Dr Oliver Mathematics
AQA GCSE Mathematics
2017 November Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Circle the fraction that is equivalent to 3.875. (1)

$$\tfrac{15}{4} \quad \tfrac{29}{8} \quad \tfrac{31}{8} \quad \tfrac{15}{8}.$$

Solution

$$\begin{aligned} 3.875 &= 3\tfrac{7}{8} \\ &= \tfrac{31}{8} \end{aligned}$$

so

$$\tfrac{15}{4} \quad \tfrac{29}{8} \quad \underline{\underline{\tfrac{31}{8}}} \quad \tfrac{15}{8}.$$

2. What is 50 as a percentage of 20? (1)
Circle your answer.

$$10\% \quad 40\% \quad 150\% \quad 250\%.$$

Solution

$$\frac{50}{20} \times 100\% = 250\%$$

so

$$10\% \quad 40\% \quad 150\% \quad \underline{\underline{250\%}}.$$

3. Circle the point that does not lie on the curve (1)

$$y = x^3.$$

$$(-\tfrac{1}{2}, -\tfrac{1}{8}) \quad (5, 125) \quad (\tfrac{1}{3}, \tfrac{1}{9}) \quad (-1, -1).$$

Solution

$$x = -\tfrac{1}{3} \Rightarrow y = -\tfrac{1}{8} \quad \checkmark$$
$$x = 5 \Rightarrow y = 125 \quad \checkmark$$
$$x = -1 \Rightarrow y = \tfrac{1}{27} \quad \times$$
$$x = -1 \Rightarrow y = -1 \quad \checkmark$$

so

$$(-\tfrac{1}{2}, -\tfrac{1}{8}) \quad (5, 125) \quad \underline{\underline{(\tfrac{1}{3}, \tfrac{1}{9})}} \quad (-1, -1).$$

4. Which **one** of these is a unit of density? (1)
Circle your answer.

$$\text{kg/m}^2 \quad \text{m}^2\text{/kg} \quad \text{kg/m}^3 \quad \text{m}^3\text{/kg}.$$

Solution

Well,

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

so

$$\text{kg/m}^2 \quad \text{m}^2\text{/kg} \quad \underline{\underline{\text{kg/m}^3}} \quad \text{m}^3\text{/kg}.$$

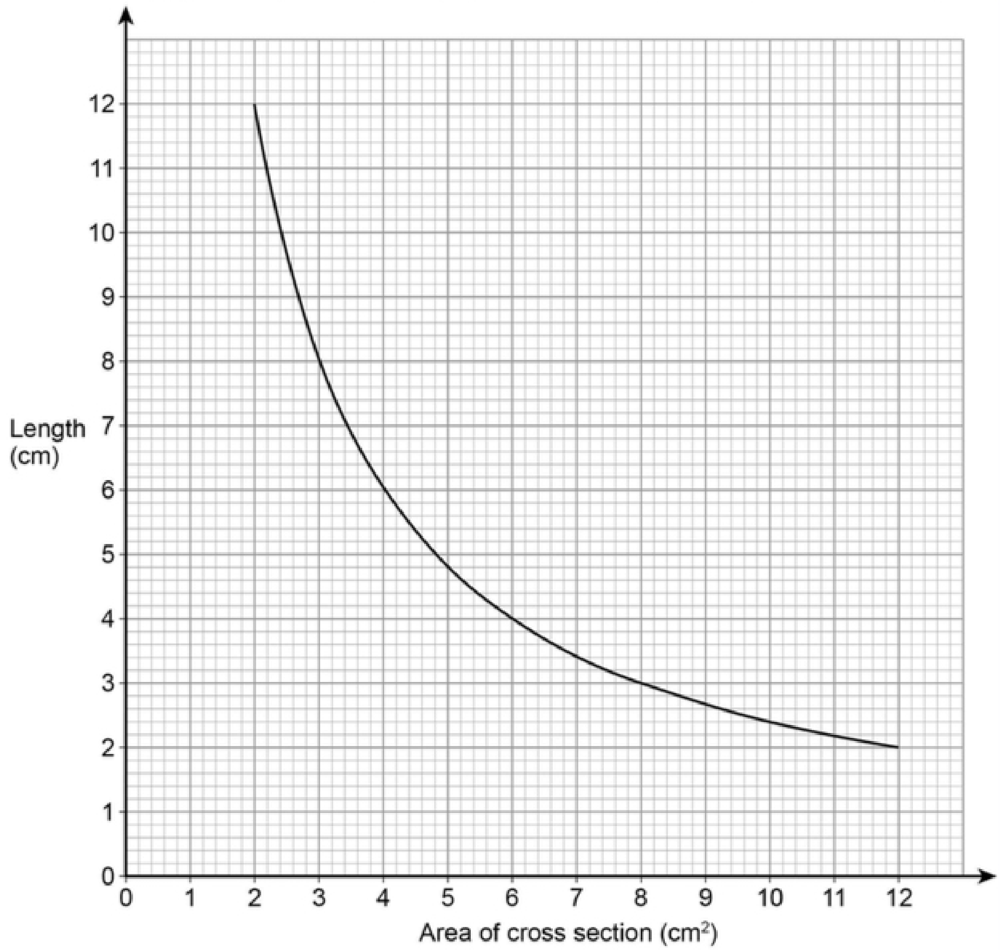
5. Solve (3)

$$4(3x - 2) = 2x - 5.$$

Solution

$$\begin{aligned} 4(3x-2) = 2x - 5 &\Rightarrow 12x - 8 = 2x - 5 \\ &\Rightarrow 10x = 3 \\ &\Rightarrow \underline{\underline{x = \tfrac{3}{10}}}. \end{aligned}$$

6. The graph shows information about prisms with the same volume.



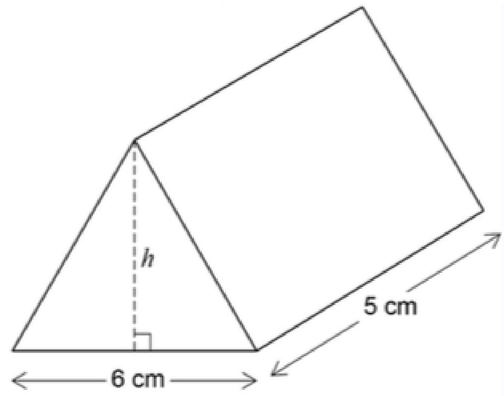
(a) Give **one** example to show the volume is 24 cm^3. (1)

Solution

Well,

$$\text{Area} = 4 \text{ cm} \Rightarrow \text{length} = \underline{\underline{6 \text{ cm}}}.$$

The diagram shows a prism with volume 24 cm^3.
The height of the triangular cross section is h.

