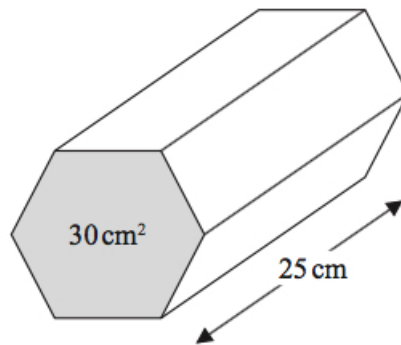


**Dr Oliver Mathematics**  
**GCSE Mathematics**  
**2016 Paper 1H: Non-Calculator**  
**1 hour 45 minutes**

The total number of marks available is 100.  
You must write down all the stages in your working.

1. The diagram shows a prism.

(3)

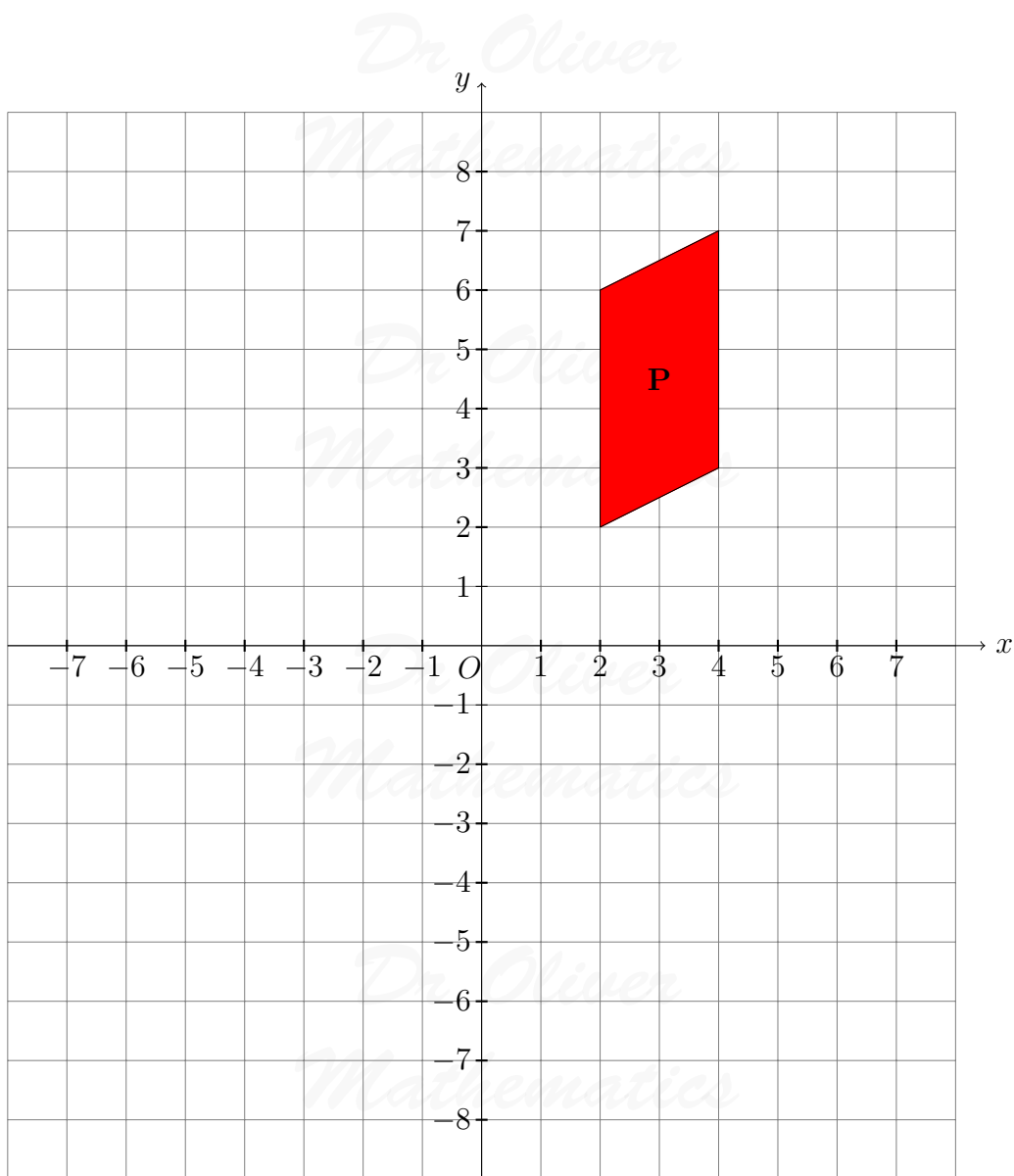


The area of the cross section of the prism is  $30 \text{ cm}^2$ .  
The length of the prism is 25 cm.  
Work out the volume of the prism.

