Dr Oliver Mathematics
GCSE Mathematics
2022 June Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Solve (2)

$$7x - 27 < 8.$$

Solution

$$7x - 27 < 8 \Rightarrow 7x < 35$$
$$\Rightarrow \underline{\underline{x < 5}}.$$

2. Write 124 as a product of its prime factors. (2)

Solution

	124
2	62
2	31
31	1

So

$$124 = 2 \times 2 \times 31 = \underline{\underline{2^2 \times 31}}.$$

3. A delivery company has a total of 160 cars and vans. (5)

The number of cars : the number of vans = 3 : 7.

Each car and each van uses electricity or diesel or petrol.

- $\frac{1}{8}$ of the cars use electricity.

- 25% of the cars use diesel.
- The rest of the cars use petrol.

Work out the number of cars that use petrol.
You must show all your working.

Solution

Well,

$$3 + 7 = 10$$

so

$$\begin{aligned}\text{the number of cars} &= \tfrac{3}{10} \times 160 \\ &= \tfrac{480}{10} \\ &= 48.\end{aligned}$$

"$\frac{1}{8}$ of the cars use electricity":

$$\frac{48}{8} = 6.$$

"25% of the cars use diesel":

$$\tfrac{1}{4} \times 48 = 12.$$

So,

$$\begin{aligned}\text{the number of cars that use petrol} &= 48 - (6 + 12) \\ &= 48 - 18 \\ &= \underline{\underline{30}}.\end{aligned}$$

4. (a) Write (1)

$$1.63 \times 10^{-3}$$

as an ordinary number.

Solution

$$1.63 \times 10^{-3} = \underline{\underline{0.001\,63}}.$$

(b) Write (1)

$$438\,000$$

in standard form.

Solution

$$438\,000 = \underline{\underline{4.38 \times 10^5}}.$$

(c) Work out (2)

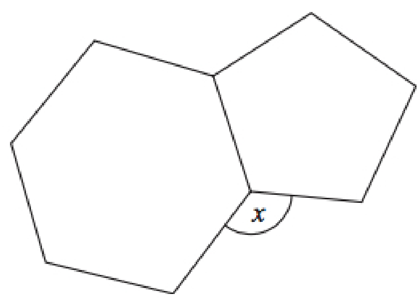
$$(4 \times 10^3) \times (6 \times 10^{-5}).$$

Give your answer in standard form.

Solution

$$\begin{aligned}(4 \times 10^3) \times (6 \times 10^{-5}) &= (4 \times 6) \times (10^3 \times 10^{-5}) \\ &= 24 \times 10^{-2} \\ &= \underline{\underline{2.4 \times 10^{-1}}}.\end{aligned}$$

5. Here is a regular hexagon and a regular pentagon. (3)



Work out the size of the angle marked x.
You must show all your working.

Solution

Well, the angle in a regular pentagon is

$$\begin{aligned}180 - \frac{360}{5} &= 180 - 72 \\ &= 108\end{aligned}$$

and the angle in a regular hexagon is

$$180 - \frac{360}{6} = 180 - 60$$
$$= 120.$$

Finally,

$$x + 108 + 120 = 360 \Rightarrow x + 228 = 360$$
$$\Rightarrow \underline{\underline{x = 132^\circ}}.$$

6. (a) Complete the table of values for (2)

$$y = x^2 - 3x + 1.$$

x	-1	0	1	2	3	4
y		1	-1			

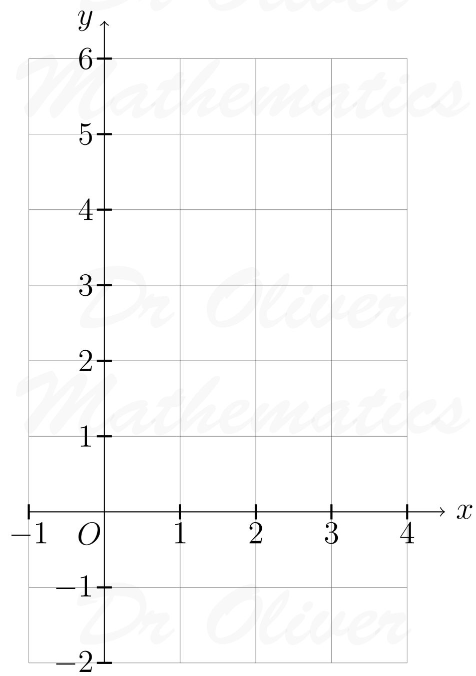
Solution

x	-1	0	1	2	3	4
y	$\underline{\underline{5}}$	1	-1	$\underline{\underline{-1}}$	$\underline{\underline{1}}$	$\underline{\underline{5}}$

