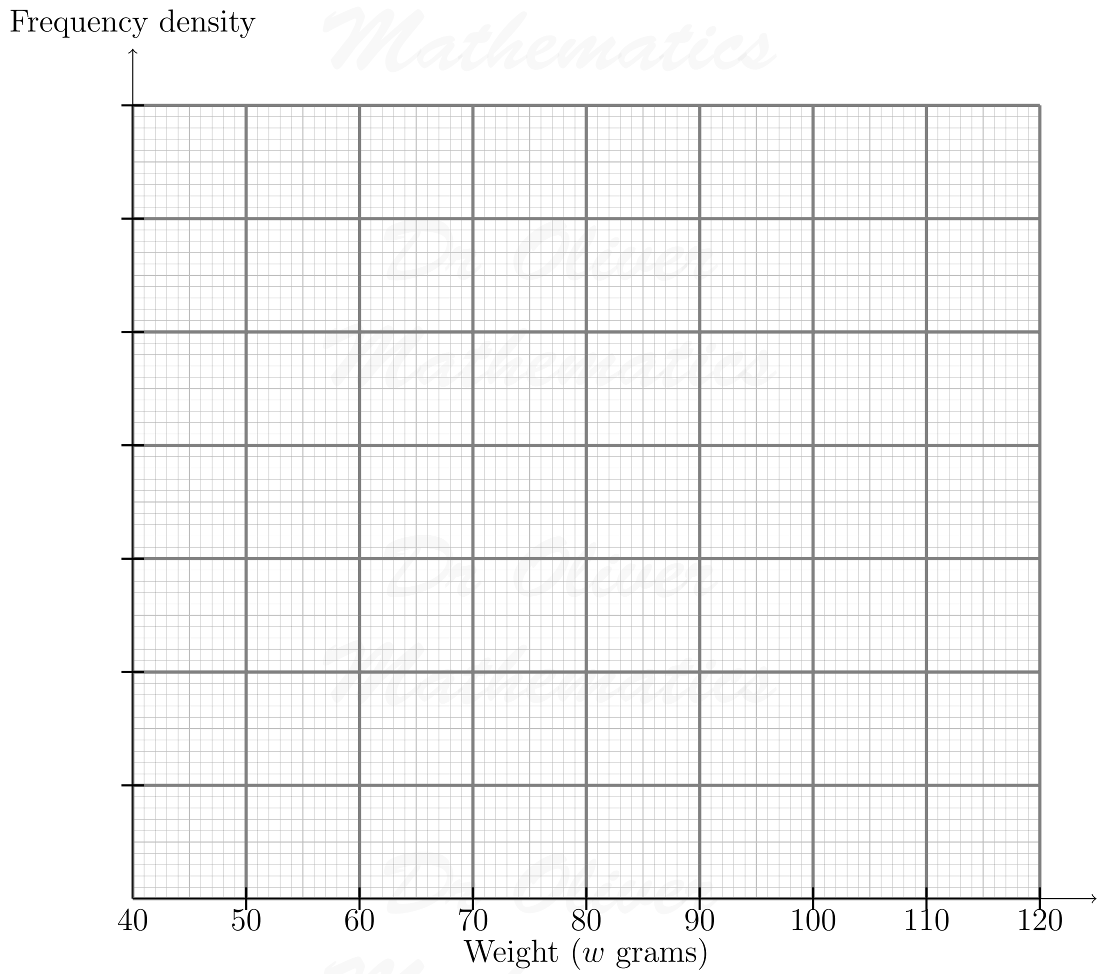
Hence,

t	100	25	$\underline{\underline{20}}$	2
p	1	$\underline{\underline{4}}$	5	$\underline{\underline{50}}$

14. The table shows information about the weights, in grams, of some potatoes. (3)

Weight (w grams)	Number of potatoes
$50 < w \leqslant 70$	20
$70 < w \leqslant 80$	50
$80 < w \leqslant 90$	60
$90 < w \leqslant 110$	30

On the grid, draw a histogram for this information.

