Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2011 November Paper 2 Variant 3: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. Solve the inequality (3)
$$x(2x-1) > 15.$$

2. (a) Given that (2)
$$y = (12-4x)^5,$$
find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

   (b) Hence find the approximate change in y as x increases from 0.5 to $(0.5+p)$, where p is small. (2)

3. (a) Find the coefficient of x^3 in the expansion of (2)
$$(1-2x)^7.$$

   (b) Find the coefficient of x^3 in the expansion of (3)
$$(1+3x^2)(1-2x)^7.$$

4. Without using a calculator, find the positive root of the equation (6)
$$(5-2\sqrt{2})x^2 - (4+2\sqrt{2})x - 2 = 0,$$
giving your answer in the form $a+b\sqrt{2}$, where a and b are integers.

5. A school council of 6 people is to be chosen from a group of 8 students and 6 teachers.

   Calculate the number of different ways that the council can be selected if

   (a) there are no restrictions, (2)

   (b) there must be at least 1 teacher on the council and more students than teachers. (3)

   After the council is chosen, a chairperson and a secretary have to be selected from the 6 council members.

(c) Calculate the number of different ways in which a chairperson and a secretary can be selected. (1)

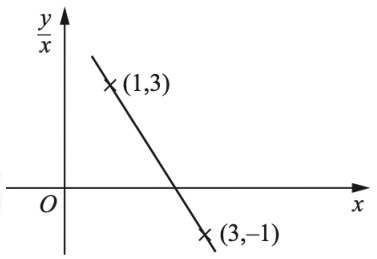
6. (a) Sketch the graph of (4)

$$y = |(2x + 3)(2x - 7)|.$$

(b) How many values of x satisfy the equation (2)

$$|(2x + 3)(2x - 7)| = 2x?$$

7. The variables x and y are related in such a way that when $\frac{y}{x}$ is plotted against x a straight line is obtained, as shown in the graph.



The line passes through the points $(1, 3)$ and $(3, -1)$.

(a) Express y in terms of x. (4)

(b) Find the value of x and of y such that (2)

$$\frac{y}{x} = -9.$$

8. A sector of a circle, of radius r cm, has a perimeter of 200 cm.

(a) Express the area, A cm^2, of the sector in terms of r. (3)

(b) Given that r can vary, find the stationary value of A. (3)

9. An aircraft, whose speed in still air is 350 kmh^{-1}, flies in a straight line from A to B, a distance of 480 km.

- There is a wind of 50 kmh^{-1} blowing from the north.
- The pilot sets a course of 130°.

(a) Calculate the time taken to fly from A to B. (5)

(b) Calculate the bearing of B from A. (3)

10. The line (9)

$$y = 2x + 10$$

intersects the curve

$$2x^2 + 3xy - 5y + y^2 = 218$$

at the points A and B.

Find the equation of the perpendicular bisector of AB.

11. (a) Solve (3)

$$4\cot\tfrac{1}{2}x = 1, \text{ for } 0^\circ < x < 360^\circ.$$

(b) Solve (5)

$$3(1 - \tan y \cos y) = 5\cos^2 y - 2, \text{ for } 0^\circ < y < 360^\circ.$$

(c) Solve (3)

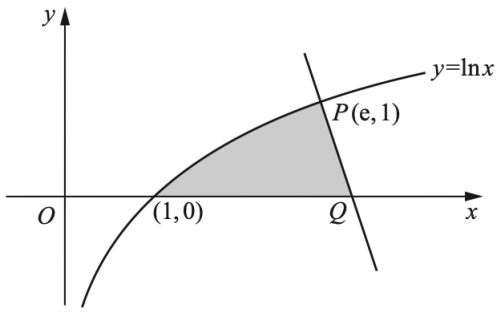
$$3\sec^2 z = 4, \text{ for } 0 < z < 2\pi \text{ radians.}$$

EITHER

12. The diagram shows part of the curve

$$y = \ln x$$

cutting the x-axis at the point $(1, 0)$.



The normal to the curve at the point $P(\mathrm{e}, 1)$ cuts the x-axis at the point Q.

(a) Show that Q is the point $\left(\mathrm{e} + \frac{1}{\mathrm{e}}, 0\right)$. (4)

(b) Show that (1)

$$\frac{\mathrm{d}}{\mathrm{d}x}(x \ln x) = 1 + \ln x.$$

(c) Hence find (5)

$$\int \ln x \, \mathrm{d}x$$

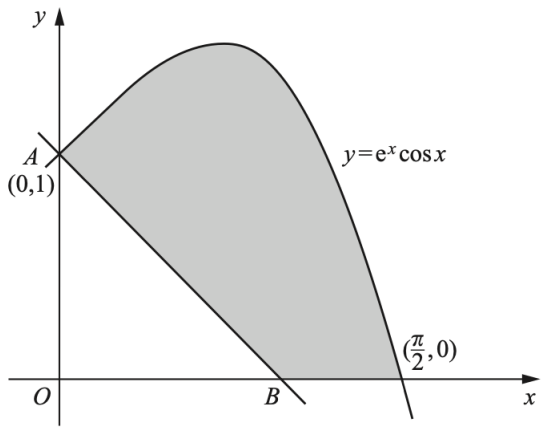
and the area of the shaded region.

OR

13. The diagram shows part of the curve

$$y = \mathrm{e}^x \cos x,$$

cutting the x-axis at the point $(\frac{1}{2}\pi, 0)$.



The normal to the curve at the point $A(0, 1)$ cuts the x-axis at the point B.

(a) Find the coordinates of B. (4)

(b) Show that (2)

$$\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{e}^x(\cos x + \sin x)) = 2\mathrm{e}^x \cos x.$$

(c) Hence find (4)

$$\int \mathrm{e}^x \cos x \, \mathrm{d}x$$

and the area of the shaded region.