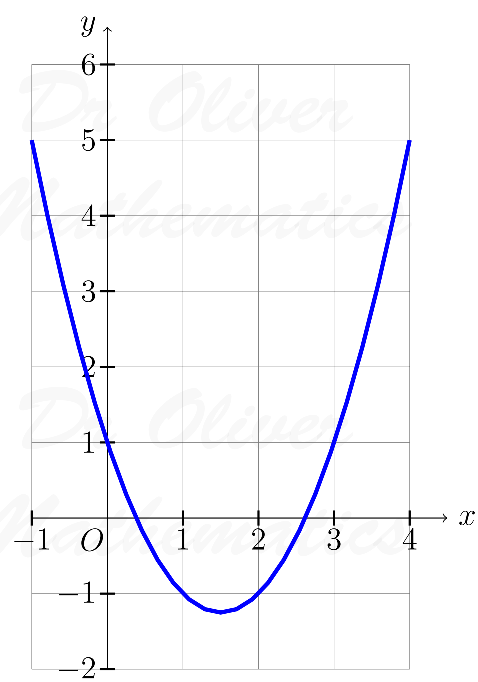
(b) On the grid, draw the graph of (2)

$$y = x^2 - 3x + 1$$

for values of x from -1 to 4.



Solution



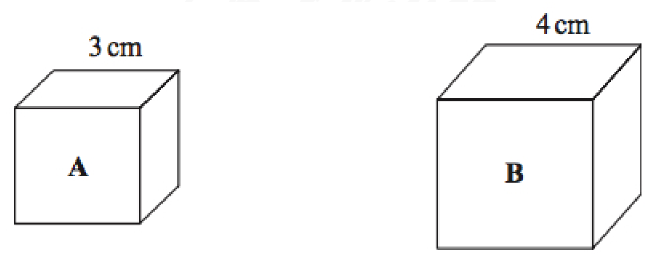
(c) Using your graph, find estimates for the solutions of the equation (2)

$$x^2 - 3x + 1 = 0.$$

Solution

Correct read-off: about $\underline{\underline{x = 0.4}}$ or $\underline{\underline{x = 2.6}}$.

7. Here are two cubes, **A** and **B**. (3)



Cube **A** has a mass of 81 g.

Cube **B** has a mass of 128 g.

Work out

$$\text{the density of cube } A : \text{the density of cube } B.$$

Give your answer in the form $a : b$, where a and b are integers.

Solution

Well,

$$\text{density} = \frac{\text{mass}}{\text{volume}}.$$

Now, the length scale factor (LSF) is

$$\tfrac{4}{3}$$

and the volume scale factor (VSF) is

$$\left(\tfrac{4}{3}\right)^3 = \tfrac{64}{27}.$$

Finally,

$$\begin{aligned}\text{the density of cube } A : \text{the density of cube } B &\Rightarrow \frac{81}{27} = \frac{128}{64} \\ &\Rightarrow \underline{\underline{3 = 2}}.\end{aligned}$$

8. The table shows the amount of snow, in cm, that fell each day for 30 days. (3)

Amount of snow (s cm)	Frequency
$0 \leqslant s < 10$	8
$10 \leqslant s < 20$	10
$20 \leqslant s < 30$	7
$30 \leqslant s < 40$	2
$40 \leqslant s < 50$	3

Work out an estimate for the mean amount of snow per day.