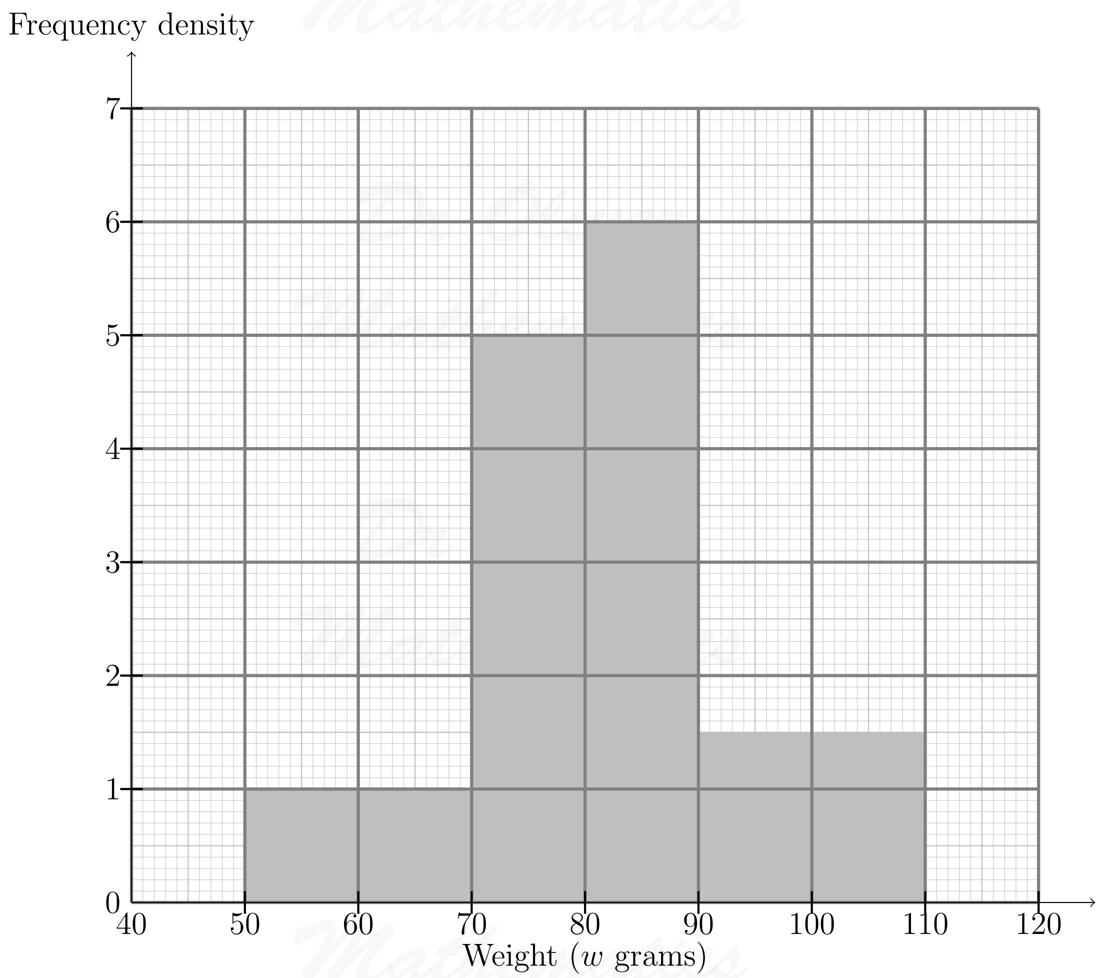


Solution

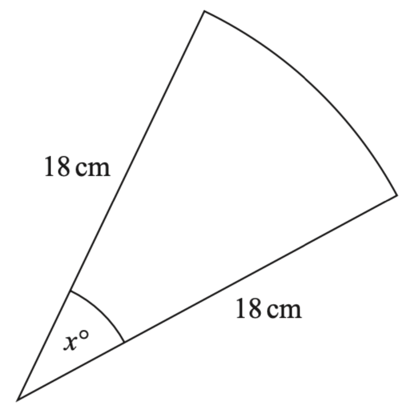
Well,

Weight (w grams)	Number of potatoes	Width	Frequency Density
$50 < w \leqslant 70$	20	20	$\frac{20}{20} = 1$
$70 < w \leqslant 80$	50	10	$\frac{50}{10} = 5$
$80 < w \leqslant 90$	60	10	$\frac{60}{10} = 6$
$90 < w \leqslant 110$	30	20	$\frac{30}{20} = 1.5$

and we can draw the histogram



15. The diagram shows a sector of a circle of radius 18 cm. (3)



The length of the arc is 4π cm.

Work out the value of x.

Solution

Now,

$$\begin{aligned}\frac{x}{360} = \frac{4\pi}{2 \times \pi \times 18} &\Rightarrow \frac{x}{360} = \tfrac{4}{36} \\ &\Rightarrow \frac{x}{360} = \tfrac{1}{9} \\ &\Rightarrow x = \tfrac{1}{9} \times 360 \\ &\Rightarrow \underline{\underline{x = 40}}.\end{aligned}$$

16. (a) Prove that (3)

$$(2m+1)^2 - (2n-1)^2 = 4(m+n)(m-n+1).$$

Solution

Well,

$\times$	$2m$	$+1$
$2m$	$4m^2$	$+2m$
$+1$	$+2m$	$+1$

and

$\times$	$2n$	$+1$