**Solution**

$$\text{Volume} = 25 \times 30 = \underline{\underline{750 \text{ cm}^3}}.$$

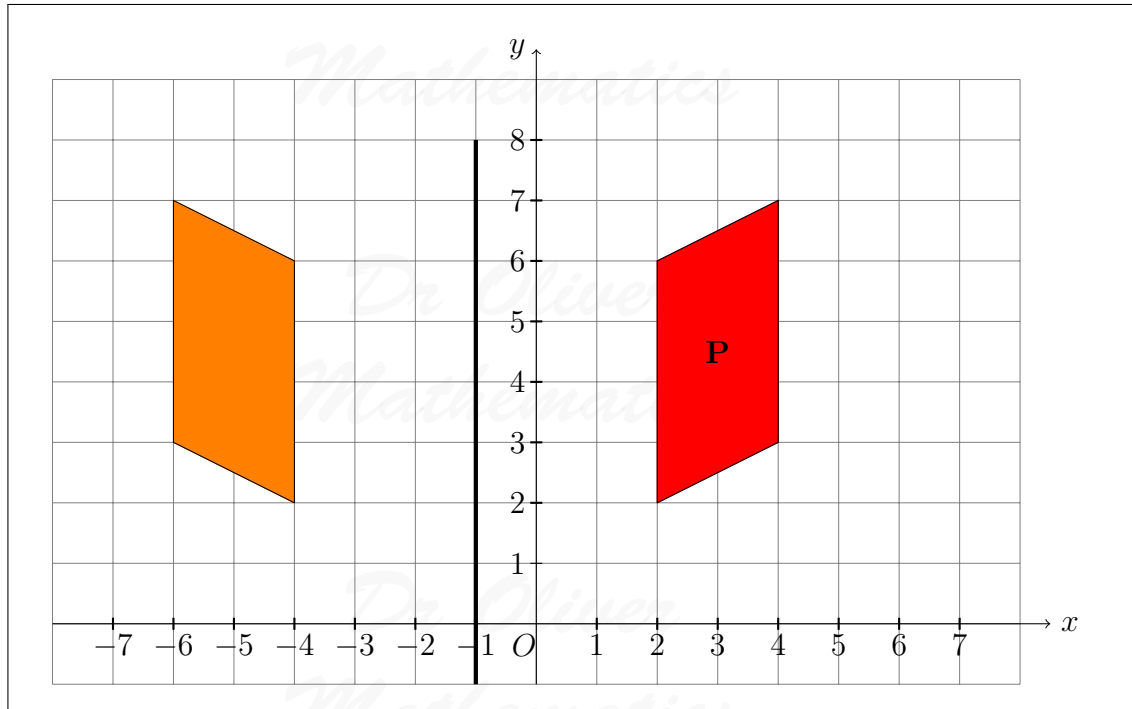
2. Here is a grid.



(a) Reflect shape **P** in the line  $x = -1$ .

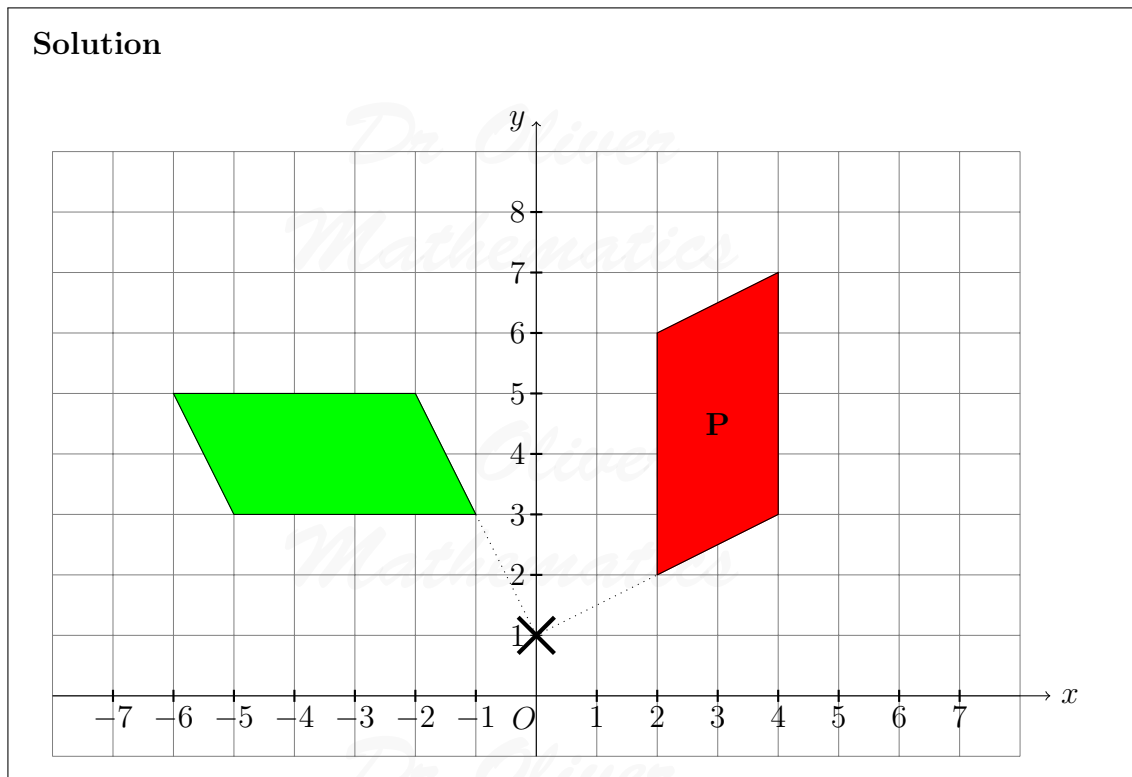
(2)

<b>Solution</b>
-----------------



(b) Rotate shape **P**  $90^\circ$  anticlockwise about  $(0, 1)$ .

(2)



3. Richard wants to find out how often people buy crisps.

He uses this question on a questionnaire.

**How often do you buy crisps?**

Often  Sometimes  Never

- (a) Write down **two** things that are wrong with this question. (2)

**Solution**

E.g., no time frame, vague response boxes, not exhaustive.

- (b) Design a better question for Richard to use on his questionnaire to find out how often people buy crisps. (2)

**Solution**

A suitable question with a time frame, e.g., “Did you buy crisps today/last week/last month? Tick the appropriate box.”

