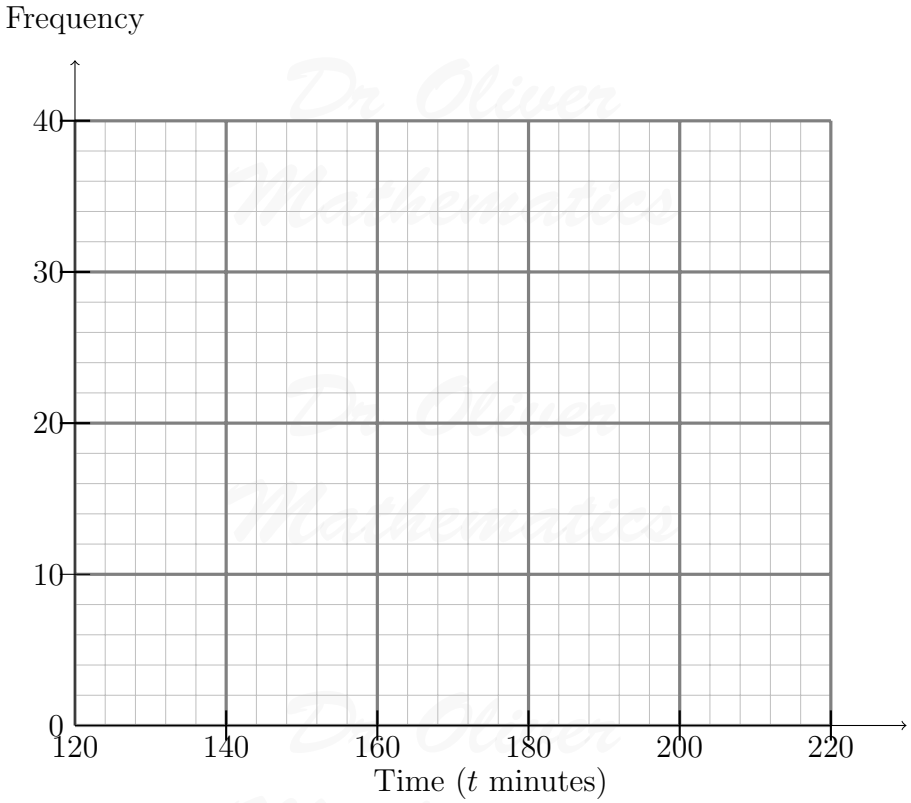
Dr Oliver Mathematics
GCSE Mathematics
2024 November Paper 3H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

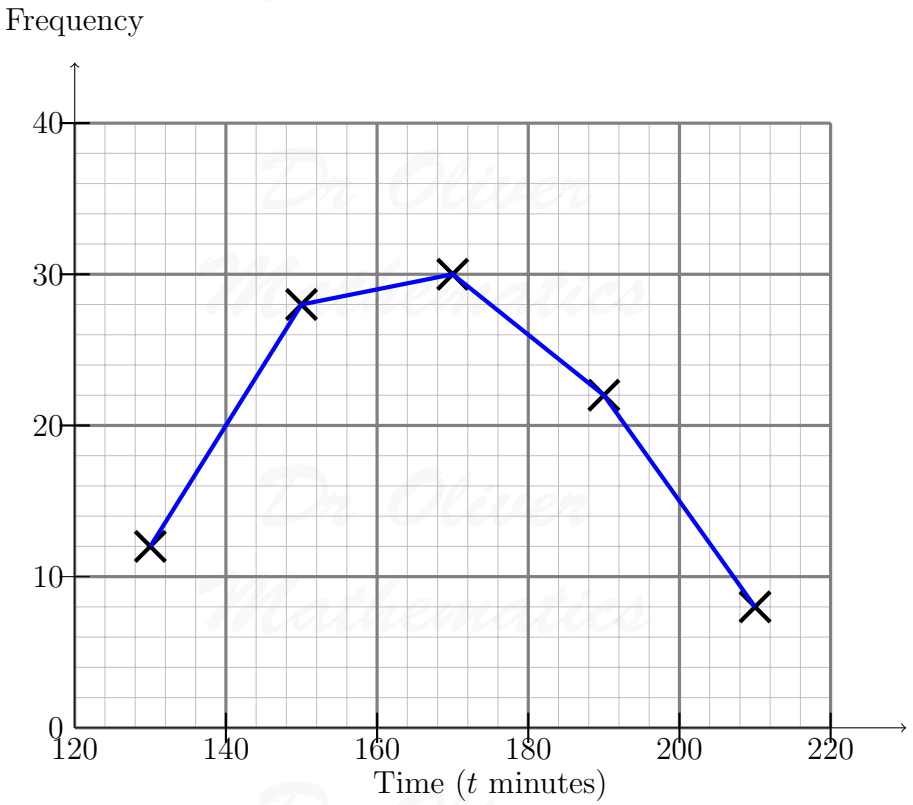
1. The table shows information about the times, in minutes, 100 people took to complete a bike race. (2)

Time (t minutes)	Frequency
$120 \leqslant t < 140$	12
$140 \leqslant t < 160$	28
$160 \leqslant t < 180$	30
$180 \leqslant t < 200$	22
$200 \leqslant t < 220$	8

On the grid below, draw a frequency polygon for this information.



Solution



2. (a) Write (1)

$$3.402 \times 10^5$$

as an ordinary number.

Solution

$3.402 \times 10^5 = \underline{\underline{340\,200}}$.