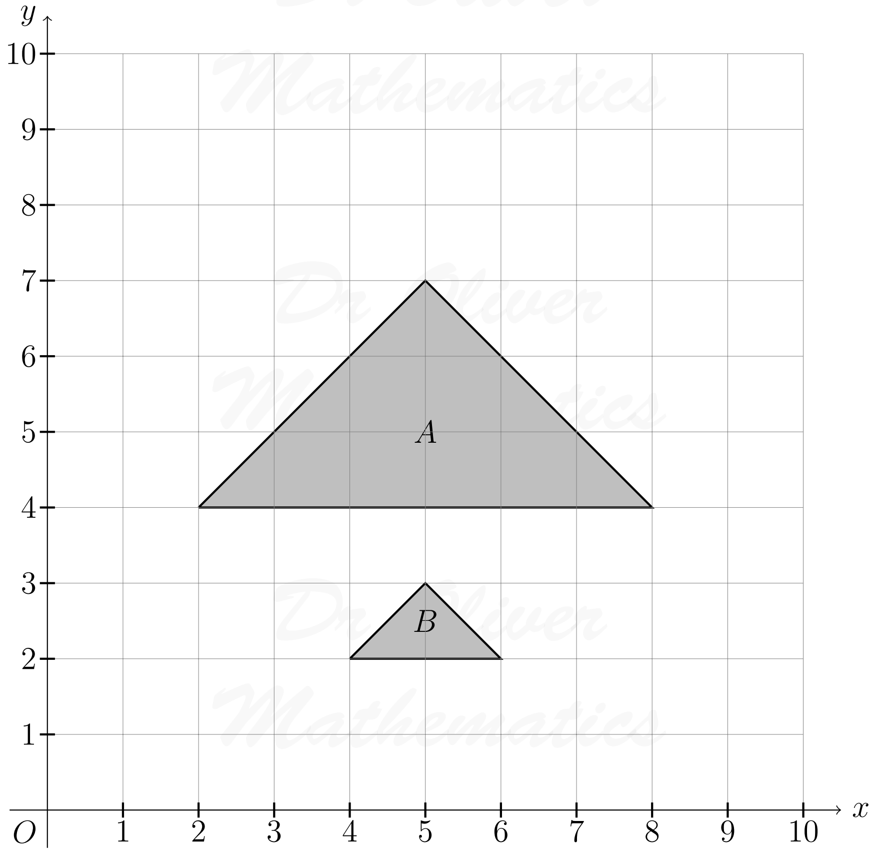


(b) Work out the height, h. (3)

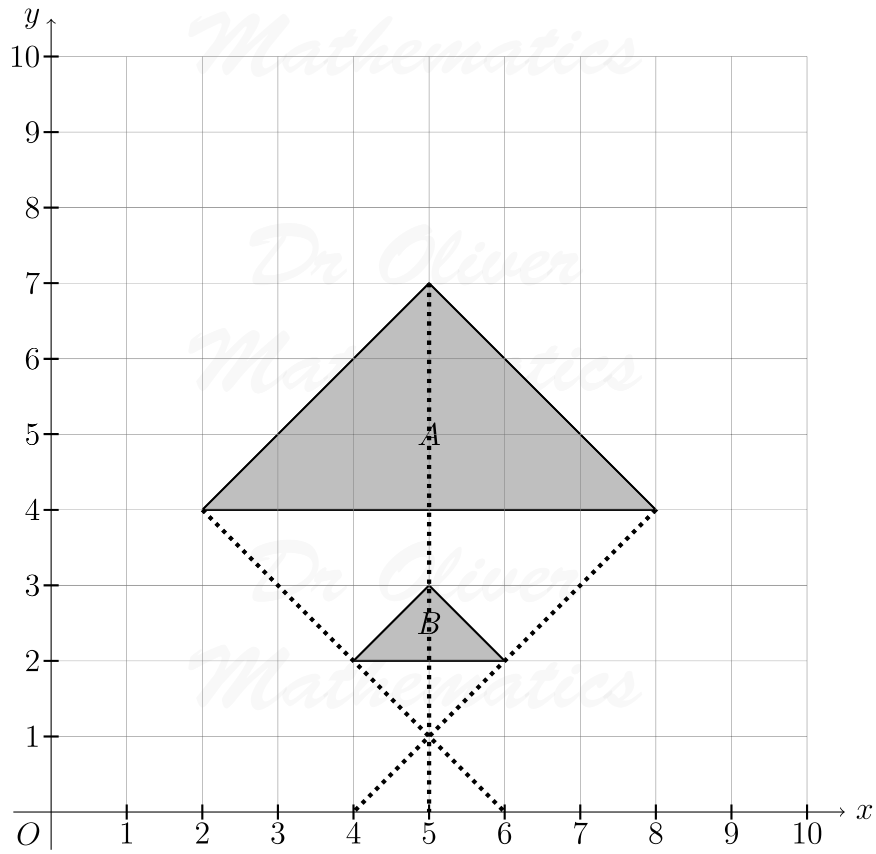
Solution
Well,

$$\begin{aligned} \text{volume} = \text{cross-sectional area} \times \text{length} &\Rightarrow 24 = (\tfrac{1}{2} \times 6 \times h) \times 5 \\ &\Rightarrow 24 = 15h \\ &\Rightarrow \underline{\underline{h = 1\tfrac{3}{5} \text{ cm}}}. \end{aligned}$$

7. Describe fully the **single** transformation that maps triangle A to triangle B. (3)



Solution



So, it is enlargement, scale factor $\frac{1}{3}$, about $(5, 1)$.