At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, 0, 1-3, 4-6, 7 or more.

Richard is going to ask the students in his maths class to answer his questionnaire.

- (c) This may **not** be a good sample to use. (1)  
Give one reason why.

**Solution**

E.g., they are all the same age, they are friends.

4. (a) Simplify  $p^2 \times p^5$ . (1)

**Solution**

$$p^2 \times p^5 = p^{2+5} = \underline{\underline{p^7}}.$$

- (b) Simplify  $g^6 \div g^4$ . (1)

**Solution**

$$g^6 \div g^4 = g^{6-4} = \underline{\underline{g^2}}.$$

- (c) Simplify  $(k^3)^2$ . (1)

**Solution**

$$(k^3)^2 = k^{3 \times 2} = \underline{\underline{k^6}}.$$

- (d) Expand and simplify  $3(m + 4) - 2(4m + 1)$ . (2)

**Solution**

$$\begin{aligned} 3(m + 4) - 2(4m + 1) &= 3m + 12 - 8m - 2 \\ &= \underline{\underline{-5m + 10}}. \end{aligned}$$

- (e) Factorise  $n^2 - 7n$ . (1)

**Solution**

$$n^2 - 7n = \underline{\underline{n(n - 7)}}.$$

5. There are 892 litres of oil in Mr Aston's oil tank. (2)  
He uses 18.7 litres of oil each day.  
**Estimate** the number of days it will take him to use all the oil in the tank.

**Solution**

Value	1 sf
892	900
18.7	20

Hence,

$$\text{number of days} \approx \frac{900}{20} = \underline{\underline{45 \text{ days}}}.$$

6. One of the teachers at a school is chosen at random. (3)  
The probability that this teacher is female is  $\frac{3}{5}$ .  
There are 36 **male** teachers at the school.  
Work out the total number of teachers at the school.

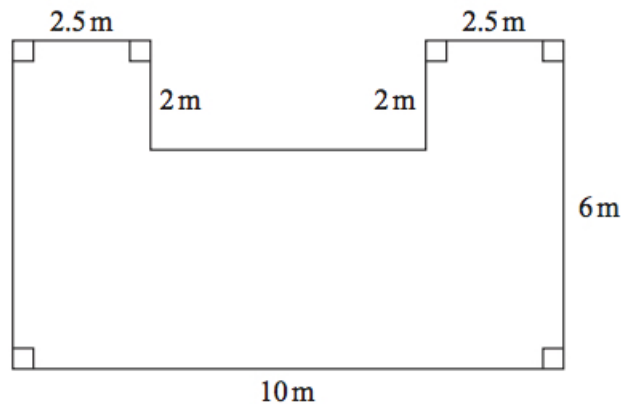
**Solution**

The number of female teachers is  $\frac{3}{5}$  of the total and so the number of male teachers is  $\frac{2}{5}$  of the total. Finally,

$$\text{the number of teachers} = \frac{5}{2} \times 36 = \frac{180}{2} = \underline{\underline{90 \text{ teachers}}}.$$

7. The diagram shows the plan of a floor.

(5)



Angie is going to varnish the floor.  
She needs 1 litre of varnish for  $5 \text{ m}^2$  of floor.  
There are 2.5 litres of varnish in each tin of varnish.  
Angie has 3 tins of varnish.  
Does she have enough varnish for all the floor?  
You must show all your working.

**Solution**

Angie needs

$$(10 \times 6) - (2 \times 5) = 60 - 10 = 50 \text{ m}^2.$$

She has

$$3 \times 2.5 \times 5 = 7.5 \times 5 = 37.5 \text{ m}^2.$$

No, she needs a fourth tin.

8. Carol spins a spinner 80 times.  
The table shows information about her results.

(3)

Dr Oliver  
Mathematics

Outcome	Frequency
<i>J</i>	39
<i>K</i>	25
<i>L</i>	16

<i>J</i>	39
<i>K</i>	25
<i>L</i>	16

Dan spins this spinner 300 times.

Work out an estimate for the number of times that Dan will get an *L*.

**Solution**

$$\begin{aligned}\text{Number of times} &= \frac{16}{80} \times 300 \\ &= \frac{1}{5} \times 300 \\ &= \underline{\underline{60 \text{ times}}}.\end{aligned}$$

9. A shop sells packets of envelopes. (3)

There are 5 envelopes in a small packet.

There are 20 envelopes in a large packet.

There is a total of  $T$  envelopes in  $x$  small packets and  $y$  large packets.

Write down a formula for  $T$  in terms of  $x$  and  $y$ .

**Solution**

$$\underline{\underline{T = 5x + 20y.}}$$

10. Point  $P$  has coordinates  $(5, 7)$ . (2)

Point  $M$  has coordinates  $(1, 2.5)$ .

Point  $M$  is the midpoint of the line  $PQ$ .

Find the coordinates of point  $Q$ .

**Solution**

$$\overrightarrow{PM} = \begin{pmatrix} 1 - 5 \\ 2.5 - 7 \end{pmatrix} = \begin{pmatrix} -4 \\ -4.5 \end{pmatrix}$$

and  $Q$  equals

$$(1 + (-4), 2.5 + (-4.5)) = \underline{\underline{(-3, -2)}}.$$

11. 66 people went on a day trip. (4)  
 Each person did only one activity on the trip.  
 Each person went skating or went to an art gallery or went bowling.  
 43 of the people are female.  
 4 of the 10 people who went skating are male.  
 20 of the people went to the art gallery.  
 10 males went bowling.  
 Work out the number of females who went to the art gallery.