(b) Write (1)

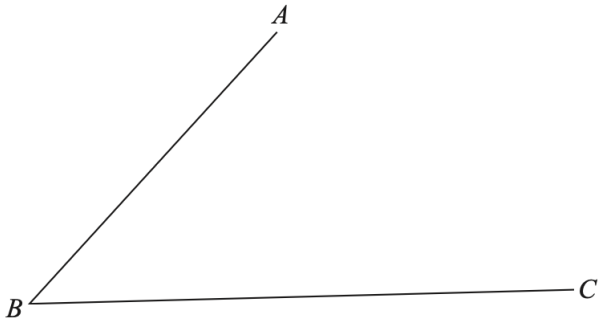
$$0.8026$$

in standard form.

Solution

$0.8026 = \underline{\underline{8.026 \times 10^{-1}}}$.

3. Use ruler and compasses to construct the bisector of angle ABC. (2)
You must show your construction lines.



Solution

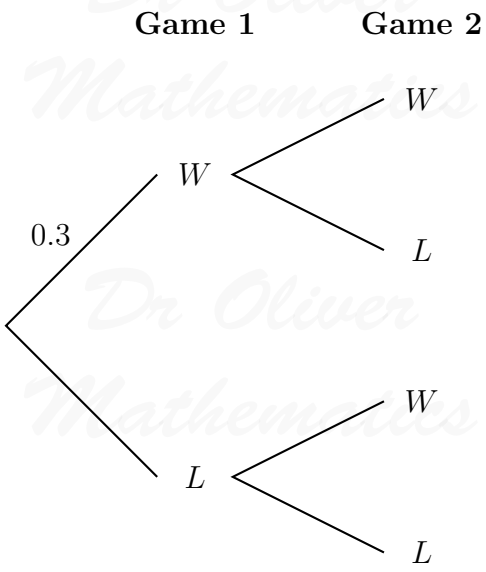
- Open your compasses to halfway between B and A.
- Starting at B, make a faint but visible arc from 12 o'clock to 3 o'clock.
- Label the points D on BA and E on BC.
- Starting at D, make a faint but visible arc from 12 o'clock to 3 o'clock.
- Starting at E, make a faint but visible arc from 12 o'clock to 3 o'clock.
- Label the point where they cross as F.
- Draw a straight line between B and F.
- And that's it!

4. Dan is playing cards.

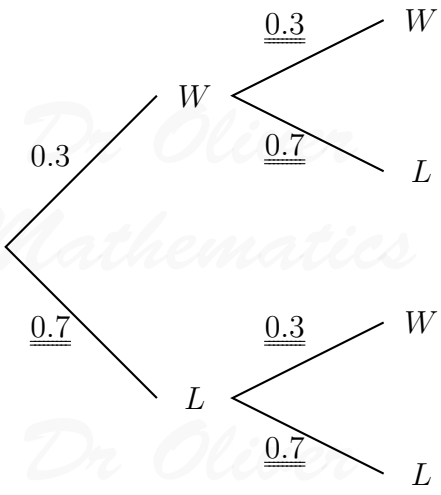
The probability that he will win a game of cards is 0.3.

Dan plays two games of cards.

(a) Complete the probability tree diagram. (2)



Solution



(b) Work out the probability that Dan does **not** win either game. (2)

Solution

Well,

$$\begin{aligned} \mathrm{P}(LL) &= 0.7 \times 0.7 \\ &= \underline{\underline{0.49}}. \end{aligned}$$

5. Robyn buys a total of 240 pens and pencils, where (5)

$$\text{number of pens} : \text{number of pencils} = 3 : 5.$$

- Robyn pays 9 p for each pen.
- She sells each pen for 11 p.
- Robyn pays 6 p for each pencil.
- She sells each pencil for 10 p.

Robyn sells all of the pens and pencils.

Work out Robyn's percentage profit.
Give your answer correct to 1 decimal place.

Solution

Well,

$$3 + 5 = 8$$

and she buys

$$\begin{aligned} \tfrac{3}{8} \times 240 &= 90 \text{ pens and} \\ \tfrac{5}{8} \times 240 &= 150 \text{ pencils.} \end{aligned}$$

She spends

$$\begin{aligned} (90 \times 9) + (150 \times 6) &= 810 + 900 \\ &= 1\,710 \text{ p} \end{aligned}$$

and her income is

$$\begin{aligned} (90 \times 11) + (150 \times 10) &= 990 + 1\,500 \\ &= 2\,490 \text{ p.} \end{aligned}$$

Finally,

$$\begin{aligned} \text{percentage profit} &= \left(\frac{2\,490 - 1\,710}{1\,710} \right) \times 100\% \\ &= 45.614\ldots \\ &= \underline{\underline{45.6\% \text{ (1 sf)}}}. \end{aligned}$$

6. The stem and leaf diagram shows the test scores of 23 students from School **A**. (4)

3	0							
4	1	2	4	4	5	7		
5	3	4	4	6	7	8	8	9
6	0	8	8	9	9			
7	1	3	9					

Key: 3|0 represents 30.

23 students from School **B** did the same test.

- Their median score was 56.
- The range of their scores was 47.

Compare the distribution of the test scores of the students from School **A** with the distribution of the test scores of the students from School **B**.

Solution

Well,

- School **A** median score was 57.
- The range of School **A** is $79 - 30 = 49$.

So,

- School **A** median was higher means that they scored higher marks.
- School **A** range was greater so there was less consistently in School **A**'s results.

