Dr Oliver Mathematics
GCSE Mathematics
2023 November Paper 2H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. (a) Expand and simplify (2)

$$3(2y - 5) + 7(y + 2).$$

Solution

$$\begin{aligned} 3(2y - 5) + 7(y + 2) &= 6y - 15 + 7y + 14 \\ &= \underline{\underline{13y - 1}}. \end{aligned}$$

(b) Factorise fully (2)

$$6x^2 + 15x.$$

Solution

$$6x^2 + 15x = \underline{\underline{3x(2x + 5)}}.$$

(c) Make g the subject of the formula (2)

$$f = 3g + 11.$$

Solution

$$\begin{aligned} f = 3g + 11 \Rightarrow 3g &= f - 11 \\ \Rightarrow \underline{\underline{g = \frac{f - 11}{3}}}. \end{aligned}$$

2. Karen is organising a party for a charity. (4)

She spends

- £100 on food,
- £120 on a hall, and
- £80 on a DJ.

Karen sells 54 tickets for the party.
Each ticket costs £7.50.

Work out the percentage profit Karen makes for the charity.

Solution

Well,

$$\begin{aligned} \text{profit} &= (54 \times 7.50) - (100 + 120 + 80) \\ &= 405 - 300 \\ &= 105 \end{aligned}$$

and so

$$\begin{aligned} \text{percentage profit} &= \left(\frac{405 - 300}{300}\right) \times 100\% \\ &= \tfrac{7}{20} \times 100\% \\ &= \underline{\underline{35\%}}. \end{aligned}$$

3. Andrew invests £4 500 in a savings account for 2 years. (2)
The account pays compound interest at a rate of 3.4% per year.

Calculate how much Andrew has in this savings account at the end of the 2 years.

Solution

$$\begin{aligned} \text{At the end of the 2 years} &= 4\,500 \times (1.034)^2 \\ &= 4\,811.202 \text{ (exact)} \\ &= \underline{\underline{\pounds 4\,811.20 \text{ (2 dp)}}}. \end{aligned}$$

4. Solve (3)

$$5x - 14 = 52 - x.$$

Solution

$$\begin{aligned} 5x - 14 = 52 - x &\Rightarrow 6x = 66 \\ &\Rightarrow \underline{\underline{x = 11}}. \end{aligned}$$

5. Chris, Debbie, and Errol share some money in the ratio 3 : 4 : 2. (4)
Debbie gets £120.

Chris then gives some of his share to Debbie and some of his share to Errol.
The money that Chris, Debbie and Errol each have is now in the ratio 2 : 5 : 3.

How much money did Chris give to Errol?

Solution

Well,

$$3 + 4 + 2 = 9$$

and so

$$\begin{aligned} \text{Chris gets} &= \tfrac{3}{4} \times 120 \\ &= 90, \\ \text{Errol gets} &= \tfrac{2}{4} \times 120 \\ &= 60. \end{aligned}$$

The whole amount is

$$120 + 90 + 60 = \pounds 270.$$

Then,

$$2 + 5 + 3 = 10,$$

after the transfer,

$$\begin{aligned} \text{Errol gets} &= \tfrac{3}{10} \times 270 \\ &= 81 \end{aligned}$$

so he gives

$$81 - 60 = \underline{\underline{\pounds 21}}$$

to Errol.

6. The bearing of port B from port A is $147°$. (2)

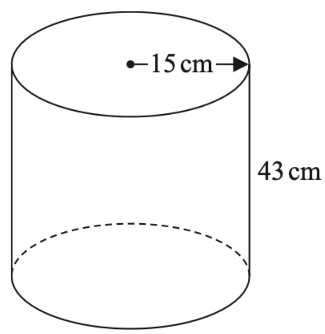
Work out the bearing of port A from port B.

Solution

Well,

$$\begin{aligned} \text{bearing} &= 360 - (180 - 147) \\ &= 360 - 33 \\ &= \underline{\underline{327°}}. \end{aligned}$$

7. The diagram shows an empty tank in the shape of a cylinder. (4)



The cylinder has radius 15 cm and height 43 cm.

Water flows into the tank at a rate of 0.47 litres per minute.

Calculate the number of minutes it will take to completely fill the tank.
Give your answer correct to the nearest minute.