**Solution**

Let's construct a table.

	M	F
Skating	4	6
Art Gallery		
Bowling	10	
Total	23	43

Men who went to the art gallery is

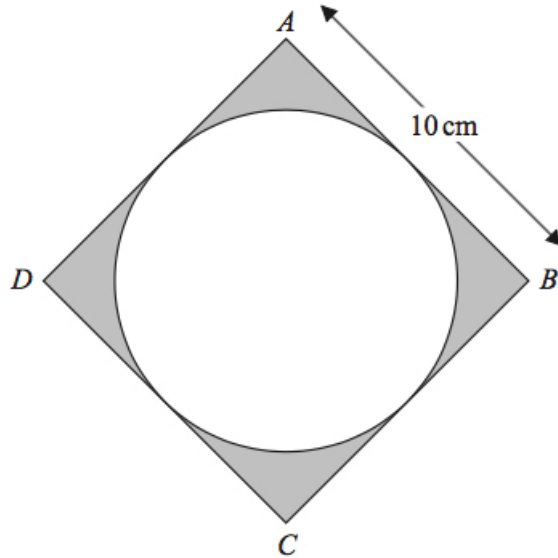
$$23 - 4 - 10 = 9$$

and the women who went to the art gallery is

$$20 - 9 = \underline{\underline{11}}.$$

12. The diagram shows a circle inside a square. (3)





$ABCD$  is a square of side 10 cm.  
 Each side of the square is a tangent to the circle.  
 Work out the total area of the shaded regions in terms of  $\pi$ .  
 Give your answer in its simplest form.

**Solution**

$$\begin{aligned} \text{Total area} &= 10^2 - \pi \times 5^2 \\ &= \underline{\underline{(100 - 25\pi) \text{ cm}^2}}. \end{aligned}$$

13. The table gives information about Ali's spending last month.

(3)

Item	Percentage of total spending
Rent	30%
Food	15%
Transport	12%
Other	43%

Ali's total spending last month was £800.  
 Next month Ali's rent, in pounds, is going to rise by 20%.  
 His total spending will still be the same.

Express the amount of money Ali will spend on rent next month as a percentage of £800.

**Solution**

This month's rent equals

$$\frac{30}{100} \times 800 = \text{£}240.$$

If it is going up by 20%,

$$\text{next month's rent} = 240 \times 1.2 = 240 + 48 = \text{£}288$$

and this leave

$$\frac{288}{800} \times 100\% = \frac{36}{100} \times 100\% = \underline{\underline{36\%}}.$$

14. (a) Use ruler and compasses to bisect the angle at  $A$ . (2)  
You must show all your construction lines.

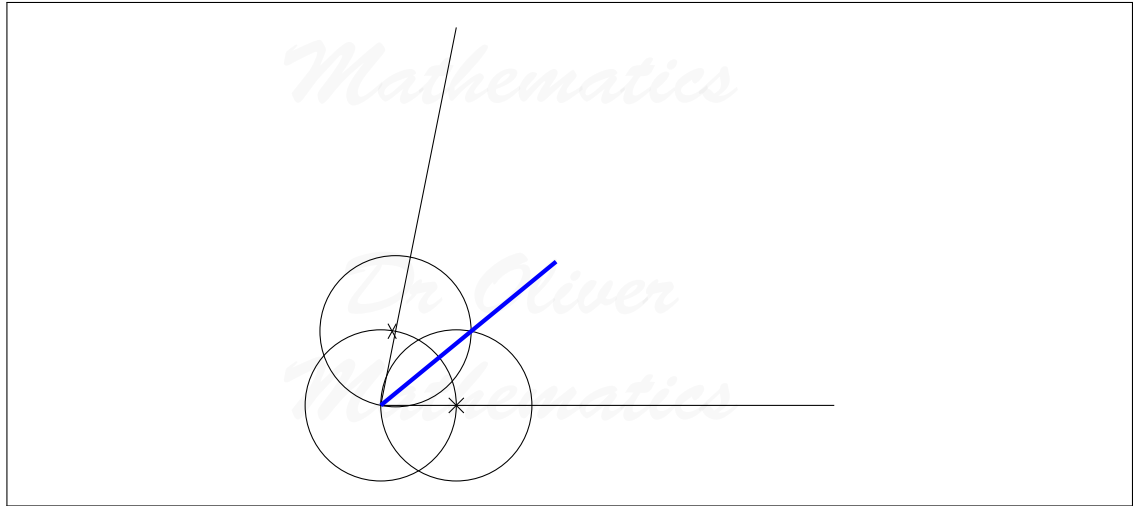


**Solution**

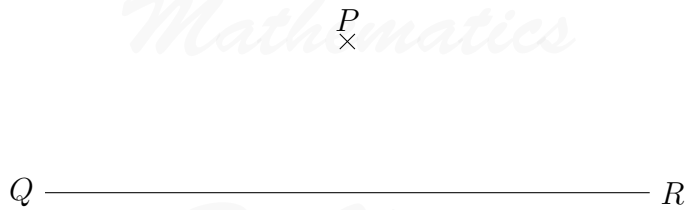
Step one: draw two arcs of the circle, centre  $A$ .

Step two: from each mark, draw a new arc.

Step three: using a ruler, draw in the straight line.