7. Jana used her calculator to find the value of a number t. (2)
The answer on her calculator began 10.2.

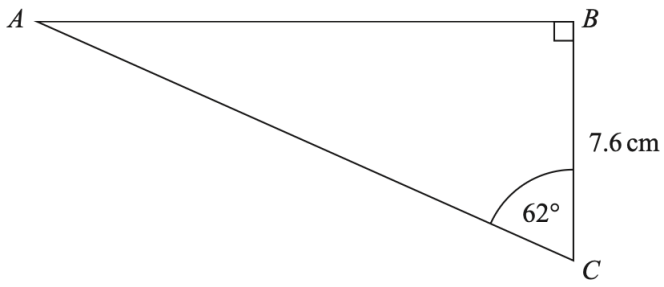
Complete the error interval for t.

$$\ldots\ldots\ldots \leqslant t < \ldots\ldots\ldots$$

Solution

$$\underline{\underline{10.2 \leqslant t < 10.3}}.$$

8. ABC is a right-angled triangle. (2)



Calculate the length of AB.
Give your answer correct to 1 decimal place.

Solution

Well,

$$\begin{aligned} \tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan 62^\circ = \frac{AB}{7.6} \\ &\Rightarrow AB = 7.6 \tan 62^\circ \\ &\Rightarrow AB = 14.293\,521\,14 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{AB = 14.3 \text{ cm (1 dp)}}}. \end{aligned}$$

9. (a) Simplify fully (2)

$$2x^3y^5 \times 7x^2y.$$

Solution

$$2x^3y^5 \times 7x^2y = \underline{\underline{14x^5y^6}}.$$

(b) Simplify (1)

$$(m^2)^{-3}.$$

Solution

$$(m^2)^{-3} = \underline{\underline{m^{-6}}}.$$

10. In a sale, the normal prices are reduced by 15%. (2)
Amina buys a dress in the sale for £46.75.

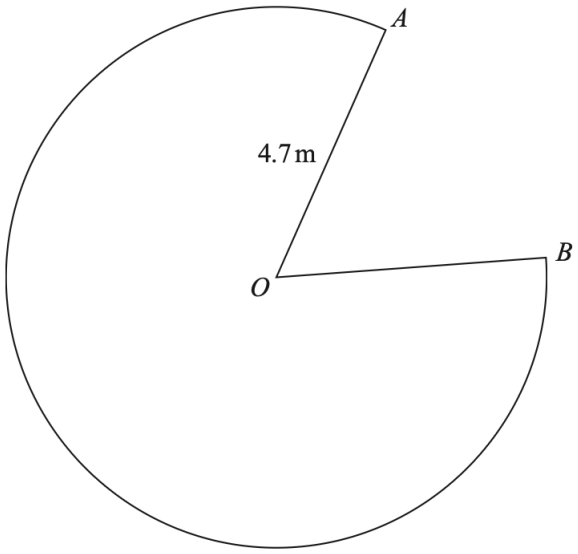
Work out the normal price of the dress.

Solution

Well,

$$\text{sale price} = (1 - 0.15) \times \text{original price} \Rightarrow 46.75 = 0.85 \times \text{original price}$$
$$\Rightarrow \text{original price} = \frac{46.75}{0.85}$$
$$\Rightarrow \text{original price} = \underline{\underline{£55}}.$$

11. OAB is a sector of a circle with centre O and radius 4.7 m. (3)



The sector has a perimeter of 34.3 m.

Find the size of the reflex angle AOB.
Give your answer correct to the nearest degree.

Solution

Well,

$$\frac{\angle AOB}{34.3 - 2 \times 4.7} = \frac{360^\circ}{2 \times \pi \times 4.7} \Rightarrow \angle AOB = \frac{360^\circ \times 24.9}{2 \times \pi \times 4.7}$$
$$\Rightarrow \angle AOB = 303.545\,725\,5 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle AOB = 304^\circ \text{ (nearest degree)}}}.$$

12. Rudi invests £4 500 in a savings account.

He gets compound interest at a rate of

- 2.4% for the first year and
- 1.8% for each extra year.

(a) Work out the value of Rudi's investment at the end of 3 years. (3)

Solution

Well,

$$\text{investment} = 4\,500 \times 1.024 \times 1.018^2$$
$$= 4\,775.380\,992 \text{ (FCD)}$$
$$= \underline{\underline{£4\,775.38 \text{ (nearest penny)}}}.$$

Bruna buys a car for £7 500.

The value of the car depreciates by x% each year.
At the end of 2 years the value of the car is £4 107.

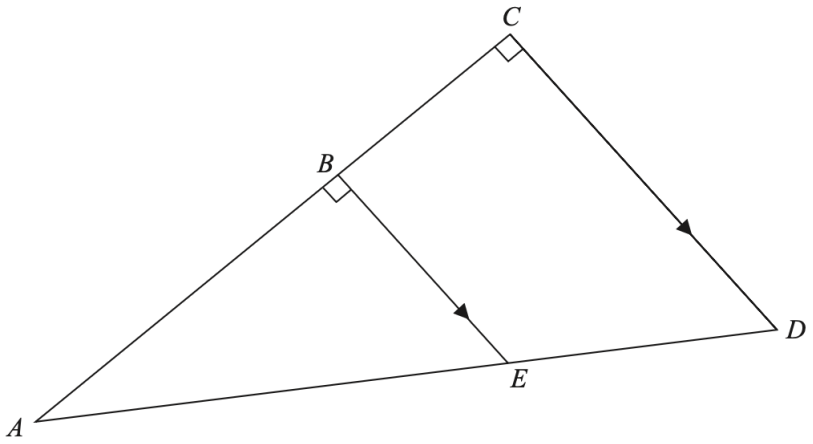
(b) Work out the value of x. (3)

Solution

Now,

$$4\,107 = \left(1 - \frac{x}{100}\right)^2 \times 7\,500 \Rightarrow 0.547\,6 = \left(1 - \frac{x}{100}\right)^2$$
$$\Rightarrow 0.74 = 1 - \frac{x}{100}$$
$$\Rightarrow \frac{x}{100} = 0.26$$
$$\Rightarrow \underline{\underline{x = 26}}.$$

13. ABC and AED are straight lines. (3)
BE and CD are parallel.



- $BE = 4.2$ cm.
- $CD = 6.3$ cm.
- $AC = 10.8$ cm.