Solution

Well, water flows into the tank at a rate of 470 cm^3 per minute.. Now,

$$\begin{aligned} \text{time taken} &= \frac{\pi \times 15^2 \times 43}{470} \\ &= 64.670\,018\,99 \text{ (FCD)} \\ &= \underline{\underline{65 \text{ mins (nearest minute)}}}. \end{aligned}$$

8. A number x is written correct to 2 significant figures. (2)

The result is 1.9.

Complete the error interval for x.

$$\ldots\ldots\ldots\ldots \leqslant x < \ldots\ldots\ldots\ldots$$

Solution

$$\underline{\underline{1.85}} \leqslant x < \underline{\underline{1.95}}$$

9. Expand and simplify (3)

$$(x+7)(x-2)(x+3).$$

Solution

Well,

$\times$	x	$+7$
x	x^2	$+7x$
-2	$-2x$	-14

so

$$(x+7)(x-2) = x^2 + 5x - 14.$$

Now,

$\times$	x^2	$+5x$	-14
x	x^3	$+5x^2$	$-14x$
$+3$	$+3x^2$	$+15x$	-42

so

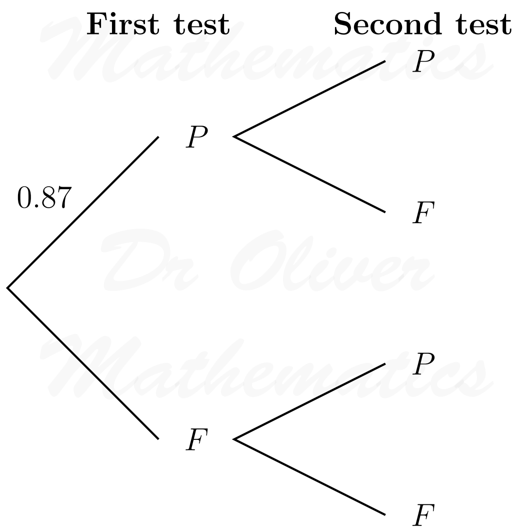
$$(x+7)(x-2)(x+3) = \underline{\underline{x^3 + 8x^2 + x - 42}}.$$

10. Shakir has to complete two tests.
He can either pass or fail each test.

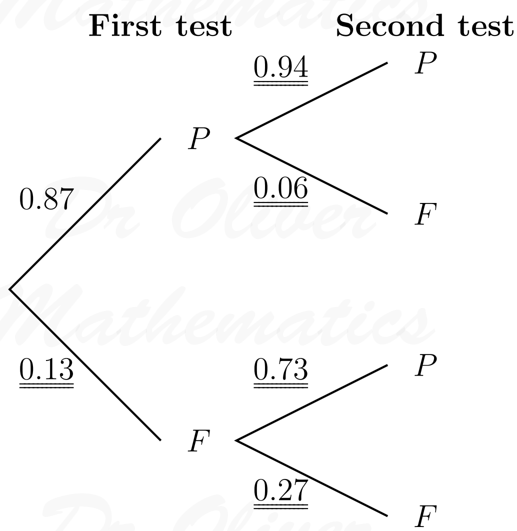
The probability that he will pass the first test is 0.87.

If he passes the first test the probability he will pass the second test is 0.94.
If he fails the first test the probability he will pass the second test is 0.73

(a) Complete the probability tree diagram for this information. (2)