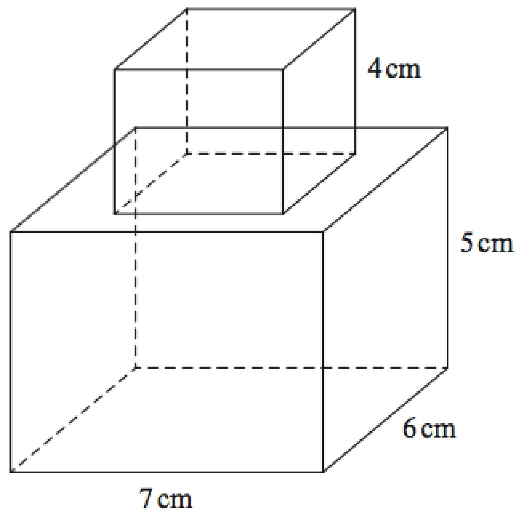
Solution

Amount of snow (s cm)	Frequency	Midpoint	Frequency×Midpoint
$0 \leqslant s < 10$	8	5	40
$10 \leqslant s < 20$	10	15	150
$20 \leqslant s < 30$	7	25	175
$30 \leqslant s < 40$	2	35	70
$40 \leqslant s < 50$	3	45	135
Total	30		570

Hence,

$$\begin{aligned} \text{mean} &= \frac{\sum fx}{\sum f} \\ &\approx \frac{570}{30} \\ &= \underline{\underline{19}}. \end{aligned}$$

9. A cube is placed on top of a cuboid, as shown in the diagram, to form a solid. (3)



The cube has edges of length 4 cm.
The cuboid has dimensions 7 cm by 6 cm by 5 cm.

Work out the total surface area of the solid.

Solution

We work out the surface area of the cuboid and the surface area of the of **four** out of the six faces on the cube (why?). Hence,

$$\begin{aligned} \text{surface area} &= 2[(5 \times 6) + (5 \times 7) + (6 \times 7)] + 4(4 \times 4) \\ &= 2[30 + 35 + 42] + 4(16) \\ &= 2(107) + 64 \\ &= 214 + 64 \\ &= \underline{\underline{278 \text{ cm}^2}}. \end{aligned}$$

10. The table shows some information about the profit made each day at a cricket club on 100 days.

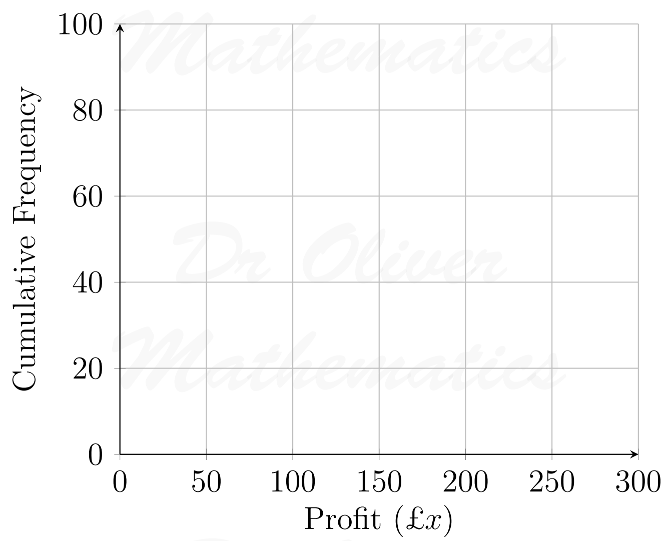
Profit (£x)	Frequency
$0 \leqslant x < 50$	10
$50 \leqslant x < 100$	15
$100 \leqslant x < 150$	25
$150 \leqslant x < 200$	30
$200 \leqslant x < 250$	5
$250 \leqslant x < 300$	15

(a) Complete the cumulative frequency table. (1)

