Dr Oliver Mathematics
Mathematics: National Qualifications N5
2023 Paper 1: Non-Calculator
1 hour

The total number of marks available is 40.
You must write down all the stages in your working.

1. Evaluate (2)

$$2\tfrac{1}{6} \div \tfrac{8}{9}.$$

Give your answer in its simplest form.

Solution

$$\begin{aligned} 2\tfrac{1}{6} \div \tfrac{8}{9} &= \tfrac{13}{6} \div \tfrac{8}{9} \\ &= \tfrac{13}{6} \times \tfrac{9}{8} \\ &= \tfrac{13}{\not{6}^{2}} \times \tfrac{\not{9}^{3}}{8} \\ &= \tfrac{39}{16} \\ &= \underline{\underline{2\tfrac{7}{16}}}. \end{aligned}$$

2. Expand and simplify (3)

$$(x + 7)^2 + 6(x^2 - 10).$$

Solution

$\times$	x	$+7$
x	x^2	$+7x$
$+7$	$+7x$	$+49$

Now,

$$\begin{aligned} (x + 7)^2 + 6(x^2 - 10) &= (x^2 + 14x + 49) + 6x^2 - 60 \\ &= \underline{\underline{7x^2 + 14x - 11}}. \end{aligned}$$

3. Solve, algebraically, the system of equations (3)

$$\begin{aligned} 2x + 3y &= 8 \\ 5x + 2y &= -2. \end{aligned}$$

Solution

$$\begin{aligned} 2x + 3y &= 8 \quad (1) \\ 5x + 2y &= -2 \quad (2) \end{aligned}$$

E.g., do $2 \times (1)$ and $3 \times (2)$:

$$\begin{aligned} 4x + 6y &= 16 \quad (3) \\ 15x + 6y &= -6 \quad (4) \end{aligned}$$

and do $(4) - (3)$:

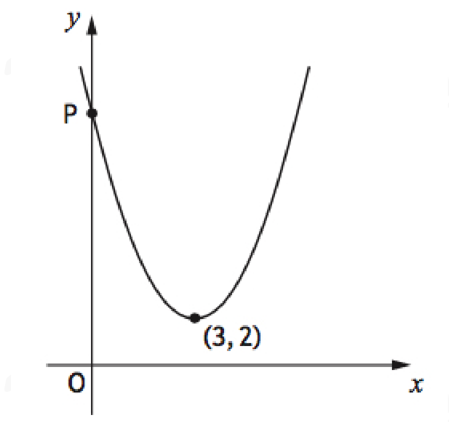
$$11x = -22 \Rightarrow \underline{\underline{x = -2}}$$

substitute into (1):

$$\begin{aligned} &\Rightarrow 2(-2) + 3y = 8 \\ &\Rightarrow -4 + 3y = 8 \\ &\Rightarrow 3y = 12 \\ &\Rightarrow \underline{\underline{y = 4}}. \end{aligned}$$

4. The graph below shows part of a parabola of the form

$$y = (x + a)^2 + b.$$



(a) (i) State the value of a. (1)

Solution
$\underline{\underline{a = -3}}$.

(ii) State the value of b. (1)

Solution
$\underline{\underline{b = 2}}$.

P is the point $(0, c)$.

(b) Find the value of c. (1)

Solution
So, the curve is $y = (x-3)^2 + 2$ and

$$\begin{aligned} x = 0 \Rightarrow c &= (0-3)^2 + 2 \\ \Rightarrow c &= 9 + 2 \\ \Rightarrow \underline{\underline{c}} &\underline{\underline{= 11}}. \end{aligned}$$

5. Determine the nature of the roots of the function (2)

$$\mathrm{f}(x) = 4x^2 + 6x - 1.$$

Solution

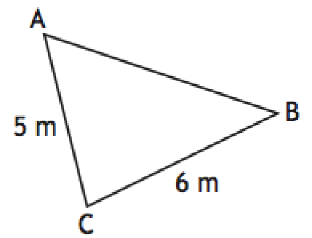
Well, $a = 4$, $b = 6$, and $c = -1$:

$$\begin{aligned} \text{discriminant} &= b^2 - 4ac \\ &= 6^2 - 4(4)(-1) \\ &= 36 + 16 \\ &= 52 \end{aligned}$$

so the roots are $\underline{\underline{\text{real}}}$ and $\underline{\underline{\text{distinct}}}$.

6. In triangle ABC: (3)

- $AC = 5$ metres,
- $BC = 6$ metres, and
- $\cos C = \frac{1}{5}$.



Calculate the length of AB.