Solution

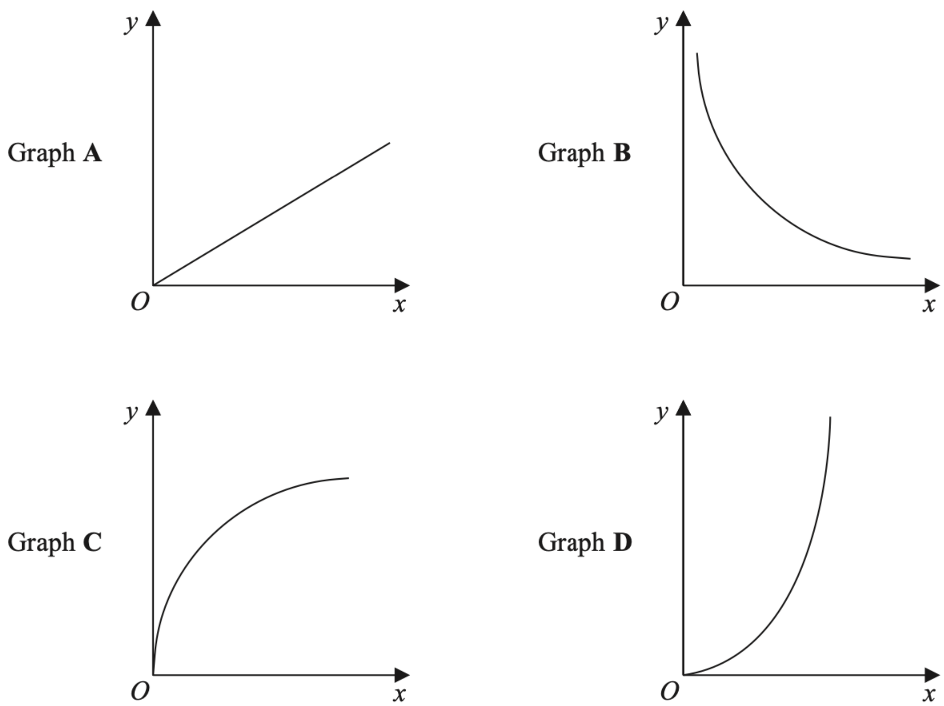


(b) Work out the probability that Shakir passes at least one of the tests. (3)

Solution

$$
\begin{aligned}
\text{P(passes at least one of the tests)} &= 1 - \text{P}(F, F)) \\
&= 1 - (0.13 \times 0.27) \\
&= 1 - 0.035\,1 \\
&= \underline{\underline{0.964\,9}}.
\end{aligned}
$$

11. The graphs of y against x represent four different types of proportionality. (2)



Match each type of proportionality in the table to the correct graph.

Proportionality	Graph
$y \propto x^2$	
$y \propto x$	
$y \propto \dfrac{1}{x}$	
$y \propto \sqrt{x}$	

Solution

Proportionality	Graph
$y \propto x^2$	$\underline{\underline{D}}$
$y \propto x$	$\underline{\underline{A}}$
$y \propto \frac{1}{x}$	$\underline{\underline{B}}$
$y \propto \sqrt{x}$	$\underline{\underline{C}}$

12. A is the point with coordinates $(7, 13)$. (3)
B is the point with coordinates $(-3, 21)$.
C is the point with coordinates $(15, 23)$.

M is the midpoint of AB.
N is the midpoint of BC.

Work out the distance between M and N.
Give your answer correct to 1 decimal place.

Solution

Well, M has coordinates

$$\left(\frac{7+(-3)}{2}, \frac{13+21}{2}\right) = (2, 17)$$

and N has coordinates

$$\left(\frac{15+(-3)}{2}, \frac{23+21}{2}\right) = (6, 22).$$

Finally,

$$\begin{aligned} MN &= \sqrt{(6-2)^2 + (22-17)^2} \\ &= \sqrt{4^2 + 5^2} \\ &= \sqrt{41} \\ &= \underline{\underline{6.4 \text{ (1 dp)}}}. \end{aligned}$$

13. Prove algebraically that $0.0\dot{7}2\dot{3}$ can be written as $\frac{241}{3\,330}$. (3)

Solution

Let $x = 0.0\dot{7}2\dot{3}$. Then

$$\begin{aligned} 10x &= 0.\dot{7}2\dot{3} \quad (1) \\ 10\,000x &= 723.\dot{7}2\dot{3} \quad (2) \end{aligned}$$

and do (2) − (1):

$$\begin{aligned} 9\,900x = 723 &\Rightarrow x = \tfrac{723}{9\,900} \\ &\Rightarrow x = \tfrac{241 \times 3}{3\,300 \times 3} \\ &\Rightarrow \underline{\underline{x = \tfrac{241}{3\,300}}}, \end{aligned}$$

as required.

14. y is proportional to x^2. (5)
$y = 3$ when $x = 0.5$.

x is inversely proportional to w.
$x = 2$ when $w = 0.2$.

Find the value of y when $w = 2$.

Solution

Now,

$$y \propto x^2 \Rightarrow y = kx^2,$$

for some constant k. Next,

$$\begin{aligned} x = 0.5, y = 3 &\Rightarrow 3 = k(0.5)^2 \\ &\Rightarrow 3 = 0.25k \\ &\Rightarrow k = 12; \end{aligned}$$

hence, $y = 12x^2$.

Now,

$$x \propto \frac{1}{w} \Rightarrow x = \frac{l}{w},$$

for some constant l. Next,

$$\begin{aligned} x = 2, w = 0.2 &\Rightarrow 2 = \frac{l}{0.2} \\ &\Rightarrow l = 0.4; \end{aligned}$$

hence, $x = \dfrac{0.4}{w}$.

