

Dr Oliver Mathematics

Points of Inflexion: Part 1

1. At which point(s) does the graph of

$$y = 5x^4 - x^5$$

has a point of inflexion?

Solution

$$\begin{aligned}y = 5x^4 - x^5 &\Rightarrow \frac{dy}{dx} = 20x^3 - 5x^4 \\ &\Rightarrow \frac{d^2y}{dx^2} = 60x^2 - 20x^3.\end{aligned}$$

Now,

$$\begin{aligned}\frac{d^2y}{dx^2} = 0 &\Rightarrow 60x^2 - 20x^3 = 0 \\ &\Rightarrow 20x^2(3 - x) = 0 \\ &\Rightarrow x = 0 \text{ or } x = 3.\end{aligned}$$

The only sign change for $\frac{d^2y}{dx^2}$ is $x = 3$ (which, after all, is the definition of a point of inflexion: around $x = 0$, we go positive-zero-positive). Finally,

$$x = 3 \Rightarrow y = 5(3^4) - 3^5 = 162;$$

hence, the point of inflexion is (3, 162).