Work out the area of trapezium $BCDE$.

Solution

Similar triangles:

$$\frac{AB}{BE} = \frac{AC}{CD} \Rightarrow \frac{AB}{4.2} = \frac{10.8}{6.3}$$
$$\Rightarrow AB = \frac{10.8 \times 4.2}{6.3}$$
$$\Rightarrow AB = 7.2 \text{ cm}.$$

Finally,

$$\begin{aligned}\text{area of } BCDE &= \text{area of } ACD - \text{area of } ABE \\ &= (\tfrac{1}{2} \times 6.3 \times 10.8) - (\tfrac{1}{2} \times 4.2 \times 7.2) \\ &= 34.02 - 15.12 \\ &= \underline{\underline{18.9 \text{ cm}^2}}.\end{aligned}$$

14. Prove algebraically that $0.4\dot{6}\dot{2}$ can be written as $\frac{229}{495}$. (3)

Solution

Let $x = 0.4\dot{6}\dot{2}$. Then

$$10x = 4.\dot{6}\dot{2} \quad (1)$$
$$1\,000x = 462.\dot{6}\dot{2} \quad (2).$$

Do (2) − (1):

$$\begin{aligned}990x = 458 &\Rightarrow x = \tfrac{458}{990} \\ &\Rightarrow x = \tfrac{229 \times 2}{495 \times 2} \\ &\Rightarrow \underline{\underline{x = \tfrac{229}{495}}},\end{aligned}$$

as required.

15. Make p the subject of the formula (4)

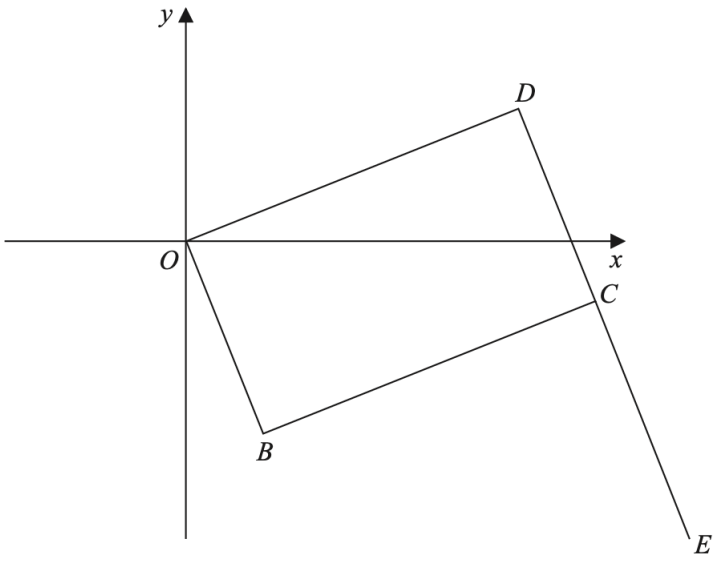
$$t = \frac{2(2p-3)}{5-2p}.$$

Solution

Well,

$$t = \frac{2(2p-3)}{5-2p} \Rightarrow t(5-2p) = 2(2p-3)$$
$$\Rightarrow 5t - 2pt = 4p - 6$$
$$\Rightarrow 5t + 6 = 4p + 2pt$$
$$\Rightarrow 5t + 6 = p(4 + 2t)$$
$$\Rightarrow \underline{\underline{p = \frac{5t+6}{4+2t}}}.$$

16. $OBCD$ is a rectangle. (5)
DCE is a straight line.



- B has coordinates $(2, -4)$.
- E has coordinates $(12, -6.5)$.

Work out the coordinates of D.
You must show all your working.

Solution

Well,

$$m_{OB} = \frac{-4-0}{2-0}$$
$$= -2,$$

and since the line goes through the origin, the equation of the line is

$$y = -2x.$$

Now, the equation of the line going though D, C, and E is

$$y = -2x + c,$$

for some constant c. Next,

$$x = 12, y = -6.5 \Rightarrow -6.5 = -24 + c$$
$$\Rightarrow c = 17.5,$$

and the the equation of the line going though D, C, and E is

$$y = -2x + 17.5 \quad (1).$$

In addition,

$$m_{OD} = \tfrac{1}{2}$$

and the equation of the line going though O and D is

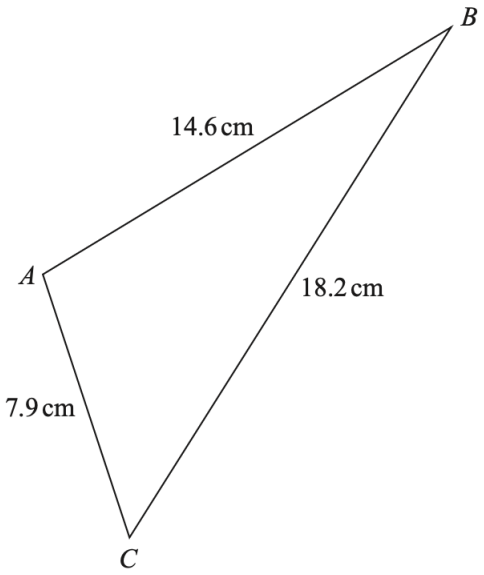
$$y = \tfrac{1}{2}x \quad (2).$$

Simultaneous equations:

$$-2x + 17.5 = \tfrac{1}{2}x \Rightarrow -2.5x = -17.5$$
$$\Rightarrow x = 7$$
$$\Rightarrow y = 3.5;$$

hence, $\underline{\underline{D(7, 3.5)}}$.