Putting it the two equations together,

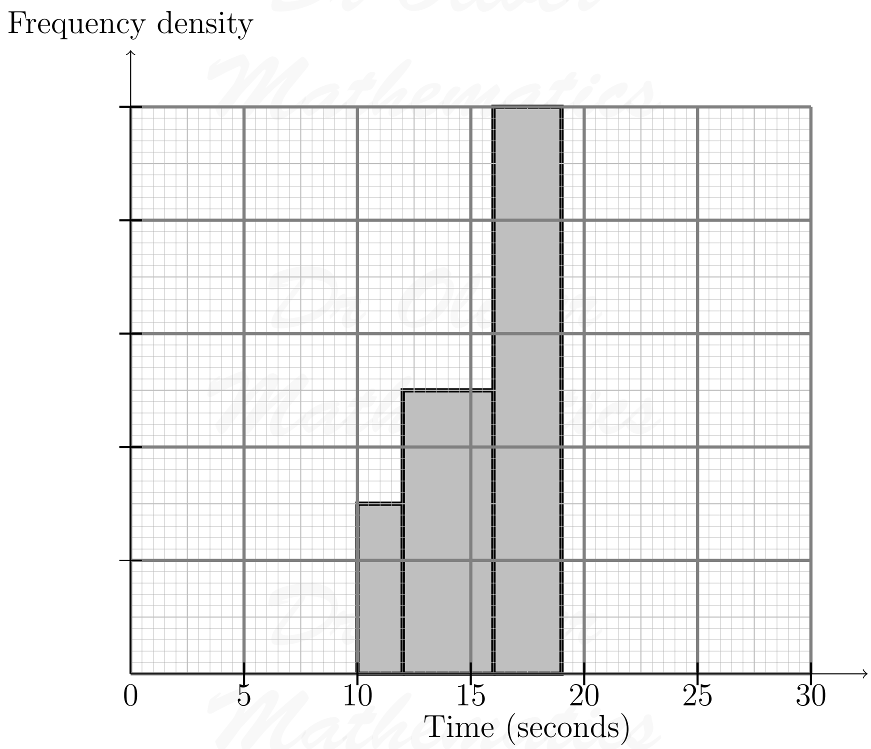
$$\begin{aligned} y &= 12x^2 \\ &= 12\left(\frac{0.4}{w}\right)^2 \\ &= 12\left(\frac{0.16}{w^2}\right) \\ &= \frac{1.92}{w^2} \end{aligned}$$

and, finally,

$$w = 2 \Rightarrow \underline{\underline{y = 0.48}}.$$

15. The incomplete table and the incomplete histogram give information about the times taken by some students to run a race.

Time (t seconds)	Frequency
$10 < t \leqslant 12$	
$12 < t \leqslant 16$	10
$16 < t \leqslant 19$	15
$19 < t \leqslant 21$	9
$21 < t \leqslant 26$	7



None of these students had a time for the race such that $t \leqslant 10$ or $t > 26$.