8. The table shows information about the distances walked by 120 students on their way to school one week. (3)

Distance, x (miles)	Frequency	
$0 < x \leqslant 5$	20	
$5 < x \leqslant 10$	48	
$10 < x \leqslant 15$	30	
$15 < x \leqslant 20$	22	
– – –	Total = 120	– – –

Work out an estimate for the mean distance.

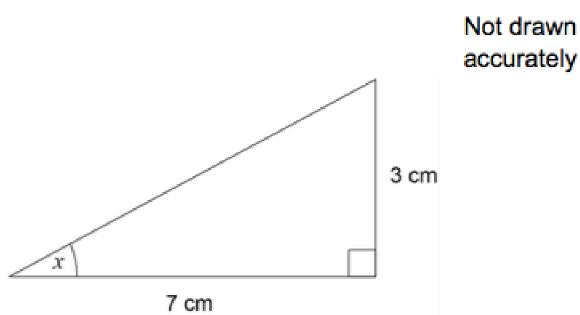
Solution

Distance, x (miles)	Frequency	Midpoint	Frequency $\times$ Midpoint
$0 < x \leqslant 5$	20	2.5	$2.5 \times 20 = 50$
$5 < x \leqslant 10$	48	7.5	$7.5 \times 48 = 360$
$10 < x \leqslant 15$	30	12.5	$12.5 \times 30 = 375$
$15 < x \leqslant 20$	22	17.5	$17.5 \times 22 = 385$
$---$	Total $= 120$	$---$	Total $= 1\,170$

Finally,

$$\begin{aligned}\text{mean distance} &= \frac{\sum fx}{\sum f} \\ &\approx \frac{1\,170}{120} \\ &= \underline{\underline{9\tfrac{3}{4}\text{ miles}}}.\end{aligned}$$

9. Work out the size of angle x. (2)



Solution

$$\begin{aligned}\tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan x = \tfrac{3}{7} \\ &\Rightarrow x = 23.198\,590\,51 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 23.2^\circ \text{ (3 sf)}}}.\end{aligned}$$

10. Work out the next term of this quadratic sequence: (2)

$$5 \quad 8 \quad 14 \quad 23 \quad \ldots$$

Solution

Let the

$$n\text{th term} = an^2 + bn + c.$$

We only need the second line of differences (why?):

$$\begin{array}{ccccc} 5 & & 8 & & 14 \\ & 3 & & 6 & \\ & & 3 & & \end{array}$$

$$\begin{array}{ccccc} a+b+c & & 4a+2b+c & & 9a+3b+c \\ & 3a+b & & 5a+b & \\ & & 2a & & \end{array}$$

We compare terms:

$$2a = 3 \Rightarrow a = \tfrac{3}{2},$$

$$\begin{aligned} 3a + b = 3 &\Rightarrow 3 \times \tfrac{3}{2} + b = 3 \\ &\Rightarrow b = -\tfrac{3}{2}, \end{aligned}$$

and

$$\begin{aligned} a + b + c = 5 &\Rightarrow \tfrac{3}{2} - \tfrac{3}{2} + c = 5 \\ &\Rightarrow c = 5; \end{aligned}$$

so

$$n\text{th term} = \tfrac{3}{2}n^2 - \tfrac{3}{2}n + 5.$$

Finally,

$$\begin{aligned} 5\text{th term} &= \tfrac{3}{2}(5^2) - \tfrac{3}{2}(5) + 5 \\ &= \underline{\underline{35}}. \end{aligned}$$

11. Circle the expression that is equivalent to (1)

$$\frac{3x^2}{6x^2+3}.$$

$$\frac{x^2}{2x^2+3} \quad \frac{x^2}{6x^2+1} \quad \frac{x^2}{2x^2+1} \quad \tfrac{1}{2}+x^2.$$

Solution

$$\begin{aligned}\frac{3x^2}{6x^2+3} &= \frac{3x^2}{3(2x^2+1)} \\ &= \frac{x^2}{2x^2+1}\end{aligned}$$

so

$$\frac{x^2}{2x^2+3} \quad \frac{x^2}{6x^2+1} \quad \underline{\underline{\frac{x^2}{2x^2+1}}} \quad \tfrac{1}{2}+x^2.$$

12. The table shows information about the UK and Germany. (3)

	Population	Area (square miles)
UK	64 000 000	95 000
Germany	82 000 000	140 000

$$\text{Population density} = \frac{\text{population}}{\text{area}}.$$

Compare the population densities of the UK and Germany.

Solution

Well,

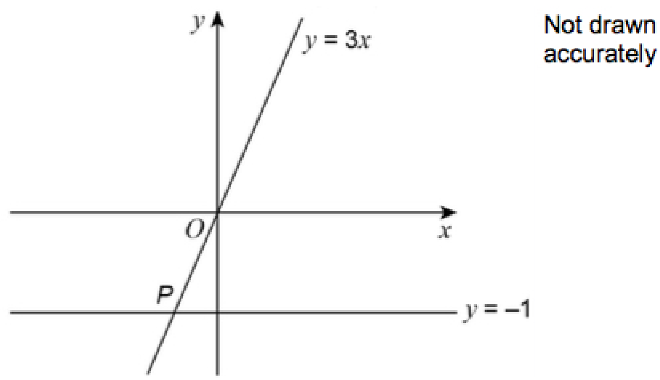
$$\begin{aligned}\text{population density}_{\text{UK}} &= \frac{64\,000\,000}{95\,000} \\ &= 673\tfrac{13}{19}\end{aligned}$$

and

$$\begin{aligned}\text{population density}_{\text{Germany}} &= \frac{82\,000\,000}{140\,000} \\ &= 585\tfrac{5}{7}.\end{aligned}$$

Hence, the $\underline{\underline{\text{UK}}}$ has a higher population density.

13. Two straight lines intersect at point P. (1)



Circle the coordinates of P.