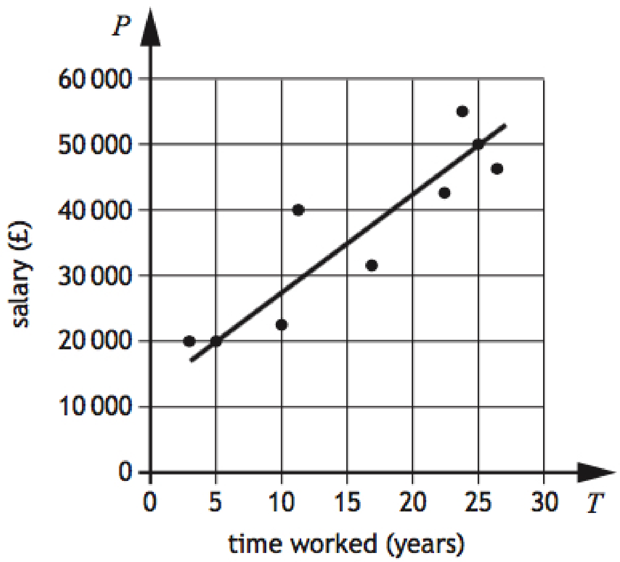
Solution

Cosine rule:

$$\begin{aligned} & AB^2 = AC^2 + BC^2 - 2 \times AC \times BC \times \cos ACB \\ \Rightarrow\ & AB^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \tfrac{1}{5} \\ \Rightarrow\ & AB^2 = 25 + 36 - 12 \\ \Rightarrow\ & AB^2 = 49 \\ \Rightarrow\ & \underline{\underline{AB = 7 \text{ cm}}}. \end{aligned}$$

7. A business recorded the salaries of a sample of its employees and the length of time they have worked for the business.

The scattergraph shows the relationship between their salary, P pounds, and the length of time, T years, they have worked.



A line of the best fit has been drawn.

(a) Find the equation of the line of best fit in terms of P and T. Give the equation in its simplest form. (3)

Solution

Well, the line of the best fit goes through $(5, 20\,000)$ and $(25, 50\,000)$:

$$\begin{aligned}\text{gradient} &= \frac{50\,000 - 20\,000}{25 - 5} \\ &= \frac{30\,000}{20} \\ &= 1\,500\end{aligned}$$

and the equation is

$$P = 1\,500T + c,$$

for some value of c. Now,

$$\begin{aligned}T = 5, P = 20\,000 &\Rightarrow 20\,000 = 1\,500(5) + c \\ &\Rightarrow 20\,000 = 7\,500 + c \\ &\Rightarrow c = 12\,500;\end{aligned}$$

so,

$$\underline{\underline{P = 1\,500T + 12\,500}}.$$

(b) Use your equation from part (a) to estimate the salary of an employee who has worked for the business for 8 years. (1)

Solution

$$\begin{aligned} T = 8 \Rightarrow P &= 1\,500(8) + 12\,500 \\ \Rightarrow P &= 12\,000 + 12\,500 \\ \Rightarrow \underline{\underline{P}} &\underline{\underline{= £24\,500}}. \end{aligned}$$

8. Express (2)

$$\frac{12}{\sqrt{15}}$$

with a rational denominator.

Give your answer in its simplest form.

Solution

$$\begin{aligned} \frac{12}{\sqrt{15}} &= \frac{12}{\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}} \\ &= \frac{12\sqrt{15}}{15} \\ &= \frac{3 \times 4\sqrt{15}}{3 \times 5} \\ &= \underline{\underline{\frac{4\sqrt{15}}{5}}}. \end{aligned}$$

9. A magazine company conducted a survey of the ages of its readers.

A sample of ten readers' ages, in years, are shown below.

33 55 38 47 36 41 42 41 35 31

(a) Calculate the median and interquartile range of the ages of readers for this sample. (3)

Solution

Sort it by increasing age:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
31	33	35	36	38	41	41	42	47	55

Now, the median is at $5\frac{1}{2}$ place:

$$\frac{38+41}{2} = \frac{79}{2} = \underline{\underline{39\tfrac{1}{2}\text{ years.}}}$$

Next, the LQ is at the

$$\frac{10+1}{4} = 2\tfrac{3}{4} \approx 3\text{th value}$$

and the UQ is at the

$$\frac{3(10+1)}{4} = 8\tfrac{1}{4} \approx 8\text{th value;}$$

so,

$$\begin{aligned} \text{IQR} &= \text{UQ} - \text{LQ} \\ &= 42 - 35 \\ &= \underline{\underline{7\text{ years}}}. \end{aligned}$$

A newspaper company also conducted a survey of the ages of its readers.

The median age of a sample of its readers was 41 years and the interquartile range was 9 years.