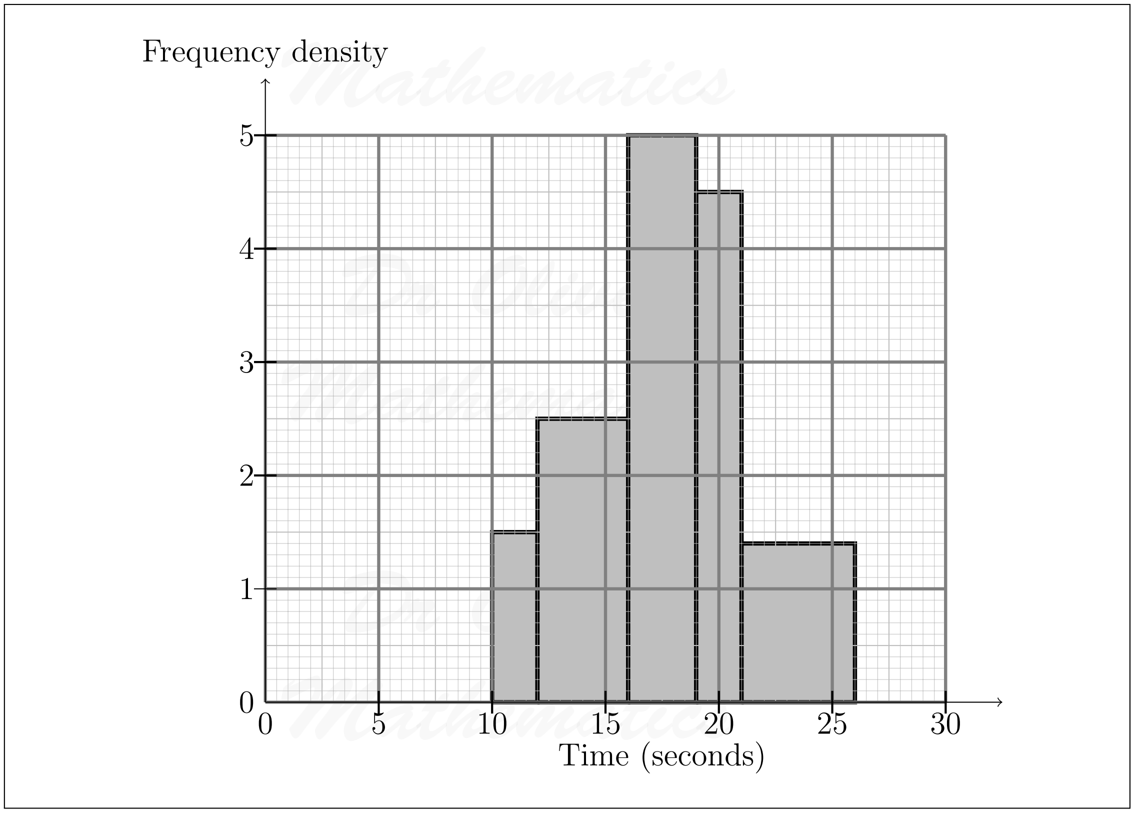
(a) Use the histogram to complete the table. (1)

Solution

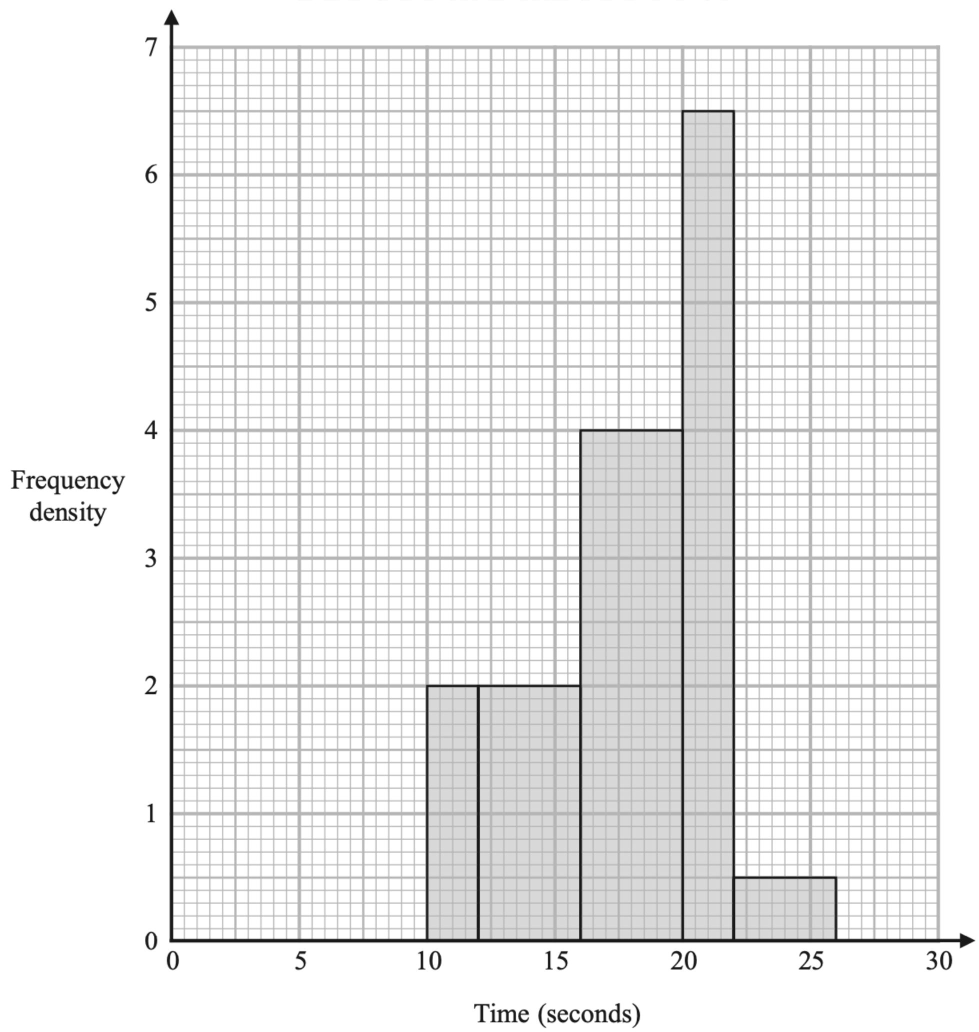
Time (t seconds)	Frequency	Width	Frequency Density
$10 < t \leqslant 12$	$2 \times 1.5 = \underline{\underline{3}}$	2	1.5
$12 < t \leqslant 16$	10	4	$\frac{10}{4} = 2.5$
$16 < t \leqslant 19$	15	3	$\frac{15}{3} = 5$
$19 < t \leqslant 21$	9	2	$\frac{9}{4} = 4.5$
$21 < t \leqslant 26$	7	5	$\frac{7}{5} = 1.4$