$2m$	$4n^2$	$-2n$
-1	$-2n$	$+1$

and so

$$\begin{aligned}(2m+1)^2-(2n-1)^2 &= (4m^2+4m+1)-(4n^2-4n+1)\\ &= 4m^2+4m-4n^2+4n\\ &= 4(m^2+m-n^2+n)\\ &= 4[(m^2-mn+m)+(mn-n^2+n)]\\ &= 4[m(m-n+1)+n(m-n+1)]\\ &= \underline{\underline{4(m+n)(m-n+1)}},\end{aligned}$$

as required.

Sophia says that the result in part (a) shows that the difference of the squares of any two odd numbers must be a multiple of 4.

(b) Is Sophia correct? (1)
You must give reasons for your answer.

Solution
Yes: the LHS are odd numbers and the RHS is $4p$, where p is some integer.

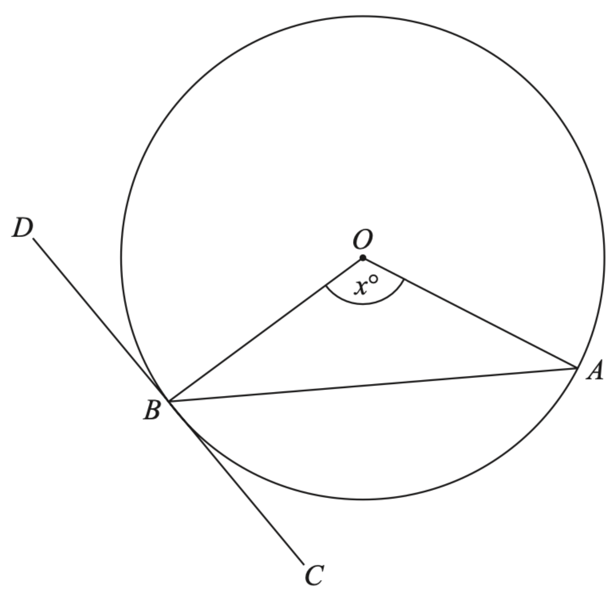
17. Work out the value of (2)

$$\left(\tfrac{8}{27}\right)^{\frac{4}{3}}.$$

Solution

$$\left(\frac{8}{27}\right)^{\frac{4}{3}} = \left[\left(\frac{8}{27}\right)^{\frac{1}{3}}\right]^4$$
$$= \left(\frac{\sqrt[3]{8}}{\sqrt[3]{27}}\right)^4$$
$$= \left(\frac{2}{3}\right)^4$$
$$= \frac{2^4}{3^4}$$
$$= \underline{\underline{\frac{16}{81}}}.$$