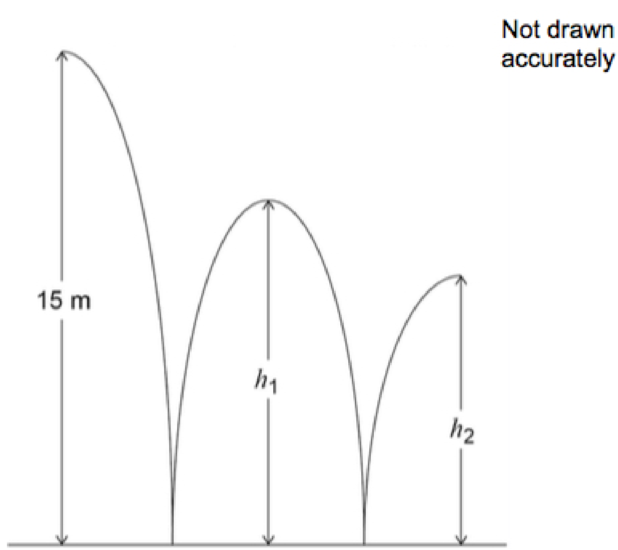
$$(-3, 1) \quad (-1, -\tfrac{1}{3}) \quad (-1, -3) \quad (-\tfrac{1}{3}, -1).$$

Solution

$$(-3, 1) \quad (-1, -\tfrac{1}{3}) \quad (-1, -3) \quad \underline{\underline{(-\tfrac{1}{3}, -1)}}.$$

14. A ball is thrown from a height of 15 metres.

It bounces to height h_1, then to height h_2 as shown.



h_1 is three quarters of the original height.

Jack expects h_2 to be three quarters of h_1.

(a) Work out the value of h_2 that he expects. (2)

Solution

$$\begin{aligned} h_1 &= 15 \times \tfrac{3}{4} \\ &= 11\tfrac{1}{4} \end{aligned}$$

and

$$\begin{aligned} h_2 &= 11\tfrac{1}{4} \times \tfrac{3}{4} \\ &= \underline{\underline{8\tfrac{7}{16}\ m}}. \end{aligned}$$

In fact, h_2 is two thirds of h_1.

(b) How does this affect the answer to part (a)? (2)
Tick a box.

☐ The ball bounced higher than he expected

☐ The ball bounced lower than he expected

Show working to support your answer.

Solution

$$\begin{aligned} h_2 &= 11\tfrac{1}{4} \times \tfrac{2}{3} \\ &= 7\tfrac{1}{2}; \end{aligned}$$

so, the ball bounced <u>lower</u> than he had expected.

15. Mirek invests £6 000 at a compound interest rate of 1.5% per year. (3)
He wants to earn more than £1 000 interest.

Work out the **least** time, in whole years, that this will take.

Solution

Year	Value
1	$6\,000 \times 1.015 = 6\,090$
2	$6\,090 \times 1.015 = 6\,181.35$
3	$6\,181.35 \times 1.015 = 6\,274.070\ldots$
4	$6\,274.070\ldots \times 1.015 = 6\,368.181\ldots$
5	$6\,368.181\ldots \times 1.015 = 6\,463.704\ldots$
6	$6\,463.704\ldots \times 1.015 = 6\,560.659\ldots$
7	$6\,560.659\ldots \times 1.015 = 6\,659.069\ldots$
8	$6\,659.069\ldots \times 1.015 = 6\,758.955\ldots$
9	$6\,758.955\ldots \times 1.015 = 6\,860.339\ldots$
10	$6\,860.339\ldots \times 1.015 = 6\,963.244\ldots$
11	$6\,963.244\ldots \times 1.015 = 7\,067.693\ldots$

Hence, 11 years.

16. (a) Factorise fully (2)

$$9y^3 - 6y.$$

Solution

$$9y^3 - 6y = 3y(2y^2 - 1)$$

difference of two squares:

$$= 3y[(\sqrt{2}y)^2 - 1]$$
$$= 3y(\sqrt{2}y + 1)(\sqrt{2}y - 1).$$

(b) Factorise (2)

$$3x^2 - 22x + 7.$$

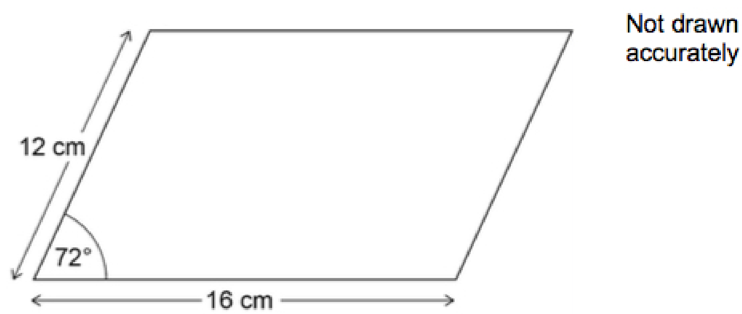
Solution

$$\left.\begin{array}{ll}\text{add to:} & -22 \\ \text{multiply to:} & (+3)\times(+7) = +21\end{array}\right\} -21,\ -1$$

E.g.,

$$\begin{aligned}3x^2 - 22x + 7 &= 3x^2 - 21x - x + 7 \\ &= 3x(x-7) - 1(x-7) \\ &= \underline{\underline{(3x-1)(x-7)}}.\end{aligned}$$

17. Work out the area of the parallelogram. (3)



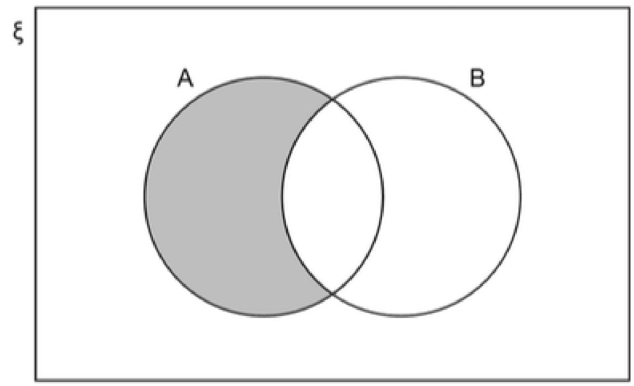
Solution

We will use

$$\text{area} = \tfrac{1}{2}ab\sin C:$$

$$\begin{aligned}\text{area} &= 2 \times \text{triange} \\ &= 2 \times \left(\tfrac{1}{2} \times 12 \times 16 \times \sin 72^\circ\right) \\ &= 182.602\,851\,1 \text{ (FCD)} \\ &= \underline{\underline{183 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

18. (a) Which of these represents the shaded region? (1)
Circle your answer.

