Dr Oliver Mathematics
Mathematics
Vectors
Past Examination Questions

This booklet consists of 27 questions across a variety of examination topics.
The total number of marks available is 315.

1. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

and line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix},$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point B and the acute angle between l_1 and l_2 is θ.

(a) Find the coordinates of B. (4)

(b) Find the value of $\cos\theta$, giving your answer as a simplified fraction. (4)

- The point A, which lies on l_1, has position vector $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
- The point C, which lies on l_2, has position vector $\mathbf{c} = 5\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.
- The point D is such that $ABCD$ is a parallelogram.

(c) Show that (3)

$$|\overrightarrow{AB}| = |\overrightarrow{BC}|.$$

(d) Find the position vector of the point D. (2)

2. The line l_1 has vector equation

$$\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ is a parameter.

- The point A has coordinates $(4, 8, a)$, where a is a constant.
- The point B has coordinates $(b, 13, 13)$, where b is a constant.

- Points A and B lie on the line l_1.

(a) Find the values of a and b. (3)

Given that the point O is the origin, and that the point P lies on l_1 such that OP is perpendicular to l_1,

(b) find the coordinates of P. (5)

(c) Hence find the distance OP, giving your answer as a simplified surd. (2)

3. The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has equation

$$\mathbf{r} = 6\mathbf{i} + 19\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}).$$

(a) Find the values of a and b. (3)

The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin.

(b) Find the position vector of point P. (6)

Given that B has coordinates $(5, 15, 1)$,

(c) show that the points A, P, and B are collinear and find the ratio $AP : PB$. (4)

4. The point A has position vector

$$\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$

and the point B has position vector

$$\mathbf{b} = \mathbf{i} + \mathbf{j} - 4\mathbf{k},$$

relative to an origin O.

(a) Find the position vector of the point C, with position vector $\mathbf{c}$, given by (1)

$$\mathbf{c} = \mathbf{a} + \mathbf{b}.$$

(b) Show that $OACB$ is a rectangle, and find its exact area. (6)

The diagonals of the rectangle, AB and OC, meet at the point D.

(c) Write down the position vector of the point D. (1)

(d) Find the size of the angle ADC. (6)

5. The line l_1 has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

and the line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}.$$

(a) Show that l_1 and l_2 do not meet. (4)

The point A is on l_1 where $\lambda = 1$ and the point B is on l_2 where $\mu = 2$.

(b) Find the cosine of the acute angle between AB and l_1. (6)

6. The points A and B have position vectors

$$2\mathbf{i} + 6\mathbf{j} - \mathbf{k} \text{ and } 3\mathbf{i} + 4\mathbf{j} + \mathbf{k},$$

respectively.

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$. (2)

(b) Find a vector equation for the line l_1. (2)

A second line l_2 passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$.

The line l_1 meets the line l_2 at the point C.

(c) Find the acute angle between and l_1 and l_2. (3)

(d) Find the position vector of the point C. (4)

7. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations:

$$l_1: \quad \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k}),$$
$$l_2: \quad \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k}),$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection. (6)

(b) Show that l_1 and l_2 are perpendicular to each other. (2)

The point A has position vector

$$5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}.$$

(c) Show that A lies on l_1. (1)

The point B is the image of A after reflection in the line l_2.

(d) Find the position vector of B. (3)

8. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations:

$$l_1: \quad \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix},$$
$$l_2: \quad \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix},$$

where λ and μ are parameters and p and q are constants.

Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$. (2)

Given further that l_1 and l_2 intersect, find

(b) the value of p, (6)

(c) the coordinates of the point of intersection. (2)

The point A lies on l_1 and has position vector

$$\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}.$$

The point C lies on l_2.

Given that a circle, with centre C, cuts the line l_1 at the points A and B,

(d) find the position vector of B. (3)

9. Relative to a fixed origin O, the point A has position vector

$$8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k},$$

the point B has position vector

$$10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k},$$

and the point C has position vector

$$9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k}.$$

The line l passes through the points A and B.

(a) Find a vector equation for the line l. (3)

(b) Find $|\overrightarrow{CB}|$. (2)

(c) Find the size of the acute angle between the line segment CB and the line l, giving your answer in degrees to 1 decimal place. (3)

(d) Find the shortest distance from the point C to the line l. (3)

The point X lies on l.

Given that the vector $\overrightarrow{CX}$ is perpendicular to l,

(e) find the area of the triangle CXB, giving your answer to 3 significant figures. (3)

10. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix},$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ.

(a) Write down the coordinates of A. (1)

(b) Find the value of $\cos\theta$. (3)

The point X lies on l_1 where $\lambda = 4$.

(c) Find the coordinates of X. (1)

(d) Find the vector $\overrightarrow{AX}$. (2)

(e) Hence, or otherwise, show that (2)

$$|\overrightarrow{AX}| = 4\sqrt{26}.$$

The point Y lies on l_2.

Given that the vector YX is perpendicular to l_1,

(f) find the length of AY, giving your answer to 3 significant figures. (3)

11. The line l_1 has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix},$$

where λ is a scalar parameter.