18. A and B are points on a circle, centre O. (3)
DBC is the tangent to the circle at B.
Angle $AOB = x°$.



Show that

$$\text{angle } ABC = \tfrac{1}{2}x°.$$

You must give a reason for each stage of your working.

Solution

Well, $\angle OBA = \angle OAB = \frac{1}{2}(180 - x)$ (base angles).
Finally,

$$\begin{aligned} \angle ABC &= 90 - \tfrac{1}{2}(180 - x) \quad (\angle OBC \text{ is a right angle}) \\ &= 90 - 90 + \tfrac{1}{2}x \\ &= \underline{\underline{\tfrac{1}{2}x^\circ}}. \end{aligned}$$

19. Solve (5)

$$\frac{1}{x} - \frac{1}{x+1} = 4.$$

Give your answer in the form $a \pm b\sqrt{2}$, where a and b are fractions.

Solution

Multiply by $x(x+1)$:

$$\begin{aligned} & \frac{1}{x} - \frac{1}{x+1} = 4 \\ \Rightarrow \quad & x(x+1) \times \frac{1}{x} - x(x+1) \times \frac{1}{x+1} = x(x+1) \times 4 \\ \Rightarrow \quad & (x+1) - x = 4x(x+1) \\ \Rightarrow \quad & 1 = 4x^2x + 4x \\ \Rightarrow \quad & 1 + 1 = 4x^2x + 4x + 1 \\ \Rightarrow \quad & 2 = (2x+1)^2 \\ \Rightarrow \quad & \pm\sqrt{2} = 2x + 1 \\ \Rightarrow \quad & -1 \pm \sqrt{2} = 2x \\ \Rightarrow \quad & \underline{\underline{x = -\tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{2}}}; \end{aligned}$$

hence, $\underline{\underline{a = -\frac{1}{2}}}$ and $\underline{\underline{b = \frac{1}{2}}}$.

20. Alfie has 11 cards. (3)

He has

- 3 blue cards
- 7 green cards and

- 1 white card.

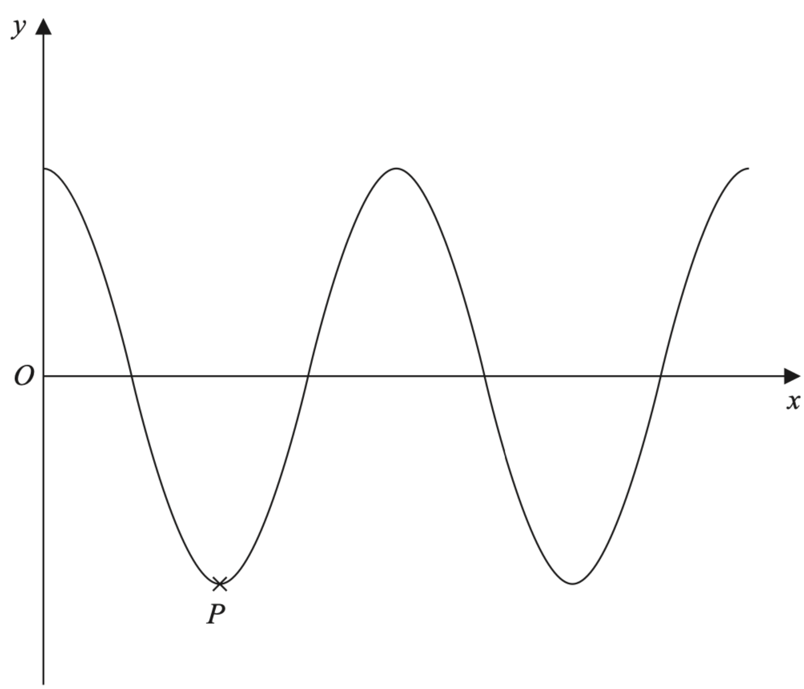
Alfie takes at random 2 of these cards.