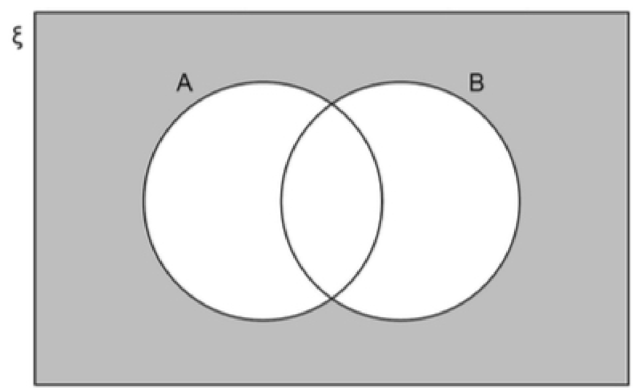


$A \quad B' \quad A \cap B' \quad A \cup B'$

Solution

$A \quad B' \quad \underline{\underline{A \cap B'}} \quad A \cup B'$

(b) Which of these represents the shaded region? (1)
Circle your answer.



$(A \cup B)' \quad (A \cap B)' \quad A' \cap B \quad A' \cup B'$

Solution

$\underline{\underline{(A \cup B)'}} \quad (A \cap B)' \quad A' \cap B \quad A' \cup B'$

19. The length of a rectangle is five times the width. (3)
The area of the rectangle is $1\,620$ cm^2.

Not drawn accurately

Work out the width of the rectangle.

> **Solution**
>
> Let the width of the rectangle be x cm, $x > 0$. Then length is $5x$ cm. Now,
>
> $$\begin{aligned} \text{area} = 1\,620 &\Rightarrow (x)(5x) = 1\,620 \\ &\Rightarrow 5x^2 = 1\,620 \\ &\Rightarrow x^2 = 324 \\ &\Rightarrow x = 18. \end{aligned}$$
>
> Hence, the width of the rectangle is 18 cm.

20. A stone is thrown upwards with a speed of v metres per second. (4)
The stone reaches a maximum height of h metres.

h is directly proportional to v^2.
When $v = 10$, $h = 5$.

Work out the maximum height reached when $v = 24$.

> **Solution**
>
> Well,
>
> $$h \propto v^2 \Rightarrow h = kv^2,$$
>
> for some constant k. Now,
>
> $$\begin{aligned} h = 5, v = 10 &\Rightarrow 5 = k(10^2) \\ &\Rightarrow 5 = 100k \\ &\Rightarrow k = \tfrac{1}{20} \end{aligned}$$

and so

$$h = \tfrac{1}{20}v^2.$$

Finally,

$$\begin{aligned} h &= \tfrac{1}{20}(24^2) \\ &= \tfrac{1}{20}(576) \\ &= \underline{\underline{28.8 \text{ m}}}. \end{aligned}$$

21. Meera is using a **graphical** method to solve

$$2x^2 - 3x = 0.$$

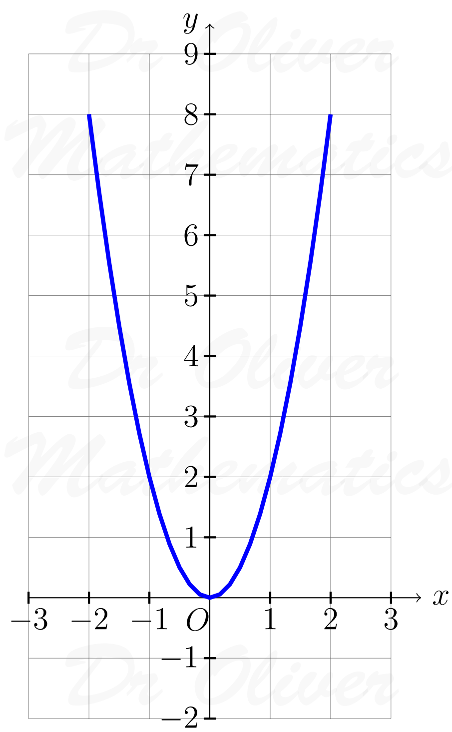
She draws the graph of

$$y = 2x^2$$

and a straight line graph on the same grid.

Here is the graph of

$$y = 2x^2.$$



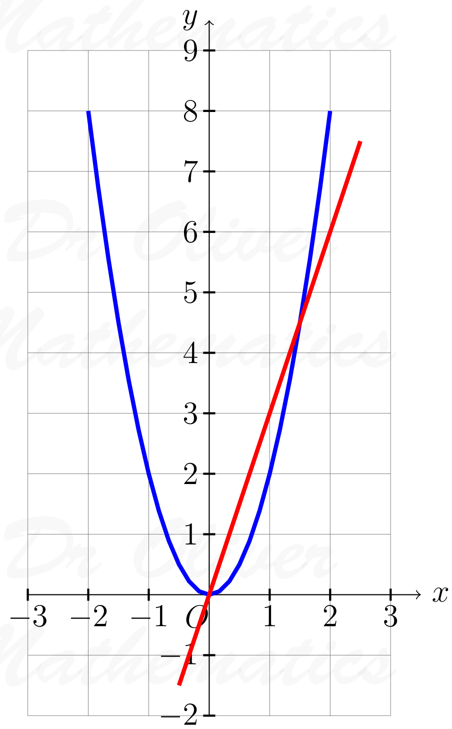
(a) Complete her method to solve (2)

$$2x^2 - 3x = 0.$$

Solution

$$2x^2 - 3x = 0 \Rightarrow 2x^2 = 3x$$

and so we need to draw on $y = 3x$:



Correct read-off: $\underline{\underline{x = 0}}$ and approximately $\underline{\underline{x = 1.4}}$.