(b) Use the table to complete the histogram. (2)

Solution


Frequency density
5
4
3
2
1
0
0
5
10
15
20
25
30
Time (seconds)

The histogram below gives information about the times taken by 43 students to run a different race.



(c) Work out an estimate for the median of the times taken by these 43 students to run the race. (3)

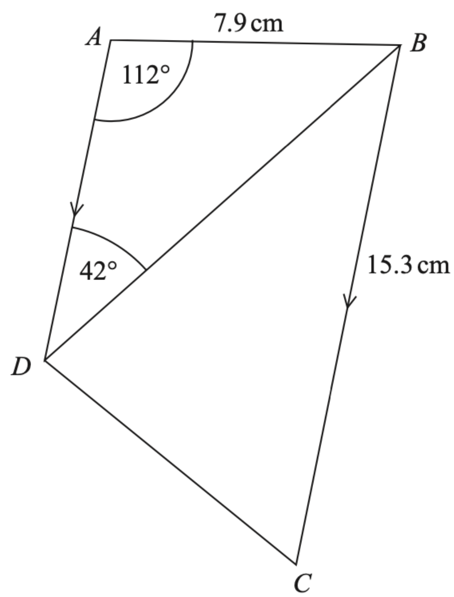
Solution

Now,

Time (t seconds)	Frequency Density	Width	Frequency	Cum Freq
$10 < t \leqslant 12$	2	2	$2 \times 2 = 4$	2
$12 < t \leqslant 16$	2	4	$2 \times 4 = 8$	$2 + 8 = 12$
$16 < t \leqslant 20$	4	4	$4 \times 4 = 16$	$12 + 16 = 28$
$20 < t \leqslant 22$	6.5	2	$6.5 \times 2 = 13$	$28 + 13 = 41$
$22 < t \leqslant 26$	0.5	4	$0.5 \times 4 = 2$	$41 + 2 = 43$

The median is at the
$$\frac{43+1}{2} = 22\text{th place}$$
and, hence,
$$\begin{aligned} \text{median} &= 16 + \left(\frac{22-12}{16}\right) \times 4 \\ &= \underline{\underline{18.5 \text{ secs}}}. \end{aligned}$$

16. $ABCD$ is a trapezium. (4)



AD is parallel to BC.

Calculate the area of triangle BCD.
Give your answer correct to 1 decimal place.

Solution

Sine rule:

$$\frac{BD}{\sin BAD} = \frac{AB}{\sin ADB} \Rightarrow \frac{BD}{\sin 112^\circ} = \frac{7.9}{\sin 42^\circ}$$
$$\Rightarrow BD = \frac{7.9 \sin 112^\circ}{\sin 42^\circ}$$
$$\Rightarrow BD = 10.949\,670\,77 \text{ (FCD)}.$$

Now, $\angle DBC = \sin 42^\circ$ (alternate angles) and we finish with the area of a triangle:

$$\begin{aligned} \text{area} &= \tfrac{1}{2} \times BD \times BC \times \angle DBC \\ &= \tfrac{1}{2} \times 10.949\ldots \times 15.3 \times \sin 42^\circ \\ &= 56.034\,356\,25 \text{ (FCD)} \\ &= \underline{\underline{56.0 \text{ cm}^2 \text{ (3 sf)}}}. \end{aligned}$$

17. (a) Show that the equation (2)

$$x^3 + 2x - 6 = 0$$

has a solution between $x = 1$ and $x = 2$.