Work out the probability that he takes cards of different colours.

Solution

$$\begin{aligned}
\mathrm{P}(\text{different colours}) &= 1 - \mathrm{P}(\text{the same}) \\
&= 1 - [\mathrm{P}(BB) + \mathrm{P}(GG)] \\
&= 1 - \left(\tfrac{3}{11} \times \tfrac{2}{10} + \tfrac{7}{11} \times \tfrac{6}{10}\right) \\
&= 1 - \left(\tfrac{6}{110} + \tfrac{42}{110}\right) \\
&= 1 - \tfrac{48}{110} \\
&= 1 - \tfrac{24}{55} \\
&= \underline{\underline{\tfrac{31}{55}}}.
\end{aligned}$$

21. The diagram shows a sketch of part of the curve with equation $y = \cos x^\circ$. (2)
P is a minimum point on the curve.

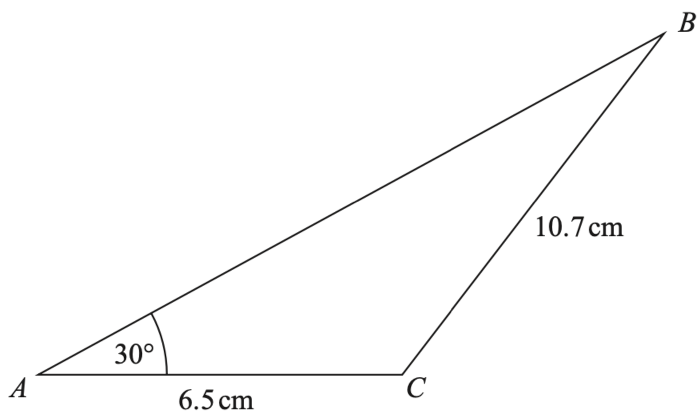


Write down the coordinates of P.

Solution

$\underline{\underline{P(180, -1)}}$.

22. Here is a triangle ABC. (4)



Work out the value of $\sin ABC$.
Give your answer in the form $\frac{m}{n}$ where m and n are integers.

Solution

$$\begin{aligned}\frac{\sin ABC}{AC} = \frac{\sin BAC}{BC} &\Rightarrow \frac{\sin ABC}{6.5} = \frac{\sin 30^\circ}{10.7} \\ &\Rightarrow \frac{\sin ABC}{6.5} = \frac{\frac{1}{2}}{10.7} \\ &\Rightarrow \sin ABC = \frac{\frac{1}{2} \times 6.5}{10.7} \\ &\Rightarrow \sin ABC = \frac{3.25}{10.7} \\ &\Rightarrow \sin ABC = \tfrac{325}{1\,070} \\ &\Rightarrow \underline{\underline{\sin ABC = \tfrac{65}{214}}}.\end{aligned}$$

23. Here are the first five terms of a geometric sequence.

$$\sqrt{5} \qquad 10 \qquad 20\sqrt{5} \qquad 200 \qquad 400\sqrt{5}.$$

(a) Work out the next term of the sequence. (2)

Solution

Well,

$$\begin{aligned} r &= \frac{10}{\sqrt{5}} \\ &= 2\sqrt{5} \end{aligned}$$

and

$$\begin{aligned} \text{6th term} &= 400\sqrt{5} \times 2\sqrt{5} \\ &= 800 \times 5 \\ &= \underline{\underline{4\,000}}. \end{aligned}$$

The 4th term of a different geometric sequence is $\frac{5\sqrt{2}}{4}$.