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix},$$

where mu is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(a) the coordinates of C. (3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(b) Find the size of the angle ACB.
Give your answer in degrees to 2 decimal places. (4)

(c) Hence, or otherwise, find the area of the triangle ABC. (5)

12. Relative to a fixed origin O, the point A has position vector

$$\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$$

and the point B has position vector

$$-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}.$$

The points A and B lie on a straight line l.

(a) Find $\overrightarrow{AB}$. (2)

(b) Find a vector equation of l. (2)

The point C has position vector

$$2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$$

with respect to O, where p is a constant.

Given that AC is perpendicular to l, find

(c) the value of p, (4)

(d) the distance AC. (2)

13. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations:

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix},$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A. (6)

(b) Find, to the nearest $0.1°$, the acute angle between l_1 and l_2. (3)

The point B has position vector

$$\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}.$$

(c) Show that B lies on l_1. (1)

(d) Find the shortest distance from B to the line l_2, giving your answer to 3 significant figures. (4)

14. Relative to a fixed origin O, the point A has position vector

$$(2\mathbf{i} - \mathbf{j} + 5\mathbf{k}),$$

the point B has position vector

$$(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}),$$

and the point D has position vector

$$(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}).$$

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$. (2)

(b) Find a vector equation for the line l. (2)

(c) Show that the size of the angle BAD is $109°$, to the nearest degree. (4)

The points A, B, and D, together with a point C, are the vertices of the parallelogram $ABCD$, where

$$\overrightarrow{AB} = \overrightarrow{DC}.$$

(d) Find the position vector of C. (2)

(e) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures. (3)

(f) Find the shortest distance from the point D to the line l, giving your answer to 3 significant figures. (2)

15. Relative to a fixed origin O, the point A has position vector

$$(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$$

and the point B has position vector

$$(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}).$$

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$. (2)

(b) Find a vector equation for the line l. (2)

The point C has position vector

$$(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}).$$

The point P lies on l.

Given that the vector $\overrightarrow{CP}$ is perpendicular to l,

(c) find the position vector of the point P. (6)

16. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = (9\mathbf{i} + 13\mathbf{j} - 3\mathbf{k}) + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$
$$l_2: \quad \mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} + \mathbf{k}),$$

where λ and μ are scalar parameters.

(a) Given that l_1 and l_2 meet, find the position vector of their point of intersection. (5)

(b) Find the acute angle between l_1 and l_2, giving your answer in degrees to 1 decimal place. (3)

Given that the point A has position vector

$$(4\mathbf{i} + 16\mathbf{j} - 3\mathbf{k})$$

and that the point P lies on l_1 such that AP is perpendicular to l_1,

(c) find the exact coordinates of P. (6)

17. With respect to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix},$$

where λ is a scalar parameter.

The point A lies on l and has coordinates $(3, -2, 6)$.

The point P has position vector

$$(-p\mathbf{i} + 2p\mathbf{k}),$$

relative to O, where p is a constant.

Given that vector $\overrightarrow{PA}$ is perpendicular to l,

(a) find the value of p. (4)

Given also that B is a point on l such that $\angle BPA = 45^\circ$,

(b) find the coordinates of the two possible positions of B. (5)

18. Relative to a fixed origin O, the point A has position vector

$$21\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$$

and the point B has position vector

$$25\mathbf{i} - 14\mathbf{j} + 18\mathbf{k}.$$

The line l has equation

$$\mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix},$$

where a, b, and c are constants and λ is a scalar parameter.

Given that the point A lies on the line l,

(a) find the value of a. (3)

Given also that the vector $\overrightarrow{AB}$ is perpendicular to l,

(b) find the value of b and c, (5)

(c) find the distance AB. (2)

The image of the point B after reflection in the line l is the point B'.

(d) Find the position vector of the point B'. (2)

19. Relative to a fixed origin O, the point A has position vector

$$\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$$

and the point B has position vector

$$\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}.$$

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$. (2)

(b) Hence find a vector equation for the line l_1. (1)

The point P has position vector

$$\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}.$$

Given that angle PBA is θ,

(c) show that $\cos\theta = \frac{1}{3}$. (3)

The line l_2 passes through the point P and is parallel to the line l_1.

(d) Find a vector equation for the line l_2. (2)

The points C and D both lie on the line l_2.

Given that

$$AB = PC = DP$$

and the x-coordinate of C is positive,

(e) find the coordinates of C and the coordinates of D. (3)

(f) find the exact area of the trapezium $ABCD$, giving your answer as a simplified surd. (4)

20. With respect to a fixed origin, the point A with position vector

$$\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

lies on the line l_1 with equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix},$$

for some scalar λ, and the point B with position vector

$$4\mathbf{i} + p\mathbf{j} + 3\mathbf{k},$$

where p is a constant, lies on the line l_2 with equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix},$$

for some scalar μ.