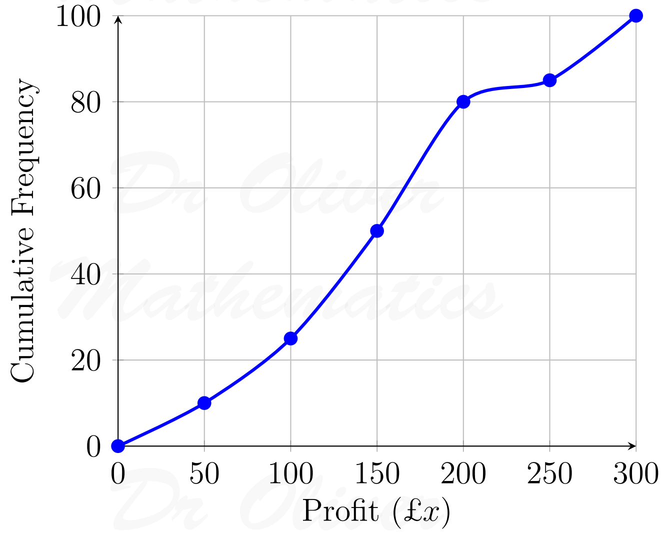
Solution

Profit (£x)	Cumulative Frequency
$0 \leqslant x < 50$	$\underline{\underline{10}}$
$0 \leqslant x < 100$	$15 + 10 = \underline{\underline{25}}$
$0 \leqslant x < 150$	$25 + 25 = \underline{\underline{50}}$
$0 \leqslant x < 200$	$50 + 30 = \underline{\underline{80}}$
$0 \leqslant x < 250$	$80 + 5 = \underline{\underline{85}}$
$0 \leqslant x < 300$	$85 + 15 = \underline{\underline{100}}$

(b) On the grid, draw a cumulative frequency graph for this information (2)

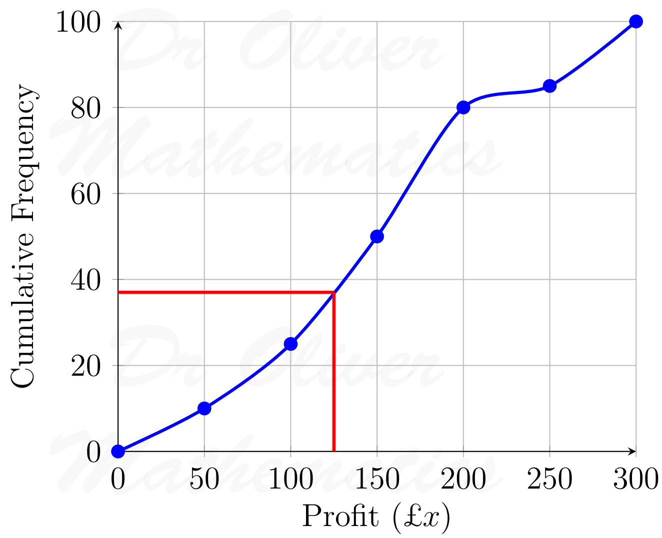


Solution



(c) Use your graph to find an estimate for the number of days on which the profit was less than £125. (1)

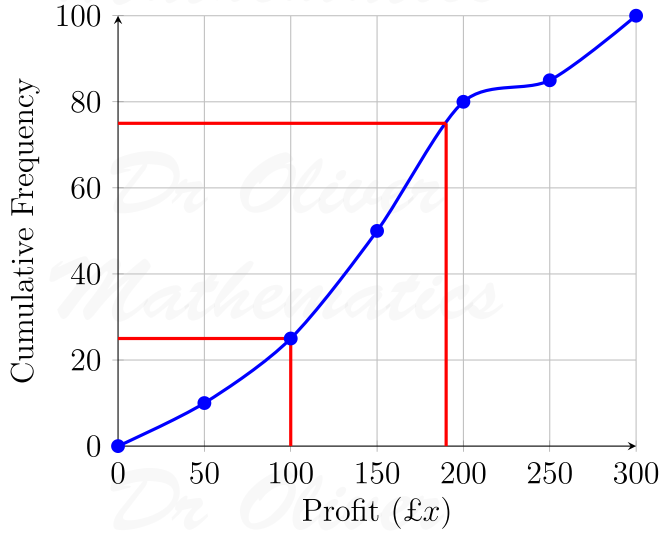
Solution



Correct read-off: about 37.

(d) Use your graph to find an estimate for the interquartile range. (2)

Solution



Hence,

$$\begin{aligned}\text{IQR} &= \text{UQ} - \text{LQ} \\ &= 190 - 100 \\ &= \underline{\underline{90}}.\end{aligned}$$

11. Cormac has some sweets in a bag. (3)
The sweets are lime flavoured or strawberry flavoured or orange flavoured.

In the bag, the number of

lime flavoured sweets : strawberry flavoured sweets : orange flavoured sweets $= 9 : 4 : x$.