The 6th term of this sequence is $\frac{5\sqrt{2}}{8}$.

Given that the terms of this sequence are all positive,

(b) work out the first term of this sequence.
You must show all your working. (3)

Solution

Now,

$$ar^3 = \frac{5\sqrt{2}}{4} \text{ and } ar^5 = \frac{5\sqrt{2}}{8}$$

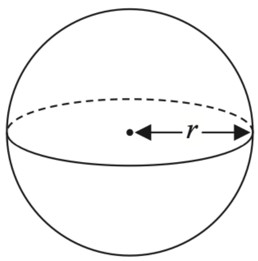
which leads to

$$\begin{aligned} r^2 = \frac{\frac{5\sqrt{2}}{8}}{\frac{5\sqrt{2}}{4}} &\Rightarrow r^2 = \frac{1}{2} \\ &\Rightarrow r = \frac{\sqrt{2}}{2} \text{ (because all terms are positive)} \\ &\Rightarrow r^3 = \frac{2\sqrt{2}}{8} \end{aligned}$$

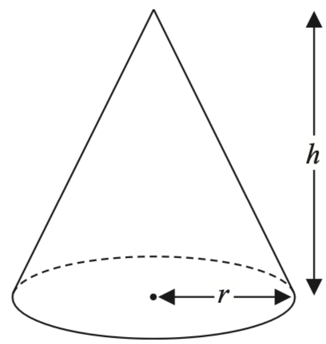
and

$$
\begin{aligned}
a &= \frac{\frac{5\sqrt{2}}{4}}{\frac{2\sqrt{2}}{8}} \\
&= \frac{5\sqrt{2}}{4} \times \frac{8}{2\sqrt{2}} \\
&= \underline{\underline{5}}.
\end{aligned}
$$

24. Here is a solid sphere and a solid cone.



Volume of sphere = $\frac{4}{3}\pi r^3$



Volume of cone = $\frac{1}{3}\pi r^2 h$

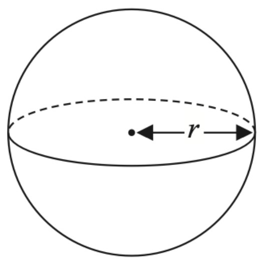
All measurements are in cm.
The volume of the sphere is equal to the volume of the cone.

(a) Find $r : h$. (2)
Give your answer in its simplest form.

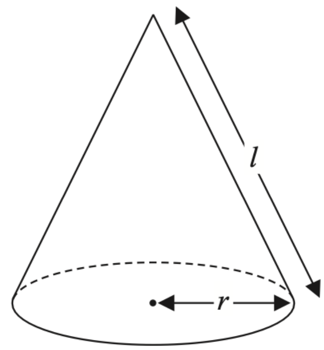
Solution
Well,

$$
\begin{aligned}
\tfrac{4}{3}\pi r^3 = \tfrac{1}{3}\pi r^2 h &\Rightarrow 4r = h \\
&\Rightarrow \underline{\underline{r : h = 1 : 4}}.
\end{aligned}
$$

Here is a different solid sphere and a different solid cone.



Surface area of sphere = $4\pi r^2$



Curved area of cone = $\pi r l$

All measurements are in cm.
The surface area of the sphere is equal to the **total** surface area of the cone.

(b) Find $r : h$. (4)
Give your answer in the form $1 : \sqrt{n}$, where n is an integer.

Solution
Well,

$$\begin{aligned}4\pi r^2 = \pi r l + \pi r^2 &\Rightarrow 3r = l \\ &\Rightarrow (3r)^2 = l^2 \\ &\Rightarrow 9r^2 = r^2 + h^2 \\ &\Rightarrow 8r^2 = h^2 \\ &\Rightarrow 2\sqrt{2}r = h \\ &\Rightarrow \underline{\underline{r : h = 1 : 2\sqrt{2}}}.\end{aligned}$$