Levi is solving

$$2x^2 + 5x = 0.$$

He uses this method.

$$\begin{aligned} 2x^2 + 5x &= 0 && \text{subtract } 5x \text{ from both sides} \\ 2x^2 &= -5x && \text{divide both sides by } x \\ 2x &= -5 && \text{divide both sides by 2} \\ x &= -2.5 \end{aligned}$$

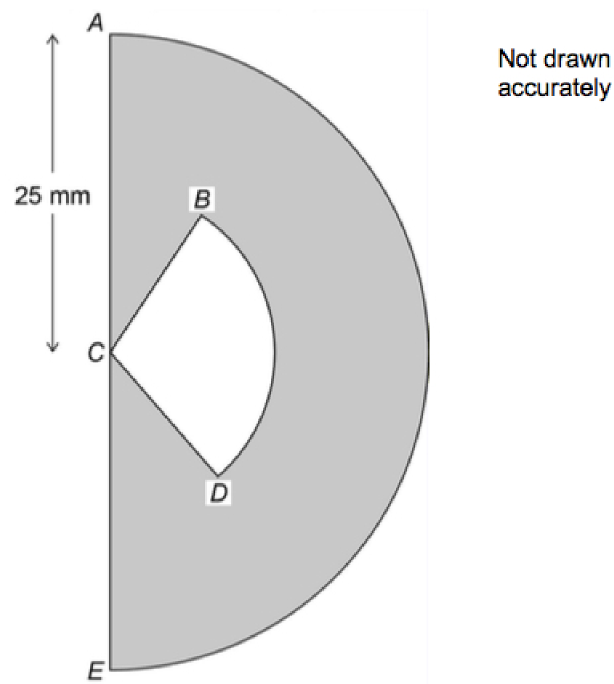
(b) Evaluate his method and his answer. (2)

Solution

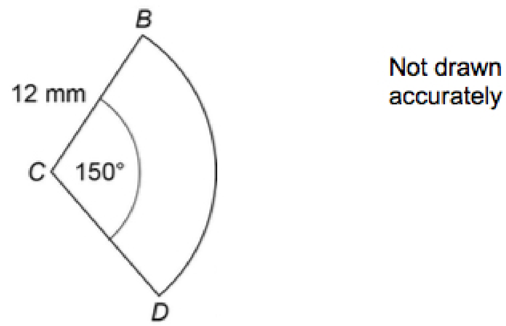
He has lost one solution:

$$\begin{aligned} 2x^2 + 5x = 0 &\Rightarrow x(2x+5) = 0 \\ &\Rightarrow x = 0 \text{ or } 2x + 5 = 0 \\ &\Rightarrow x = 0 \text{ or } x = -2.5. \end{aligned}$$

22. The cross section of an earring is a semi-circle, centre C, radius 25 mm. (5)
The earring is black and white.
The shaded area is black.



Sector BCD is white and has radius 12 mm.



Is more than 20% of the semi-circle white?
You **must** show your working.

Solution

Well,

$$\begin{aligned}\text{area}_{\text{white}} &= \tfrac{150}{360} \times \pi \times 12^2 \\ &= 60\pi\end{aligned}$$

and

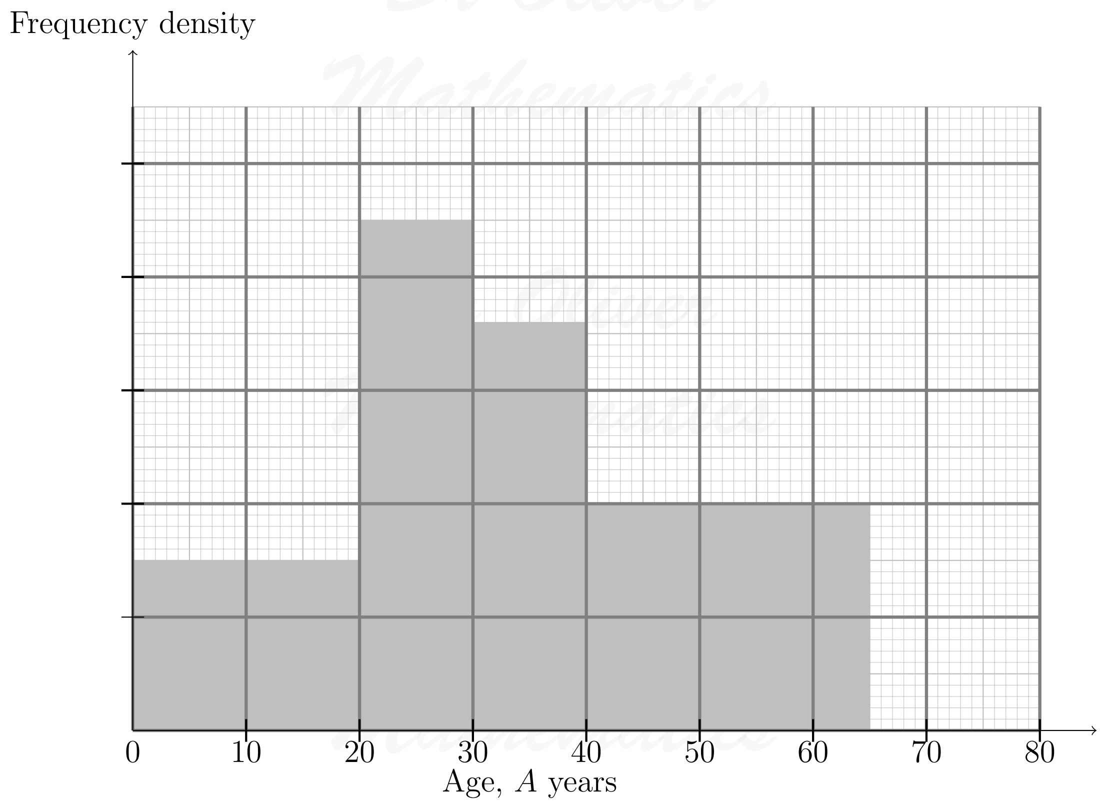
$$\begin{aligned}\text{area}_{\text{black}} &= \tfrac{1}{2} \times \pi \times 25^2 \\ &= \tfrac{625}{2}\pi.\end{aligned}$$

Finally,

$$\begin{aligned}\text{percentage}_{\text{white}} &= \left(\frac{60\pi}{\frac{625}{2}\pi}\right) \times 100\% \\ &= 19.2\%;\end{aligned}$$

hence, it is less than 20% of the semi-circle.