17. Here is triangle ABC. (4)



Work out the area of triangle ABC.
Give your answer correct to 3 significant figures.

Solution

Cosine rule:

$$\begin{aligned} & BC^2 = AC^2 + AB^2 - 2 \times AC \times AB \times \cos BAC \\ \Rightarrow \quad & 18.2^2 = 7.9^2 + 14.6^2 - 2 \times 7.9 \times 14.6 \times \cos BAC \\ \Rightarrow \quad & 331.24 = 62.41 + 213.16 - 230.068 \cos BAC \\ \Rightarrow \quad & 230.068 \cos BAC = -55.67 \\ \Rightarrow \quad & \cos BAC = -0.241\,972\,938\,7 \text{ (FCD)} \\ \Rightarrow \quad & \angle BAC = 104.002\,955\,1^\circ \text{ (FCD)}. \end{aligned}$$

Finally,

$$\begin{aligned} \text{area of triangle } ABC &= \tfrac{1}{2} \times AC \times AB \times \sin BAC \\ &= \tfrac{1}{2} \times 7.9 \times 14.6 \times \sin 104.002\ldots \\ &= 55.956\,234\,9 \text{ (FCD)} \\ &= \underline{\underline{56.0 \text{ cm}^2 \text{ (3 sf)}}}. \end{aligned}$$

18. Maria wants to find an estimate for the number of frogs in a lake.

- On Saturday she catches 40 of the frogs.
- She puts a tag on each frog and releases them.
- On Monday she catches 55 of the frogs.
- 11 of the frogs have tags.

(a) Work out an estimate for the total number of frogs in the lake. (3)

Solution

Let x be the number of frogs in a lake. Now,

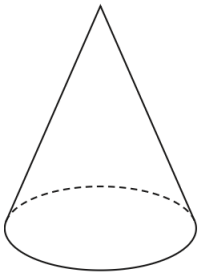
$$\frac{x}{40} = \frac{55}{11} \Rightarrow x = 40 \times 5$$
$$\Rightarrow \underline{\underline{x = 200}}.$$

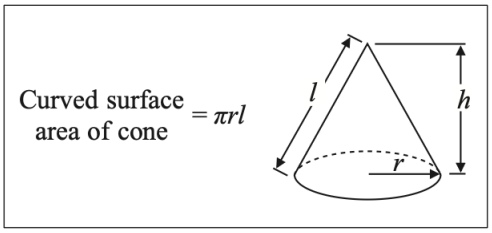
(b) State one assumption you have made. (1)

Solution

E.g., the sample of frogs taken was a random sample.

19. The diagram shows a cone. (4)





- The radius of the base of the cone is $\frac{3}{4}$ of the height of the cone.
- The total surface area of the cone is 54π cm^2.

Work out the height of the cone.

Solution

Pythagoras' theorem:

$$\begin{aligned} l^2 = r^2 + h^2 &\Rightarrow l^2 = r^2 + (\tfrac{3}{4}r)^2 \\ &\Rightarrow l^2 = r^2 + \tfrac{9}{16}r^2 \\ &\Rightarrow l^2 = \tfrac{25}{16}r^2 \\ &\Rightarrow l = \tfrac{5}{4}r. \end{aligned}$$

Now, the **total** surface area of the cone is 54π cm^2 which means

$$\begin{aligned} (\pi \times \tfrac{3}{4}h \times \tfrac{5}{4}h) + [\pi \times (\tfrac{3}{4}h)^2] = 54\pi &\Rightarrow \tfrac{15}{16}h^2 + \tfrac{9}{16}h^2 = 54 \\ &\Rightarrow \tfrac{24}{16}h^2 = 54 \\ &\Rightarrow h^2 = 36 \\ &\Rightarrow \underline{\underline{h = 6 \text{ cm}}}. \end{aligned}$$

20. Solve the simultaneous equations: (4)

$$y^2 = 3x^2 + 4$$
$$y + 2x = 7.$$

Give your solutions correct to 3 significant figures.

Solution

Well,

$$y + 2x = 7 \Rightarrow y = 7 - 2x$$

$\times$	7	$-2x$
7	49	$-14x$
$-2x$	$-14x$	$+4x^2$

$$\Rightarrow y^2 = 49 - 28x + 4x^2.$$

Now,

$$\begin{aligned} y^2 = 3x^2 + 4 &\Rightarrow 49 - 28x + 4x^2 = 3x^2 + 4 \\ &\Rightarrow x^2 - 28x = -45 \end{aligned}$$