Solution

Let

$$\mathrm{f}(x)x^3 + 2x - 6.$$

Now,

$$\begin{aligned} \mathrm{f}(1) &= 1 + 2 - 6 = -3 \\ \mathrm{f}(2) &= 8 + 4 - 6 = 6; \end{aligned}$$

as $\mathrm{f}(x)$ is continuous it follows that $\mathrm{f}(x)$ a solution between $x = 1$ and $x = 2$.

(b) Show that the equation (1)

$$x^3 + 2x - 6 = 0$$

can be rearranged to give

$$x = \frac{6}{x^2 + 2}.$$

Solution

Well,

$$\begin{aligned} x^3 + 2x - 6 = 0 &\Rightarrow x^3 + 2x = 6 \\ &\Rightarrow x(x^2+2) = 6 \\ &\Rightarrow \underline{\underline{x = \frac{6}{x^2+2}}}, \end{aligned}$$

as required.

(c) Starting with $x_0 = 1.45$, use the iteration formula (3)

$$x_{n+1} = \frac{6}{x_n^2 + 2}$$

twice to find an estimate for the solution of

$$x^3 + 2x - 6 = 0.$$

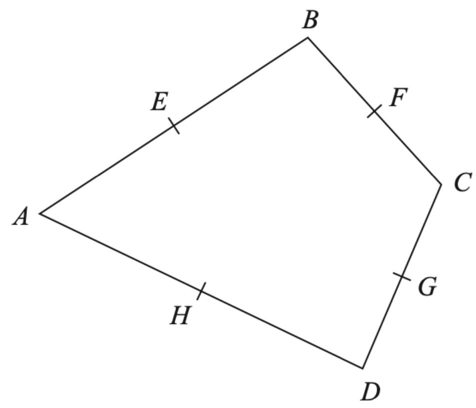
Give your answer correct to 4 decimal places.

Solution

Now,

$$\begin{aligned} x_1 &= \frac{6}{x_0^2+2} \\ &= \frac{6}{1.45^2+2} \\ &= 1.462\,522\,852 \text{ (FCD)}, \\ x_2 &= \frac{6}{x_1^2+2} \\ &= \frac{6}{1.462\ldots^2+2} \\ &= 1.449\,634\,937 \text{ (FCD)} \\ &= \underline{\underline{1.449\,6 \text{ (4 dp)}}}. \end{aligned}$$

18. $ABCD$ is a quadrilateral. (4)



- E, F, G, and H are the midpoints of AB, BC, CD, and DA.
- $\overrightarrow{AH} = \mathbf{a}$.
- $\overrightarrow{AE} = \mathbf{b}$.
- $\overrightarrow{DG} = \mathbf{c}$.

Prove, using vectors, that $EFGH$ is a parallelogram.

Solution

Well,

$$\begin{aligned}\overrightarrow{HE} &= \overrightarrow{HA} + \overrightarrow{AE}\\ &= -\overrightarrow{AH} + \overrightarrow{AE}\\ &= -\mathbf{a} + \mathbf{b},\end{aligned}$$

$$\begin{aligned}\overrightarrow{AC} &= \overrightarrow{AD} + \overrightarrow{DC}\\ &= 2\overrightarrow{AH} + 2\overrightarrow{DG}\\ &= 2\mathbf{a} + 2\mathbf{c},\end{aligned}$$

$$\begin{aligned}\overrightarrow{AB} &= 2\overrightarrow{AE}\\ &= 2\mathbf{b},\end{aligned}$$

$$\begin{aligned}\overrightarrow{CF} &= \tfrac{1}{2}\overrightarrow{CB}\\ &= \tfrac{1}{2}(\overrightarrow{CA} + \overrightarrow{AB})\\ &= \tfrac{1}{2}(-\overrightarrow{AC} + \overrightarrow{AB})\\ &= \tfrac{1}{2}(-2\mathbf{a} - 2\mathbf{c} + 2\mathbf{b})\\ &= -\mathbf{a} + \mathbf{b} - \mathbf{c},\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{GF} &= \overrightarrow{GC} + \overrightarrow{CF} \\ &= \mathbf{c} + (-\mathbf{a} + \mathbf{b} - \mathbf{c}) \\ &= -\mathbf{a} + \mathbf{b}.\end{aligned}$$

So, $\overrightarrow{HE} = \overrightarrow{GF}$ and, hence, $EFGH$ is a parallelogram.

