Dr Oliver Mathematics
Mathematics
Factors
Past Examination Questions

This booklet consists of 17 questions across a variety of examination topics.
The total number of marks available is 89.

1. Factorise completely (3)

$$x^3 - 4x^2 + 3x.$$

2. Given that

$$f(x) \equiv (x^2 - 6x)(x - 2) + 3x,$$

   (a) express $f(x)$ in the form $x(ax^2 + bx + c)$, where a, b, and c are constants. (3)

   (b) Hence factorise $f(x)$ completely. (2)

   (c) Sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes. (3)

3. (a) On the same axes sketch the graphs of the curves with equations and indicate on your sketches the coordinates of all the points where the curves cross the x-axis.

   (i) $y = x^2(x - 2)$, (3)

   (ii) $y = x(6 - x)$. (3)

   (b) Use algebra to find the coordinates of the points where the graphs intersect. (7)

4. The curve C has equation

$$y = (x + 3)(x - 1)^2.$$

   (a) Sketch C showing clearly the coordinates of the points where the curve meets the coordinate axes. (4)

   (b) Show that the equation of C can be written in the form (2)

$$y = x^3 + x^2 - 5x + k,$$

   where k is a positive integer, and state the value of k.

5. Factorise completely (3)

$$x^3 - 9x.$$

6. (a) Factorise completely $x^3 - 6x^2 + 9x$. (3)

(b) Sketch the curve with equation (4)

$$y = x^3 - 6x^2 + 9x,$$

showing the coordinates of the points at which the curve meets the x-axis.

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation (2)

$$y = (x-2)^3 - 6(x-2)^2 + 9(x-2),$$

showing the coordinates of the points at which the curve meets the x-axis.

7. (a) Factorise completely $x^3 - 4x$. (3)

(b) Sketch the curve C with equation (3)

$$x^3 - 4x,$$

showing the coordinates of the points at which the curve meets the x-axis.

8. (a) On the axes below sketch the graphs of the following two graphs, showing clearly the coordinates of the points where the curves cross the coordinate axes. (5)

(i) $y = x(4-x)$,

(ii) $y = x^2(7-x)$.

(b) Show that the x-coordinates of the points of intersection of (3)

$$y = x(4-x) \text{ and } y = x^2(7-x)$$

are given by the solutions to the equation

$$x(x^2 - 8x + 4) = 0.$$

9. Sketch the graph of (3)

$$y = x(x+2)(3-x).$$

10. Sketch (4)

$$y = (x+1)(x+3)^2,$$

showing the coordinates of the points at which it meets the axes.

11. The curve C has equation $y = x(5-x)$ and the line L has equation $2y = 5x + 4$.

(a) Use algebra to show that C and L do not intersect. (4)

(b) Sketch C and L on the same diagram, showing the coordinates of the points at which C and L meet the axes. (4)

12. The curve C_1 has equation

$$y = x^2(x+2).$$

(a) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis. (3)

The curve C_2 has equation

$$y = (x-k)^2(x-k+2),$$

where k is a constant and $k > 2$.

(b) Sketch C_2, showing the coordinates of the points where C_2 meets the x- and y-axes. (3)

13. Factorise completely $x - 4x^3$. (3)

14. Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.

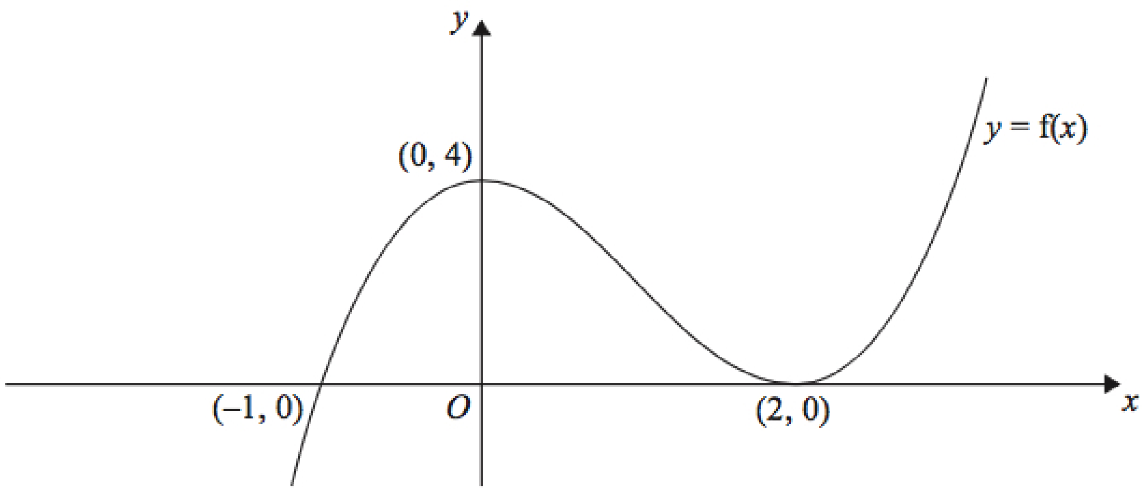

Figure 1: $y = \mathrm{f}(x)$

The curve C passes through the point $(-1, 0)$ and touches the x-axis at the point $(2, 0)$. The curve C has a maximum at the point $(0, 4)$. The equation of the curve C can be written in the form

$$y = x^3 + ax^2 + bx + c,$$

where a, b, and c are integers. Calculate the values of a, b, and c

15. The curve C has equation (3)

$$y = x^2(x-2),$$

for all real values of x. Sketch a graph of curve C. Show on the sketch the coordinates of each point where the curve C crosses the coordinate axes.

16. Factorise fully $25x - 9x^3$.

17. (a) Factorise completely $9x - 4x^3$. (3)

(b) Sketch the curve C with equation (3)

$$y = 9x - 4x^3.$$

Show on your sketch the coordinates at which the curve meets the x-axis.