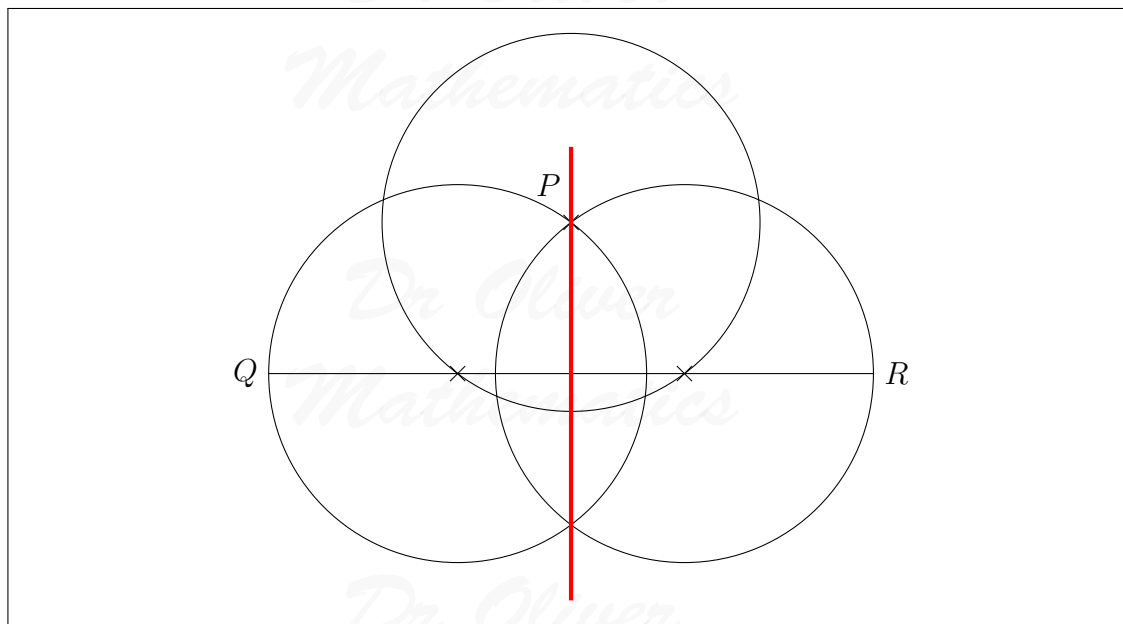


- (b) Use ruler and compasses to construct the perpendicular from the point  $P$  to the line  $QR$ . (2)  
 You must show all your construction lines.

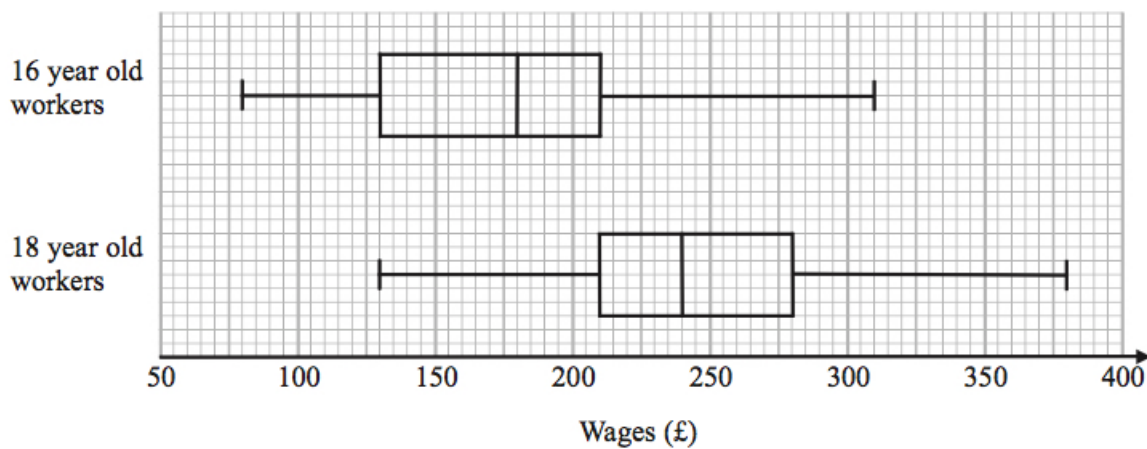


**Solution**

- Step one: draw one arc of the circle, centre  $P$ .  
 Step two: from each mark, draw a new arc.  
 Step three: using a ruler, draw in the straight line.



15. The box plots give information about the wages of a group of 16 year old workers and a group of 18 year old workers.



- (a) Compare the distribution of the wages of the 16 year old workers with the distribution of the wages of the 18 year old workers. (3)

**Solution**

E.g., the median for the 18 year old workers is £240 whereas the median for the 16 year old workers is £180.

The interquartile range is bigger for the 16 year old workers (£80) than it is for the 18 year old workers (£70).

The range is bigger for the 18 year old workers (£250) than it is for the 16 year

old workers (£230).

The 18 year old workers are positively skewed whereas the 16 year old workers are negatively skewed.

There are 200 workers who are 16 years old.

- (b) Work out an estimate for the number of these workers whose wages are £130 or more. (2)

**Solution**

£130 is the lower quartile for the 16 year old workers so the percentage is 75%.  
So, the

$$\text{estimate} = 200 \times 75\% = \underline{150}.$$

16. Work out the value of (2)

$$(3.5 \times 10^6) \div (5 \times 10^{-3}).$$

Give your answer in standard form.