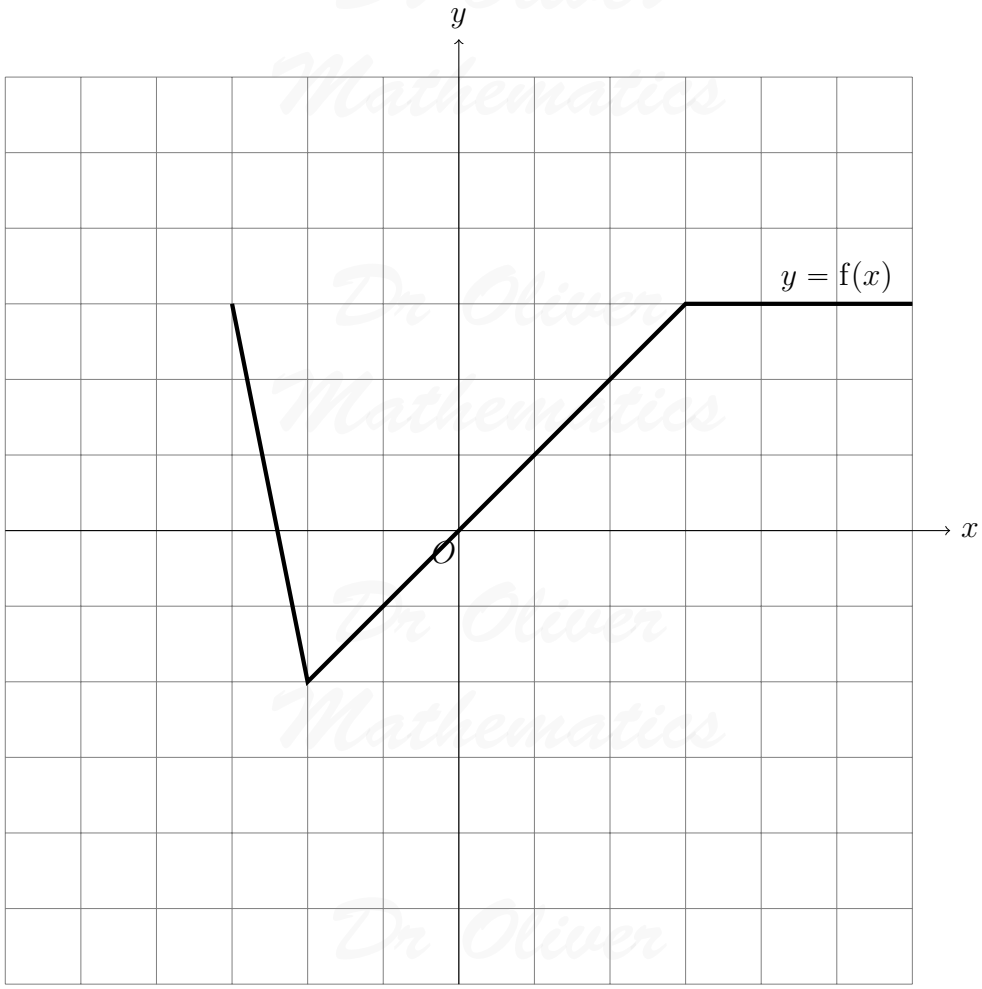
coefficient of x : -28
half it: -14
square it: $(-14)^2 = +196$

$$\Rightarrow x^2 - 28x + 196 = -45 + 196$$
$$\Rightarrow (x-14)^2 = 151$$
$$\Rightarrow x - 14 = \pm\sqrt{151}$$
$$\Rightarrow x = 14 \pm \sqrt{151}$$
$$\Rightarrow y = 7 - 2(14 \pm \sqrt{151})$$
$$\Rightarrow y = 7 - 28 \mp \sqrt{151}$$
$$\Rightarrow y = -21 \mp 2\sqrt{151};$$

hence, the solutions are

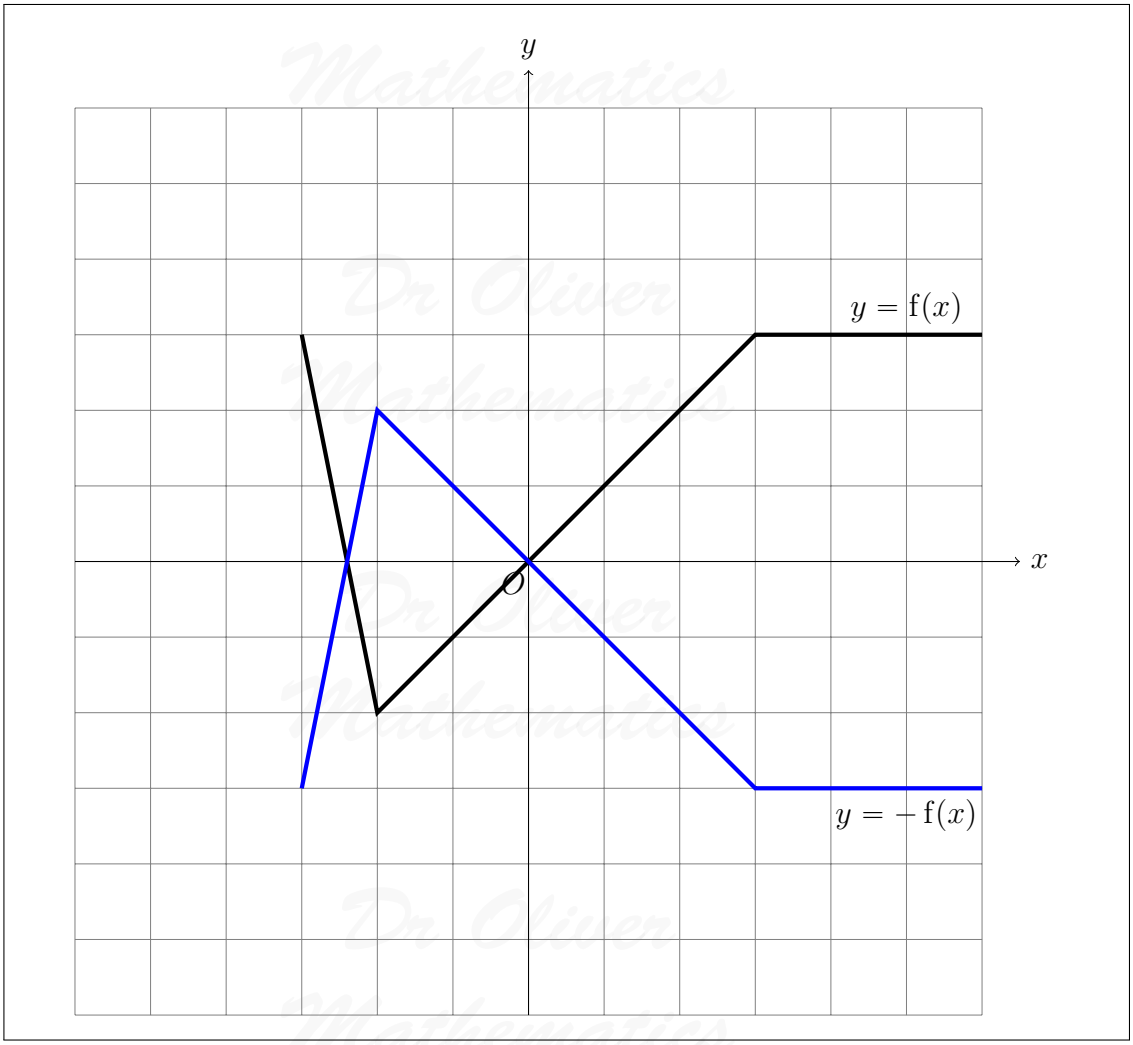
$$\underline{\underline{x = 1.71, y = -45.6 \text{ or } x = 26.3, y = 3.58 \text{ (3 sf)}.}}$$