(a) Find the value of the constant p. (1)

(b) Show that l_1 and l_2 intersect and find the position vector of their point of intersection, C. (4)

(c) Find the size of the angle ACB, giving your answer in degrees to 3 significant figures. (3)

(d) Find the area of the triangle ABC, giving your answer to 3 significant figures. (2)

21. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations:

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -13 \end{pmatrix},$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix},$$

where λ and μ are scalar parameters and p is a constant.

The lines l_1 and l_2 intersect at the point A.

(a) Find the coordinates of A. (2)

(b) Find the value of the constant p. (3)

(c) Find the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places. (3)

The point B lies on l_2 where $\mu = 1$.

(d) Find the shortest distance from the point B to the line l_1, giving your answer to 3 significant figures. (3)

22. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix},$$

where μ is scalar parameter.

The point A lies on l_1 where $\mu = 1$.

(a) Find the coordinates of A. (1)

The point P has position vector

$$\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}.$$

The line l_2 passes through the point P and is parallel to the line l_1.

(b) Write down a vector equation for the line l_2. (2)

(c) Find the exact value of the distance AP. (2)
Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

The acute angle between AP and l_2 is θ.

(d) Find the value of $\cos\theta$. (3)

A point E lies on the line l_2.

Given that $AP = PE$,

(e) find the area of triangle APE, (3)

(f) find the coordinates of the two possible positions of E. (5)

23. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \quad \mathbf{r} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix},$$

$$l_2 : \quad \mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix},$$

where λ and μ are scalar parameters.

The lines l_1 and l_2 intersect at the point X.

(a) Find the coordinates of the point X. (3)

(b) Find the size of the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places. (3)

The point A lies on l_1 and has position vector

$$\begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix}.$$

(c) Find the distance AX, giving your answer as a surd in its simplest form. (2)

The point Y lies on l_2.

Given that the vector $\overrightarrow{YA}$ is perpendicular to the line l_1,

(d) find the distance YA, giving your answer to one decimal place. (2)

The point B lies on l_1 where

$$|\overrightarrow{AX}| = 2|\overrightarrow{AB}|.$$

(e) Find the two possible position vectors of B. (3)

24. The point A with coordinates $(-3, 7, 2)$ lies on a line l_1.

The point B also lies on the line l_1.

Given that

$$\overrightarrow{AB} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix},$$

(a) find the coordinates of point B. (2)

The point P has coordinates $(9, 1, 8)$.

(b) Find the cosine of the angle PAB, giving your answer as a simplified surd. (3)

(c) Find the exact area of triangle PAB, giving your answer in its simplest form. (3)

The line l_2 passes through the point P and is parallel to the line l_1.

(d) Find a vector equation for the line l_2. (2)

The point Q lies on the line l_2.

Given that the line segment AP is perpendicular to the line segment BQ,

(e) find the coordinates of the point Q. (5)

25. Figure 1 shows a sketch of triangle OAB.

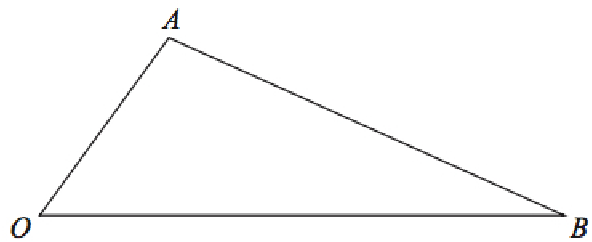

Figure 1: triangle OAB

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.
The point M is the midpoint of AB.
The straight line through C and M cuts OB at the point N.
Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Find $\overrightarrow{CM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (2)

(b) Show that (2)

$$\overrightarrow{ON} = (2 - \tfrac{3}{2}\lambda)\mathbf{a} + \tfrac{1}{2}\lambda\mathbf{b},$$

where λ is a scalar constant.

(c) Hence prove that $ON : NB = 2 : 1$. (2)

26. Figure 2 shows a sketch of triangle ABC.

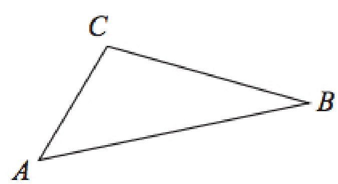

Figure 2: a sketch of triangle ABC

Given that

- $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$ and
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$,

(a) find $\overrightarrow{AC}$, (2)

(b) show that (3)

$$\cos ABC = \tfrac{9}{10}.$$

27. Figure 3 shows a sketch of a parallelogram $PQRS$.

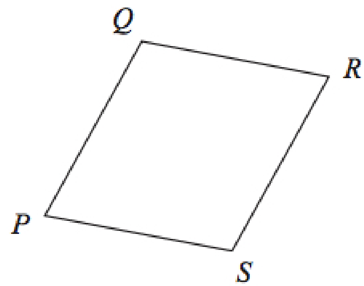

Figure 3: a parallelogram $PQRS$

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and
- $\overrightarrow{QR} = 5\mathbf{i} - 2\mathbf{k}$,

(a) show that parallelogram $PQRS$ is a rhombus. (2)

(b) Find the exact area of the rhombus $PQRS$. (4)