**Solution**

$$\begin{aligned} \frac{3.5 \times 10^6}{5 \times 10^{-3}} &= 0.7 \times 10^9 \\ &= \underline{7 \times 10^8}. \end{aligned}$$

17. (a) Solve (2)

$$3x - 5 < 16.$$

**Solution**

$$\begin{aligned} 3x - 5 < 16 &\Rightarrow 3x < 21 \\ &\Rightarrow \underline{x < 7}. \end{aligned}$$

- (b) Solve (3)

$$\frac{11 - w}{4} = 1 + w.$$

**Solution**

$$\begin{aligned}\frac{11-w}{4} = 1+w &\Rightarrow 11-w = 4(1+w) \\ &\Rightarrow 11-w = 4+4w \\ &\Rightarrow 7 = 5w \\ &\Rightarrow \underline{\underline{w = 1\frac{2}{5}}}\end{aligned}$$

18. (a) Work out

$$1\frac{1}{5} \times 2\frac{1}{3}.$$

(3)

Give your answer as a mixed number in its simplest form.

**Solution**

$$\begin{aligned}1\frac{1}{5} \times 2\frac{1}{3} &= \frac{6}{5} \times \frac{7}{3} \\ &= \frac{2}{5} \times \frac{7}{1} \\ &= \frac{14}{5} \\ &= \underline{\underline{2\frac{4}{5}}}\end{aligned}$$

(b) Work out

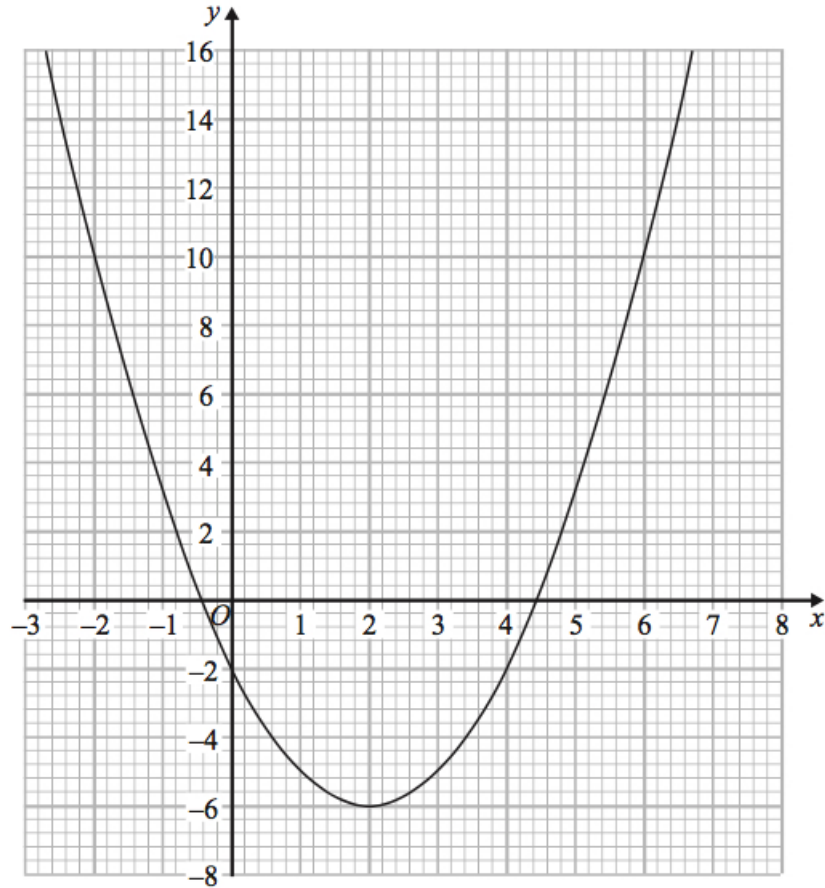
$$2\frac{7}{15} - 1\frac{2}{3}.$$

(3)

**Solution**

$$\begin{aligned}2\frac{7}{15} - 1\frac{2}{3} &= (2-1) + \frac{7-10}{15} \\ &= 1 - \frac{3}{15} \\ &= \frac{12}{15} \\ &= \underline{\underline{\frac{4}{5}}}\end{aligned}$$

19. The diagram shows the graph of  $y = x^2 - 4x - 2$ .



- (a) Use the graph to find estimates for the solutions of (3)  
 (i)  $x^2 - 4x - 2 = 0$ ,

**Solution**

Approximately  $x = -0.4$  and  $x = 4.4$ .

- (ii)  $x^2 - 4x - 6 = 0$ .

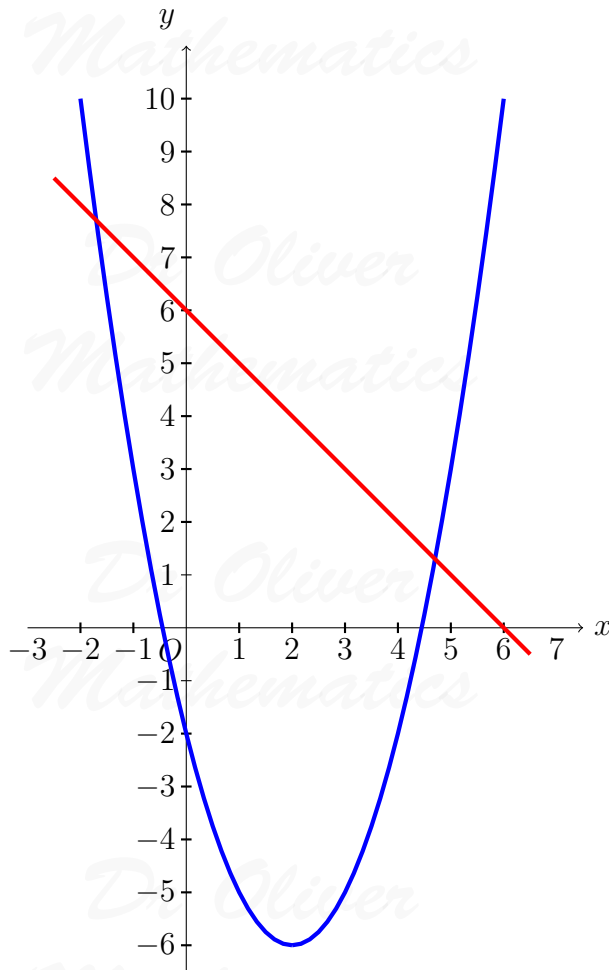
**Solution**

$$x^2 - 4x - 6 = 0 \Rightarrow x^2 - 4x - 2 = 4$$

so we need  $y = 4$ : approximately  $x = -1.2$  and  $x = 5.2$ .