21. Here is the graph of $y = \mathrm{f}(x)$.

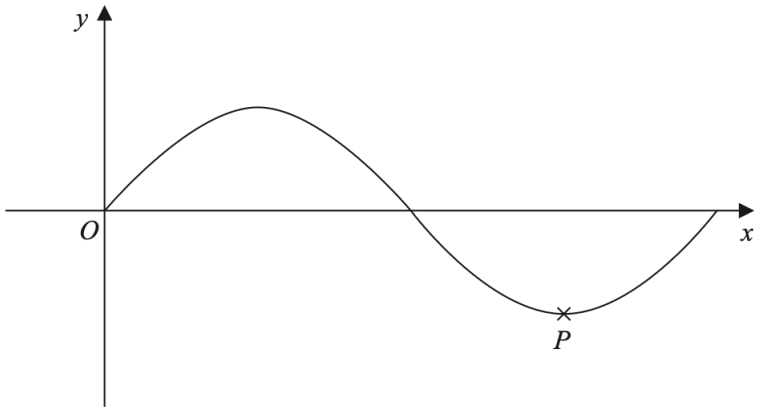


(a) On the grid, draw the graph of $y = -\mathrm{f}(x)$. (1)

Solution



Here is a sketch of the graph of $y = \sin x^\circ$.



The point marked P is a turning point on the graph.

The graph of

$$y = \sin x^\circ$$

is translated to give the graph of

$$y = \sin(x + 180)^\circ + 4.$$

Following the translation the point P, shown on the graph above, moves to point R.

(b) Find the coordinates of R. (3)

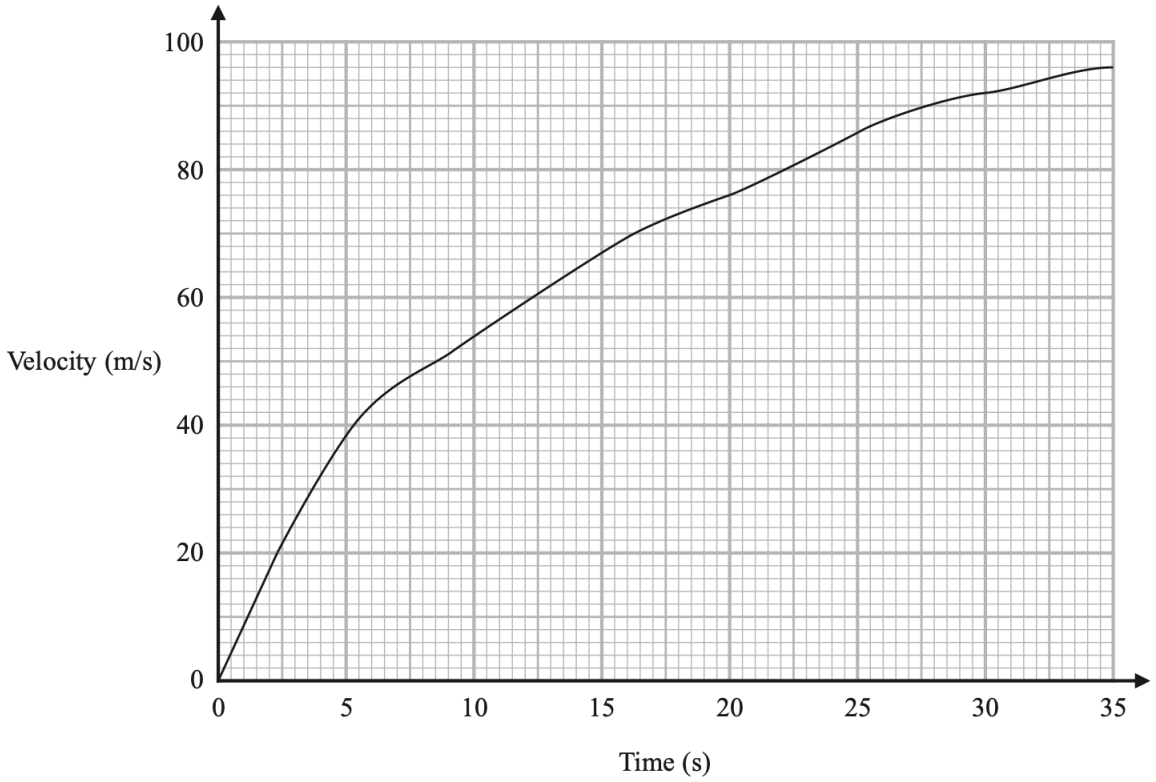
Solution

Well, $P(270, -1)$ and

$$y = -1 + 4 \Rightarrow y = 3;$$

hence, $\underline{\underline{R(90, 3)}}$.