Cormac is going to take at random a sweet from the bag.

The probability that he takes a lime flavoured sweet is $\frac{3}{7}$.

Work out the value of x.

Solution

Here,

$$9 + 4 + x = 13 + x$$

and

$$\frac{9}{13+x} = \frac{3}{7} \Rightarrow 63 = 3(13+x)$$
$$\Rightarrow 21 = 13 + x$$
$$\Rightarrow \underline{\underline{x = 8}}.$$

12. Express (3)

$$0.1\dot{1}\dot{7}$$

as a fraction.

You must show all your working.

Solution

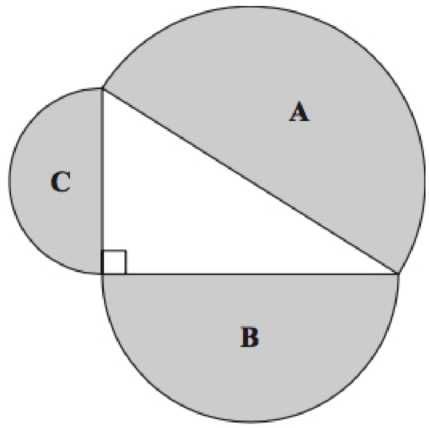
Let $x = 0.1\dot{1}\dot{7}$. Then

$$10x = 1.\dot{1}\dot{7} \quad (1)$$
$$1\,000x = 117.\dot{1}\dot{7} \quad (2).$$

Do (2) − (1):

$$990x = 116 \Rightarrow x = \tfrac{116}{990}$$
$$\Rightarrow x = \tfrac{2\times 58}{2\times 495}$$
$$\Rightarrow \underline{\underline{x = \tfrac{58}{495}}}.$$

13. A right-angled triangle is formed by the diameters of three semicircular regions, **A**, **B**, and **C** as shown in the diagram. (3)



Show that

$$\text{area of region } \mathbf{A} = \text{area of region } \mathbf{B} + \text{area of region } \mathbf{C}.$$

Solution

Pythagoras' theorem:

$$a^2 = b^2 + c^2.$$

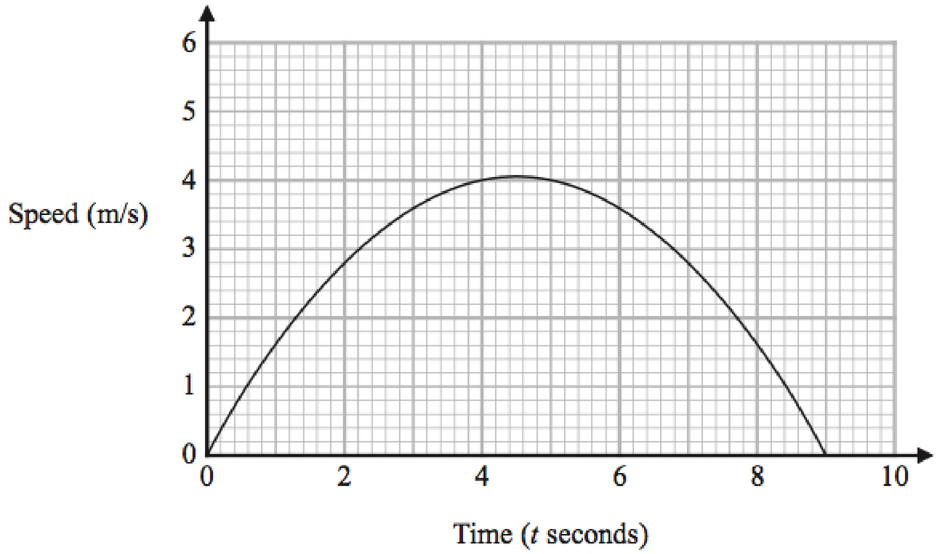
Now, the opposite and adjacent sides are $\frac{1}{2}b$ and $\frac{1}{2}c$ and so