- (b) Use the graph to find estimates for the values of  $x$  that satisfy the simultaneous equations  $y = x^2 - 4x - 2$  and  $x + y = 6$ . (3)

**Solution**



It is where the blue curve and the red line cross: approximately  $x = -1.8$  and  $x = 4.8$ .

20.  $P$ ,  $M$ , and  $S$  are points on a circle, centre  $O$ .

(5)



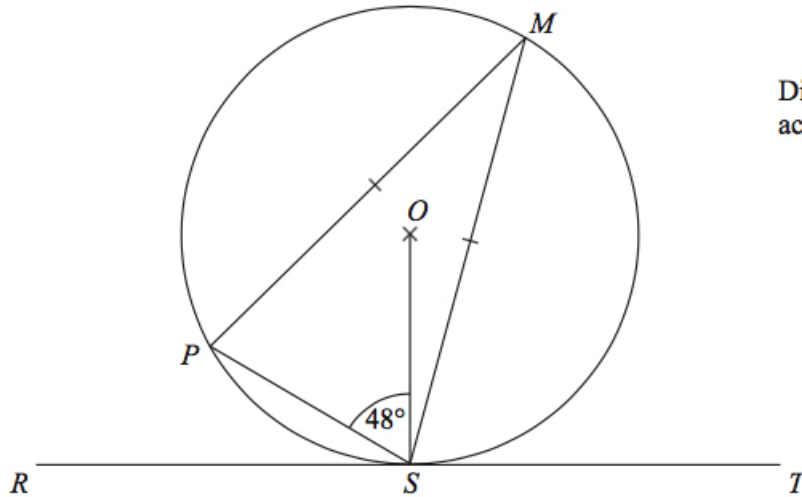


Diagram **NOT**  
accurately drawn

$RST$  is a tangent to the circle.

Angle  $PSO = 48^\circ$ .

$MP = MS$ .

Work out the size of angle  $MST$ .

Give reasons for each stage of your working.

### Solution

$\angle SPO = 48^\circ$  (base angles of an isosceles triangle are equal)

$\angle POS = 180 - 48 - 48 = 84^\circ$  (the third angle)

$\angle PMS = \frac{1}{2} \times 84 = 42^\circ$  (angle at the centre of a circle is twice the angle at the circumference)

$\angle OSM = \angle OMS = 21^\circ$  (half the angle)

$\angle MST = 90 - 21 = \underline{\underline{69^\circ}}$  (complementary angle)

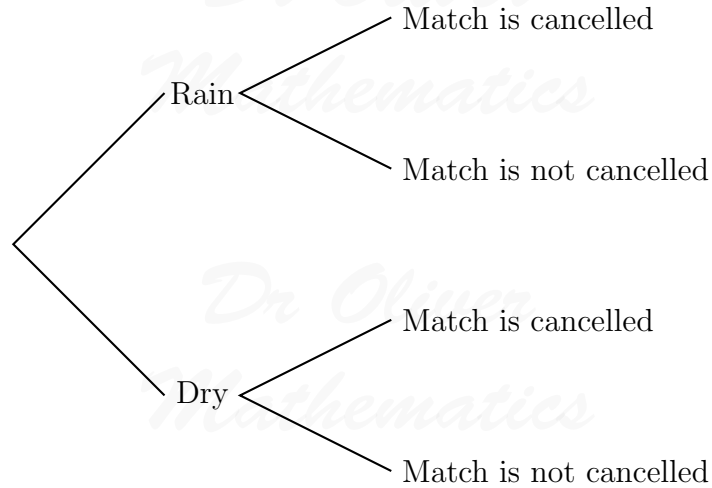
21. The probability that it will rain on a day in June is 0.2.

When it rains the probability that my tennis match is cancelled is 0.7.

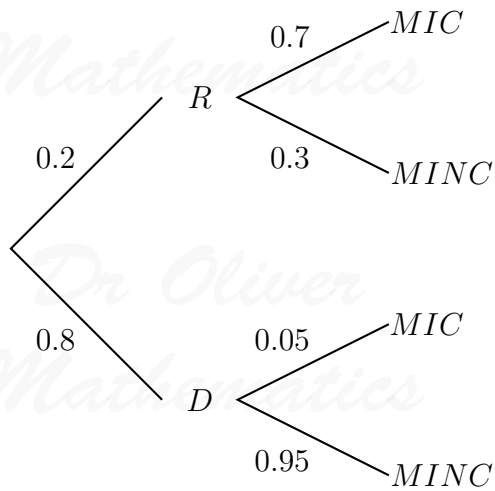
When it does not rain, the probability that my tennis match is **not** cancelled is 0.95.

(a) Complete the probability tree diagram for this information.