19. The functions f and g are such that

$$\mathrm{f}(x) = (2x+3)^2 \text{ and } \mathrm{g}(x) = 2x - 1.$$

(a) Find $\mathrm{g}\,\mathrm{f}(-3)$. (2)

Solution
Now,

$$\begin{aligned}\mathrm{g}\,\mathrm{f}(-3) &= \mathrm{g}(\mathrm{f}(-3)) \\ &= \mathrm{g}(9) \\ &= 17.\end{aligned}$$

(b) Find $\mathrm{g}^{-1}(x)$. (2)

Solution
Well,

$$\begin{aligned}y = 2x - 1 &\Rightarrow y + 1 = 2x \\ &\Rightarrow \frac{y+1}{2} = x,\end{aligned}$$

and so

$$\mathrm{g}^{-1}(x) = \frac{x+1}{2}.$$

20. Write (4)

$$\frac{14}{3x-21} + \left[(x+4) \div \frac{2x^2-6x-56}{2x+3}\right]$$

in the form

$$\frac{ax+d}{cx+d},$$

where a, b, c, and d are integers.

Solution

Well,

$$2x^2 - 6x - 56 = 2(x^2 - 3x - 28)$$

$$\left.\begin{array}{ll}\text{add to:} & -3 \\ \text{multiply to:} & -28\end{array}\right\} -7,\ +4$$

$$= 2(x-7)(x+4).$$

Now,

$$\begin{aligned}(x+4) \div \frac{2x^2-6x-56}{2x+3} &= (x+4) \div \frac{2(x-7)(x+4)}{2x+3} \\ &= (x+4) \times \frac{2x+3}{2(x-7)(x+4)} \\ &= \frac{2x+3}{2(x-7)}.\end{aligned}$$

Finally,

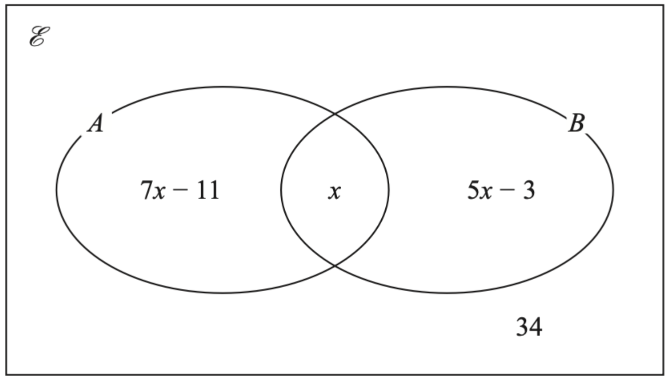
$$\begin{aligned}\frac{14}{3x-21} + \left[(x+4) \div \frac{2x^2-6x-56}{2x+3}\right] &= \frac{14}{3(x-7)} + \frac{2x+3}{2(x-7)} \\ &= \frac{28}{6(x-7)} + \frac{6x+9}{6(x-7)} \\ &= \underline{\underline{\frac{6x+37}{6x-42}}};\end{aligned}$$

hence, $\underline{\underline{a=6}}$, $\underline{\underline{b=37}}$, $\underline{\underline{c=6}}$, and $\underline{\underline{d=-42}}$.

21. Vicky has a collection of medals. (4)

The V enn diagram gives information about the number of medals in her collection where

- $\mathscr{E}$ = {all medals},
- A = {English medals}, and
- B = {gold medals}.



Vicky is going to take at random a medal from her collection.

Given that the medal is gold, the probability that the medal is English is $\frac{2}{11}$.

Work out the number of medals in Vicky's collection.

Solution

Well,

$$\begin{aligned}\frac{x}{x+(5x-3)} = \tfrac{2}{11} \Rightarrow \frac{x}{6x-3} &= \tfrac{2}{11} \\ \Rightarrow 11x &= 2(6x-3) \\ \Rightarrow 11x &= 12x - 6 \\ \Rightarrow x &= 6.\end{aligned}$$

Finally,

$$\begin{aligned}\text{medals} &= (7\times 6 - 11) + 6 + (5 \times 6 - 3) + 34 \\ &= 31 + 6 + 27 + 34 \\ &= \underline{\underline{98}}.\end{aligned}$$