$$\begin{aligned}\tfrac{1}{2}\pi(\tfrac{1}{2}b)^2 + \tfrac{1}{2}\pi(\tfrac{1}{2}c)^2 &= \tfrac{1}{2}\pi\left[(\tfrac{1}{2}b)^2 + (\tfrac{1}{2}c)^2\right] \\ &= \tfrac{1}{2}\pi\left[\tfrac{1}{4}b^2 + \tfrac{1}{4}c^2\right] \\ &= \tfrac{1}{2}\pi\left[\tfrac{1}{4}(b^2 + c^2)\right] \\ &= \tfrac{1}{2}\pi\left[\tfrac{1}{4}a^2\right] \\ &= \tfrac{1}{2}\pi(\tfrac{1}{2}a)^2,\end{aligned}$$

as required. Hence,

$$\underline{\underline{\text{area of region } \mathbf{A} = \text{area of region } \mathbf{B} + \text{area of region } \mathbf{C}.}}$$

14. Here is a speed-time graph.



(a) Work out an estimate of the gradient of the graph at $t = 2$. (3)

Solution

Well, the tangent goes through $(0, 1)$ and $(3, 4)$:

$$\begin{aligned} \text{gradient} &= \frac{4-1}{3-0} \\ &= \frac{3}{3} \\ &= \underline{\underline{1}}. \end{aligned}$$

(b) What does the area under the graph represent? (1)

Solution

E.g., the distance travelled.

15. A, B, and C are three points such that

- $\overrightarrow{AB} = 3\mathbf{a} + 4\mathbf{b}$ and
- $\overrightarrow{AC} = 15\mathbf{a} + 20\mathbf{b}$.

(a) Prove that A, B, and C lie on a straight line. (2)

Solution

$$\begin{aligned}\overrightarrow{AC} &= 15\mathbf{a} + 20\mathbf{b} \\ &= 5(3\mathbf{a} + 4\mathbf{b}) \\ &= 5\overrightarrow{AB}\end{aligned}$$

and so AB is parallel to AC.

Hence, A, B, and C lie on a straight line.

D, E, and F are three points on a straight line such that

- $\overrightarrow{DE} = 3\mathbf{e} + 6\mathbf{f}$ and
- $\overrightarrow{EF} = -10.5\mathbf{e} - 21\mathbf{f}$.

(b) Find the ratio (3)

$$\text{length of } DF : \text{length of } DE.$$

Solution

Well,

$$\begin{aligned}\overrightarrow{DF} &= \overrightarrow{DE} + \overrightarrow{EF} \\ &= (3\mathbf{e} + 6\mathbf{f}) + (-10.5\mathbf{e} - 21\mathbf{f}) \\ &= -7.5\mathbf{e} - 15\mathbf{f} \\ &= -2.5(3\mathbf{e} + 6\mathbf{f}) \\ &= -2.5\overrightarrow{DE}.\end{aligned}$$

Hence,

$$\begin{aligned}\text{length of } DF : \text{length of } DE &= 2.5 : 1 \\ &= 5 : 2.\end{aligned}$$

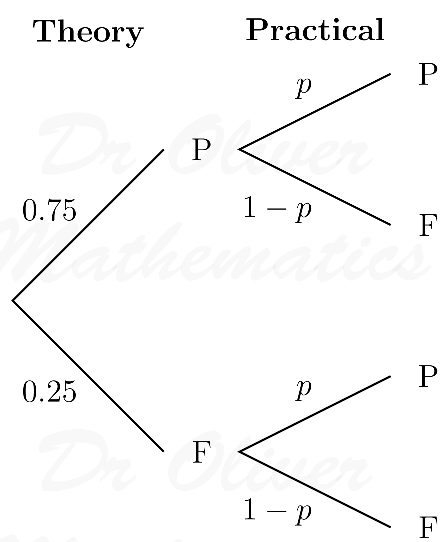
16. A first aid test has two parts, a theory test and a practical test. (4)
The probability of passing the theory test is 0.75.
The probability of passing only one of the two parts is 0.36.

The two events are independent.

Work out the probability of passing the practical test.