23. Here is some information about a tennis club.



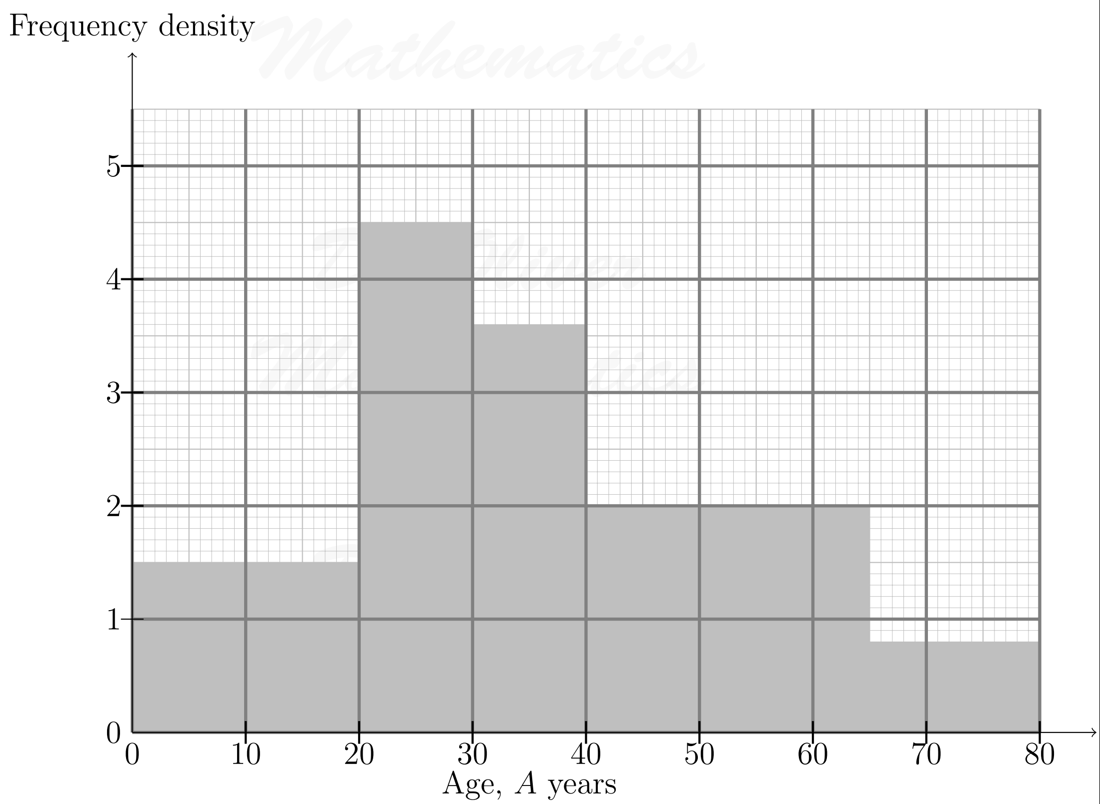
- There are 30 members with $A < 20$.
- There are 12 members with $65 \leqslant A < 80$.
- There are no members with $A \geqslant 80$.

(a) Complete the histogram. (3)

Solution

We will make up a table, scale the vertical axis, and draw the histogram:

Age	Frequency	Width	Frequency Density
$0 \leqslant A < 20$	$20 \times 1.5 = 30$	20	1.5
$20 \leqslant A < 30$	$10 \times 4.5 = 45$	10	4.5
$30 \leqslant A < 40$	$10 \times 3.6 = 36$	10	3.6
$40 \leqslant A < 65$	$25 \times 2 = 50$	25	2
$65 \leqslant A < 80$	12	15	$\frac{12}{15} = 0.8$

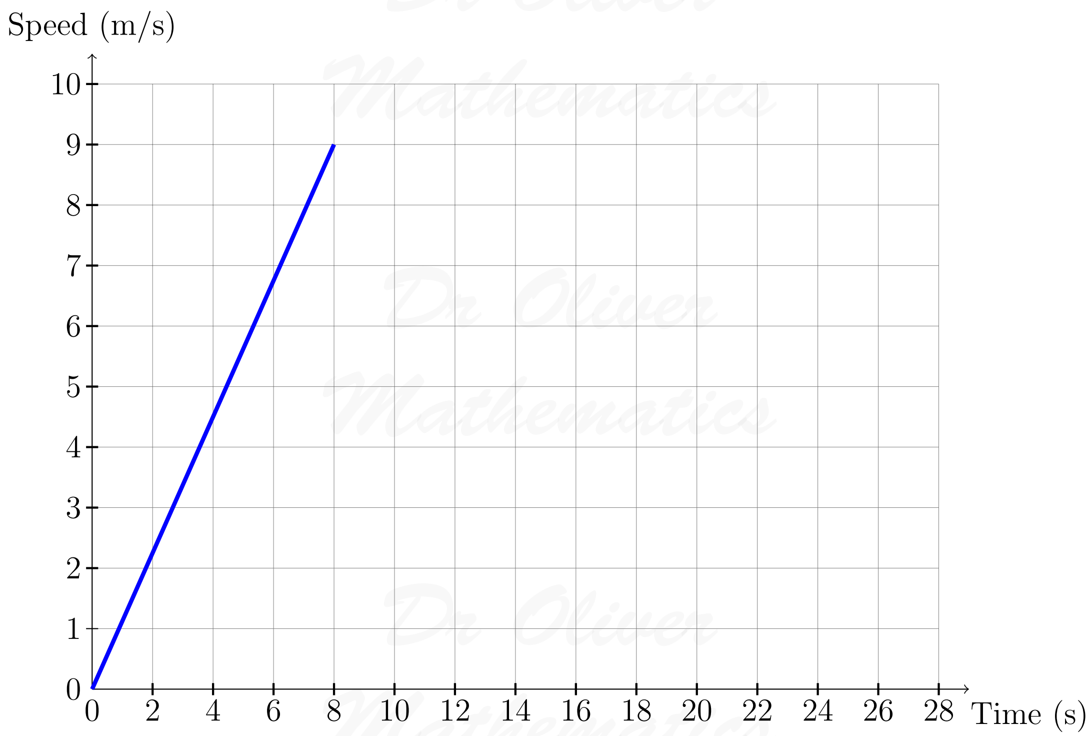


(b) Work out the total number of members of the club. (2)