(3)



**Solution**



- (b) Work out the probability that, on a day in June, it does **not** rain and my tennis match is cancelled. (2)

**Solution**

$$0.8 \times 0.05 = \underline{\underline{0.04}}$$

22. Solve

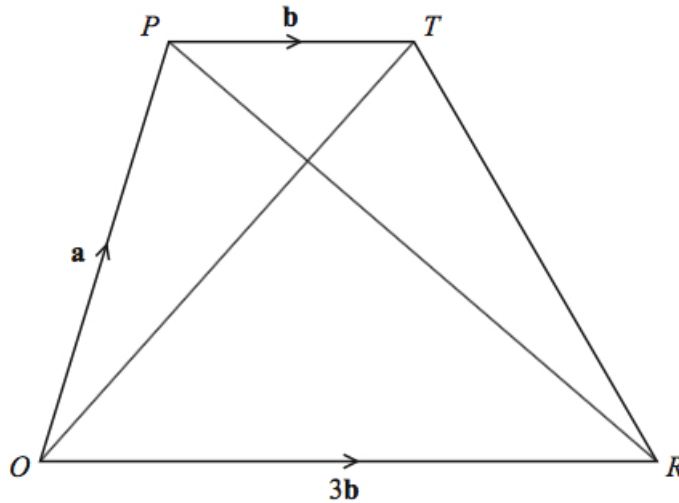
$$x^2 = 4(x - 3)^2.$$

(3)

**Solution**

$$\begin{aligned}
 x^2 &= 4(x - 3)^2 \Rightarrow x^2 = 4(x^2 - 6x + 9) \\
 &\Rightarrow x^2 = 4x^2 - 24x + 36 \\
 &\Rightarrow 3x^2 - 24x + 36 = 0 \\
 &\Rightarrow x^2 - 8x + 12 = 0 \\
 &\Rightarrow (x - 2)(x - 6) = 0 \\
 &\Rightarrow \underline{x = 2 \text{ or } x = 6.}
 \end{aligned}$$

23.  $OPTR$  is a trapezium.



$$\begin{aligned}
 \overrightarrow{OP} &= \mathbf{a}. \\
 \overrightarrow{PT} &= \mathbf{b}. \\
 \overrightarrow{OR} &= 3\mathbf{b}.
 \end{aligned}$$

(a) (i) Find  $\overrightarrow{OT}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

(2)

**Solution**

$$\overrightarrow{OT} = \overrightarrow{OP} + \overrightarrow{PT} = \underline{\underline{\mathbf{a} + \mathbf{b}}}.$$

(ii) Find  $\overrightarrow{PR}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

Give your answer in its simplest form.

**Solution**

$$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} = \underline{\underline{3\mathbf{b} - \mathbf{a}}}.$$

$S$  is the point on  $PR$  such that  $PS : SR = 1 : 3$ .

(b) Find  $\overrightarrow{OS}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

Give your answer in its simplest form.

(2)

**Solution**

$$\begin{aligned}\overrightarrow{OS} &= \overrightarrow{OP} + \overrightarrow{PS} \\ &= \mathbf{a} + \frac{1}{4}\overrightarrow{PR} \\ &= \mathbf{a} + \frac{1}{4}(3\mathbf{b} - \mathbf{a}) \\ &= \frac{3}{4}\mathbf{a} + \frac{3}{4}\mathbf{b} \\ &= \underline{\underline{\frac{3}{4}(\mathbf{a} + \mathbf{b})}}.\end{aligned}$$

(c) What does your answer to part (b) tell you about the position of point  $S$ ?

(2)

**Solution**

$$\overrightarrow{OS} = \frac{3}{4}(\mathbf{a} + \mathbf{b}) = \frac{3}{4}\overrightarrow{OT}$$

and

$$\underline{\underline{OS = \frac{3}{4}OT.}}$$

24. Given that  $y \propto \frac{1}{x^2}$ , complete this table of values.

(4)

$x$	1	2	5	10
$y$				1

**Solution**

$$y = \frac{k}{x^2}$$

for some constant  $k$ . Then

$$1 = \frac{k}{10^2} \Rightarrow k = 100$$

and

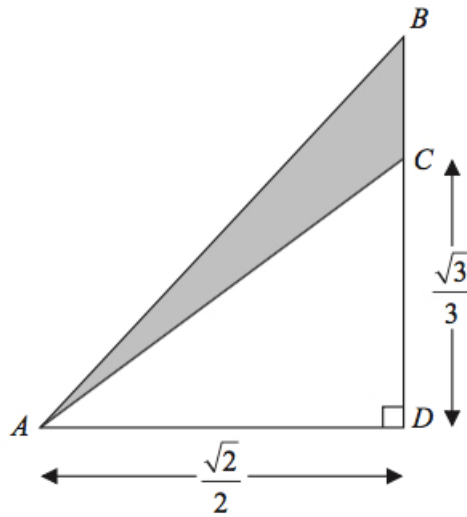
$$y = \frac{100}{x^2}.$$

And so the table is

$x$	1	2	5	10
$y$	<u>100</u>	<u>25</u>	<u>4</u>	1

25.  $ABD$  is a right angled triangle.

(3)



All measurements are given in centimetres.

$C$  is the point on  $BD$  such that  $CD = \frac{\sqrt{3}}{3}$ .

$AD = BD = \frac{\sqrt{2}}{2}$ .

Work out the exact area, in  $\text{cm}^2$ , of the shaded region.

**Solution**

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{3} \\ &= \frac{\sqrt{2}}{4} \left( \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3} \right) \\ &= \frac{\sqrt{2}}{4} \left( \frac{3\sqrt{2} - 2\sqrt{3}}{6} \right) \\ &= \frac{6 - 2\sqrt{6}}{24} \\ &= \frac{3 - \sqrt{6}}{12}. \end{aligned}$$