Solution

Let the probability of passing the the practical test be p. We make up a tree diagram:



Now,

$$0.75 \times (1-p) + 0.25 \times p = 0.36 \Rightarrow 0.75 - 0.75p + 0.25p = 0.36$$
$$\Rightarrow 0.39 = 0.5p$$
$$\Rightarrow \underline{\underline{p = 0.78}}.$$

17. y is directly proportional to the square root of t. (4)
$y = 15$ when $t = 9$.

t is inversely proportional to the cube of x.
$t = 8$ when $x = 2$,

Find a formula for y in terms of x.
Give your answer in its simplest form.

Solution

$$y \propto \sqrt{t} \Rightarrow y = k\sqrt{t},$$

for some constant k. Now,

$$\begin{aligned} y = 15, t = 9 &\Rightarrow 15 = k\sqrt{9} \\ &\Rightarrow 15 = 3k \\ &\Rightarrow k = 5 \end{aligned}$$

and so

$$y = 5\sqrt{t}.$$

Next,

$$t \propto \frac{1}{x^3} \Rightarrow t = \frac{l}{x^3},$$

for some constant l. Now,

$$\begin{aligned} t = 8, x = 2 &\Rightarrow 8 = \frac{l}{2^3} \\ &\Rightarrow 8 = \frac{l}{8} \\ &\Rightarrow l = 64 \end{aligned}$$

and so

$$t = \frac{64}{x^3}.$$

Finally,

$$\begin{aligned} y &= 5\sqrt{t} \\ &= 5\sqrt{\frac{64}{x^3}} \\ &= 5\left(\frac{8}{x^{\frac{3}{2}}}\right) \\ &= \underline{\underline{\frac{40}{x^{\frac{3}{2}}}}}. \end{aligned}$$

18. Work out the value of (4)

$$\frac{\left(5\tfrac{4}{9}\right)^{-\frac{1}{2}} \times \left(4\tfrac{2}{3}\right)}{2^{-3}}.$$

You must show all your working.

Solution

$$\begin{aligned}\frac{\left(5\frac{4}{9}\right)^{-\frac{1}{2}}\times\left(4\frac{2}{3}\right)}{2^{-3}} &\Rightarrow \frac{\left(\frac{49}{9}\right)^{-\frac{1}{2}}\times\left(\frac{14}{3}\right)}{\frac{1}{8}}\\ &\Rightarrow \frac{\left(\frac{9}{49}\right)^{\frac{1}{2}}\times\left(\frac{14}{3}\right)}{\frac{1}{8}}\\ &\Rightarrow \frac{\left(\frac{3}{7}\right)\times\left(\frac{14}{3}\right)}{\frac{1}{8}}\\ &= 2\times 8\\ &= \underline{\underline{16}}.\end{aligned}$$

19. Solve (4)

$$\frac{1}{2x-1}+\frac{3}{x-1}=1.$$

Give your answer in the form

$$\frac{p\pm\sqrt{q}}{2},$$

where p and q are integers.

Solution

Multiply by $(2x-1)(x-1)$:

$$\begin{aligned}&\frac{1}{2x-1}+\frac{3}{x-1}=1\\ \Rightarrow\quad &(x-1)+3(2x-1)=(2x-1)(x-1)\end{aligned}$$

$\times$	x	-1
$2x$	$2x^2$	$-2x$
-1	$-x$	$+1$

$$\begin{aligned}
&\Rightarrow \quad x - 1 + 6x - 3 = 2x^2 - 3x + 1 \\
&\Rightarrow \quad 2x^2 - 10x + 5 = 0 \\
&\Rightarrow \quad x^2 - 5x + \tfrac{5}{2} = 0 \\
&\Rightarrow \quad x^2 - 5x = -\tfrac{5}{2} \\
&\Rightarrow \quad x^2 - 5x + \tfrac{25}{4} = -\tfrac{5}{2} + \tfrac{25}{4} \\
&\Rightarrow \quad (x - \tfrac{5}{2})^2 = -\tfrac{10}{4} + \tfrac{25}{4} \\
&\Rightarrow \quad (x - \tfrac{5}{2})^2 = \tfrac{15}{4} \\
&\Rightarrow \quad x - \tfrac{5}{2} = \pm\tfrac{\sqrt{15}}{2} \\
&\Rightarrow \quad \underline{\underline{x = \tfrac{5\pm\sqrt{15}}{2}}};
\end{aligned}$$

hence, $\underline{\underline{p = 5}}$ and $\underline{\underline{q = 15}}$.