Solution

$$30 + 45 + 36 + 50 + 12 = \underline{\underline{173}}.$$

24. Beth ran a 200 metre race. (3)
Here is a graph of the first 8 seconds of her race.
She completed the race at a constant speed of 9 m/s.



Amy completed the race in 27 seconds.

Did Beth finish before Amy?
You **must** show your working.

Solution

Well,

$$\begin{aligned} \text{distance} &= \tfrac{1}{2} \times 8 \times 9 \\ &= 36 \text{ m} \end{aligned}$$

and so Beth has

$$200 - 36 = 154 \text{ m}$$

to go. At 9 m/s, that is

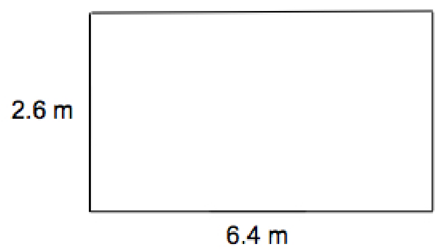
$$\frac{154}{9} = 17\tfrac{1}{9} \text{ s}$$

and the total time for Beth is

$$8 + 17\tfrac{1}{9} = 25\tfrac{1}{9} \text{ s.}$$

Hence, Beth finished before Amy.

25. The dimensions of a rectangular floor are to the nearest 0.1 metres. (5)



Not drawn accurately

A force of 345 Newtons is applied to the floor.
The force is to the nearest 5 Newtons.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the upper bound of the pressure.
Give your answer to 4 significant figures.
You **must** show your working.

Solution

Well,

$$342.5 \leqslant \text{force} < 347.5$$

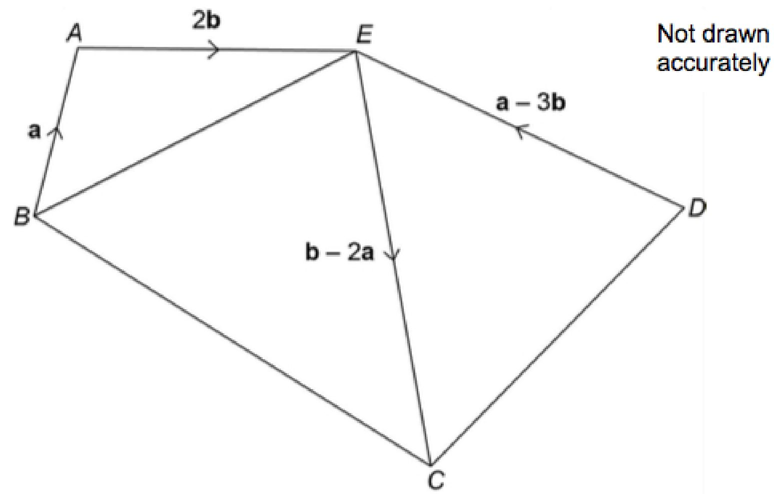
and

$$2.55 \times 6.35 \leqslant \text{metres} < 2.65 \times 6.45.$$

Now,

$$\begin{aligned} \text{upper bound} &= \frac{347.5}{2.55 \times 6.35} \\ &= 21.460\,552\,73 \text{ (FCD)} \\ &= \underline{\underline{21.5 \text{ N/m}^2}}. \end{aligned}$$

26. $ABCDE$ is a pentagon. (3)



Show that $BCDE$ is a parallelogram.

Solution

Well,

$$\begin{aligned}\overrightarrow{CB} &= \overrightarrow{CE} + \overrightarrow{EA} + \overrightarrow{AB} \\ &= -\overrightarrow{EC} - \overrightarrow{AE} - \overrightarrow{BA} \\ &= -(\mathbf{b} - 2\mathbf{a}) - 2\mathbf{b} - \mathbf{a} \\ &= -\mathbf{b} + 2\mathbf{a} - 2\mathbf{b} - \mathbf{a} \\ &= \mathbf{a} - 3\mathbf{b} \\ &= \overrightarrow{DE}.\end{aligned}$$

Hence, $BCDE$ is a parallelogram because CB is equal and parallel to DE.

27. Solve (6)

$$\frac{x}{4} - \frac{2x}{x+2} = 1.$$

Give your solutions to 2 decimal places.
You **must** show your working.

Solution

Multiply be $4(x+2)$:

$$\begin{aligned}
& \frac{x}{4} - \frac{2x}{x+2} = 1 \\
\Rightarrow \quad & 4(x+2) \times \frac{x}{4} - 4(x+2) \times \frac{2x}{x+2} = 4(x+2) \times 1 \\
\Rightarrow \quad & x(x+2) - 8x = 4(x+2) \\
\Rightarrow \quad & x^2 - 6x = 4x + 8 \\
\Rightarrow \quad & x^2 - 10x = 8
\end{aligned}$$

we'll complete the square:

$$\begin{aligned}
\Rightarrow \quad & x^2 - 10x + 25 = 8 + 25 \\
\Rightarrow \quad & (x-5)^2 = 33 \\
\Rightarrow \quad & x - 5 = \pm\sqrt{33} \\
\Rightarrow \quad & x = 5 \pm \sqrt{33} \\
\Rightarrow \quad & \underline{\underline{x = -0.74 \text{ or } x = 10.74 \text{ (2 dp)}}}.
\end{aligned}$$