(b) Make two valid comments comparing the ages of the readers of the magazine and the ages of the readers of the newspaper. (2)

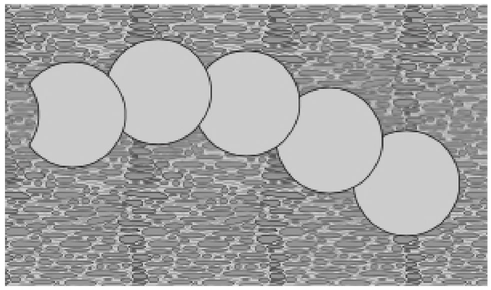
Solution

Average: Since the median for the newspaper (41) was higher than the median for the magazine ($39\frac{1}{2}$), the newspaper has an older readership.

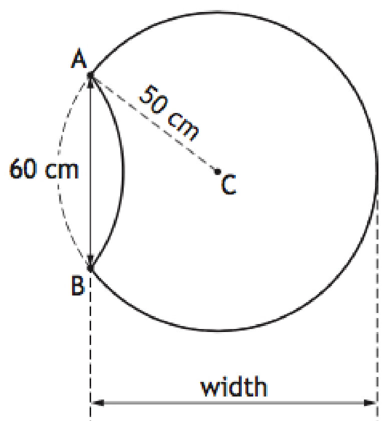
IQR: Since the IQR for the newspaper (9) was higher than the than the IQR for the magazine (7), the newspaper were more varied in years.

10. Alan buys some identical paving slabs to make a path. (4)

Each slab is part of a circle.



The diagram below shows a single slab.



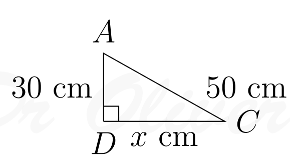
The circle, centre C, has a radius of 50 centimetres.

Length AB is 60 centimetres.

Calculate the width of the paving slab.

Solution

Let D be the midpoint of AB and x cm be the 'height' of CD:



Pythagoras' theorem:

$$\begin{aligned} AC^2 = AD^2 + CD^2 &\Rightarrow 50^2 = 30^2 + CD^2 \\ &\Rightarrow 2\,500 = 900 + CD^2 \\ &\Rightarrow CD^2 = 1\,600 \\ &\Rightarrow CD = 40 \text{ cm}. \end{aligned}$$

Finally,

$$\begin{aligned} \text{width} &= CD + \text{radius} \\ &= 40 + 50 \\ &= \underline{\underline{90 \text{ cm}}}. \end{aligned}$$

11. Given that (1)

$$\sin 30^\circ = 0.5,$$

state the value of $\sin 330^\circ$.

Solution

Well,

$$\begin{aligned} \sin 330^\circ &= -\sin(360 - 330)^\circ \\ &= -\sin 30^\circ \\ &= \underline{\underline{-0.5}}. \end{aligned}$$

12. Simplify (3)

$$\frac{5c^{-2}}{c^3 \times c^4}.$$

Give your answer with a **positive power**.

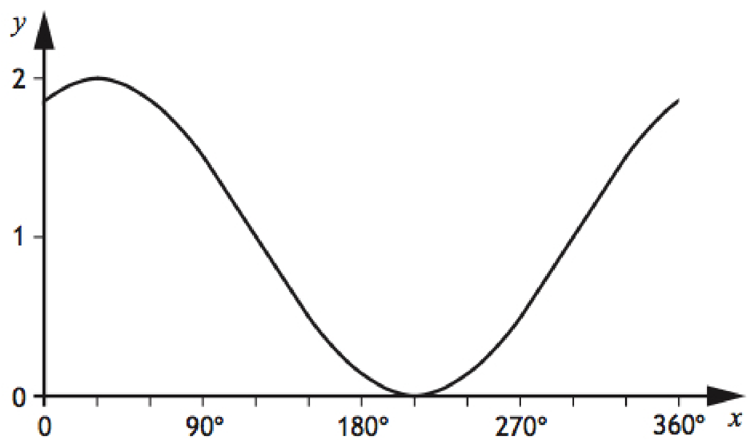
Solution

$$\frac{5c^{-2}}{c^3 \times c^4} = \frac{5c^{-2}}{c^7} = \frac{5}{c^2 \times c^7} = \underline{\underline{\frac{5}{c^9}}}.$$

13. Part of the graph of

$$y = \cos(x + a)^\circ + b$$

is shown.



(a) State the value of a. (1)

Solution

$\underline{\underline{a = -30}}$.

(b) State the value of b. (1)

Solution

$\underline{\underline{b = 1}}$.

14. Solve, algebraically, the inequation (3)

$$\frac{x+1}{3} - 2 > \frac{3x}{5}.$$

Solution

Multiply by 15:

$$\begin{aligned}\frac{x+1}{3} - 2 > \frac{3x}{5} &\Rightarrow 5(x+1) - 30 > 3(3x)\\ &\Rightarrow 5x + 5 - 30 > 9x\\ &\Rightarrow -4x > 25\\ &\Rightarrow \underline{\underline{x < 6\tfrac{1}{4}}}.\end{aligned}$$