20. The centre of a circle is the point with coordinates $(-1, 3)$. (4)

The point A with coordinates $(6, 8)$ lies on the circle.

Find an equation of the tangent to the circle at A.
Give your answer in the form

$$ax + by + c = 0,$$

where a, b, and c are integers.

Solution

Well, the gradient of the two points is

$$\begin{aligned}
m &= \frac{8 - 3}{6 - (-1)} \\
&= \tfrac{5}{7}
\end{aligned}$$

and the gradient of the tangent line

$$m_{\text{tangent}} = -\frac{1}{\frac{5}{7}} = -\tfrac{7}{5}.$$

Hence, an equation of the tangent to the circle at A is

$$\begin{aligned} y - 8 = -\tfrac{7}{5}(x-6) &\Rightarrow 5y - 40 = -7(x-6) \\ &\Rightarrow 5y - 40 = -7x + 42 \\ &\Rightarrow \underline{\underline{7x + 5y - 82 = 0}}; \end{aligned}$$

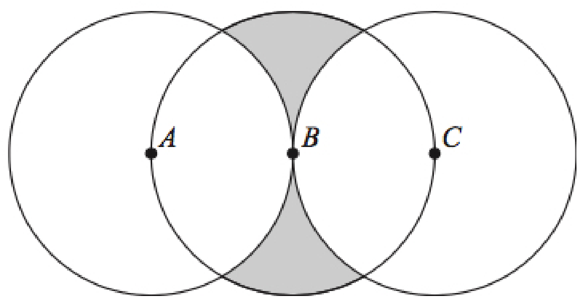
so, $\underline{\underline{a = 7}}$, $\underline{\underline{b = 5}}$, and $\underline{\underline{c = -82}}$.

21. The diagram shows three circles, each of radius 4 cm. (5)

The centres of the circles are A, B, and C such that ABC is a straight line and

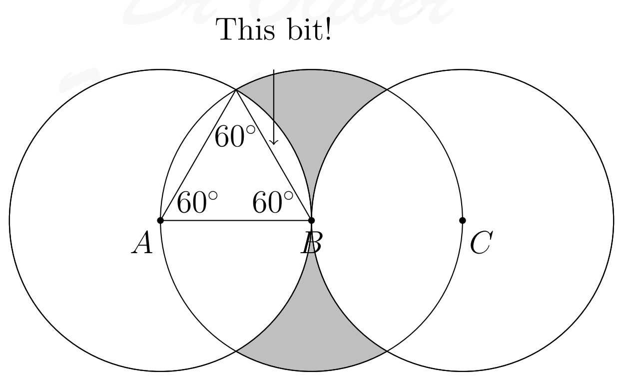
$$AB = BC = 4 \text{ cm}.$$

.



Work out the total area of the two shaded regions.
Give your answer in terms of π.

Solution



Now, we have a 60° angle (why?) and the triangle is equilateral:

$$\begin{aligned}\text{area of the segment} &= \text{area of the sector} - \text{area of the triangle}\\ &= (\tfrac{60}{360} \times \pi \times 4^2) - (\tfrac{1}{2} \times 4 \times 4 \times \sin 60^\circ)\\ &= (\tfrac{1}{6} \times \pi \times 16) - (8 \times \tfrac{\sqrt{3}}{2})\\ &= \tfrac{8}{3}\pi - 4\sqrt{3}.\end{aligned}$$

Finally,

$$\begin{aligned}\text{total area} &= \text{area of a circle} - (4 \times \text{total of the sector}) - (4 \times \text{total of the segment})\\ &= (\pi \times 4^2) - 4(\tfrac{8}{3}\pi) - 4(\tfrac{8}{3}\pi - 4\sqrt{3})\\ &= 16\pi - \tfrac{32}{3}\pi - \tfrac{32}{3}\pi + 16\sqrt{3}\\ &= \underline{\underline{16\sqrt{3} - \tfrac{16}{3}\pi}}.\end{aligned}$$