22. Here is a velocity-time graph for an aeroplane. (3)



Work out an estimate for the distance the aeroplane travelled in the first 30 seconds.
Use 3 strips of equal width.

Solution

Well,

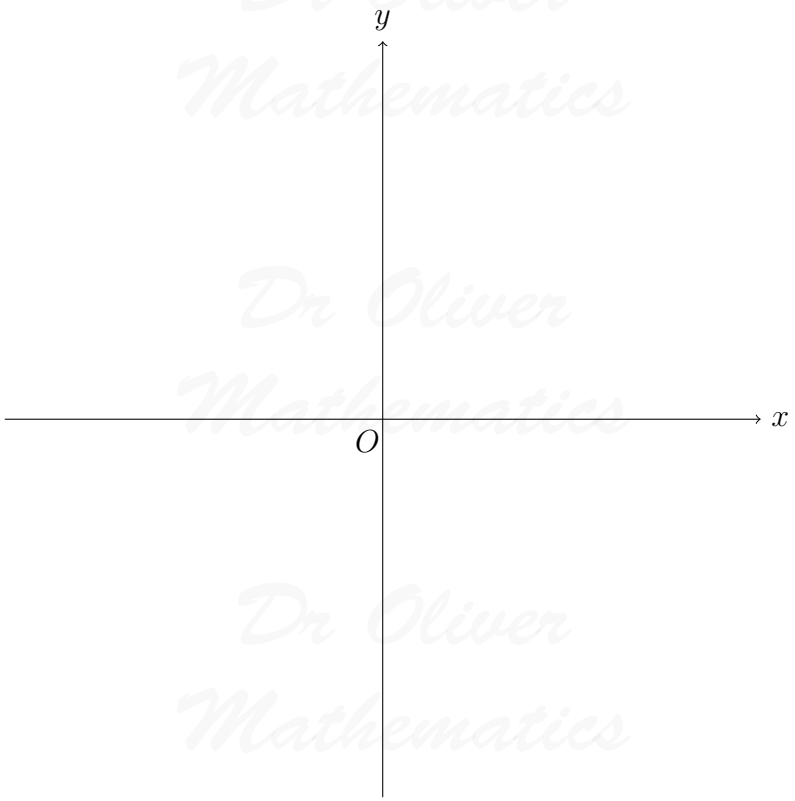
$$\begin{aligned}\text{distance} &= \text{0-10} + \text{10-20} + \text{20-30} \\ &= (\tfrac{1}{2} \times 10 \times 54) + [\tfrac{1}{2} \times 10 \times (54 + 76)] + [\tfrac{1}{2} \times 10 \times (76 + 92)] \\ &= 270 + 650 + 840 \\ &= \underline{\underline{1\,760 \text{ m}}}.\end{aligned}$$

23. Sketch the graph of (5)

$$y = x^2 - 6px - 7, \text{ where } p > 0,$$

showing the coordinates of the turning point, in terms of p, and the coordinates of the intercept with the y-axis.

You must show all your working.



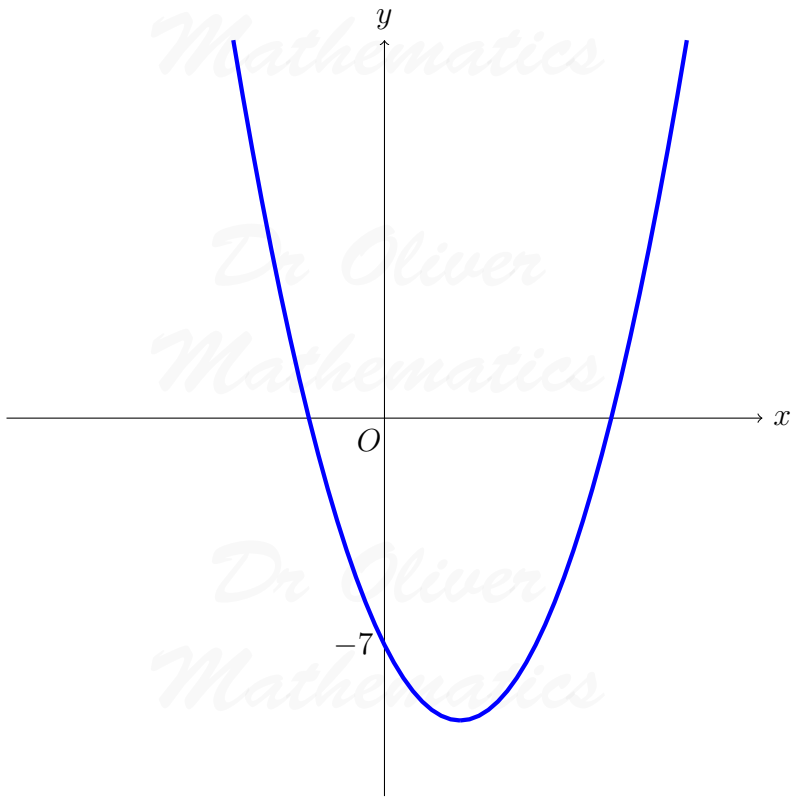
Solution

Well,

coefficient of x :	$-6p$
half it:	$-3p$
square it:	$(-3p)^2 = +9p^2$

and

$$\begin{aligned} y &= [x^2 - 6px] - 7 \\ &= [(x^2 - 6px + 9p^2) - 9p^2] - 7 \\ &= (x - 3p)^2 - 9p^2 - 7. \end{aligned}$$



The turning point is $\underline{\underline{(3p, -9p^2 - 7)}}$.