Dr Oliver Mathematics
AQA GCSE Mathematics
2017 November Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Work out (1)

$$\sqrt{2^6 + 6^2}.$$

Circle your answer.

$$10 \quad 14 \quad 50 \quad 100.$$

Solution

Now,

$$\begin{aligned} \sqrt{2^6 + 6^2} &= \sqrt{64 + 36} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

so

$$\underline{\underline{10}} \quad 14 \quad 50 \quad 100.$$

2. What is (1)

$$800 \text{ million}$$

in standard form?
Circle your answer.

$$800 \times 10^6 \quad 8 \times 10^8 \quad 8 \times 10^9 \quad 0.8 \times 10^{10}.$$

Solution

Well,

$$\begin{aligned} 800 \text{ million} &= 800 \times 1 \times 10^6 \\ &= 8 \times 10^8 \end{aligned}$$

so

$$800 \times 10^6 \quad \underline{\underline{8 \times 10^8}} \quad 8 \times 10^9 \quad 0.8 \times 10^{10}.$$

3. Circle the expression that is equivalent to (1)

$$(4a^5)^2.$$

$$16a^{10} \quad 16a^7 \quad 8a^{10} \quad 8a^7.$$

Solution

Well,

$$\begin{aligned} (4a^5)^2 &= (4^2) \times (a^5)^2 \\ &= 16 \times a^{10} \\ &= 16a^{10} \end{aligned}$$

so

$$\underline{\underline{16a^{10}}} \quad 16a^7 \quad 8a^{10} \quad 8a^7.$$

4. (1)

$$y = \frac{10}{x}.$$

If the value of x doubles, what happens to the value of y?
Circle your answer.

$$\div 2 \quad \times 2 \quad \div 5 \quad \times 5.$$

Solution

Well,

$$\frac{10}{2x} = \tfrac{1}{2}\left(\frac{10}{x}\right) = \tfrac{1}{2}y$$

so

$$\underline{\underline{\div 2}} \quad \times 2 \quad \div 5 \quad \times 5.$$

5. (a) Factorise (1)

$$x^2 - 100.$$

Solution

Difference of two squares:

$$x^2 - 100 = (x)^2 - (10)^2 = \underline{\underline{(x+10)(x-10)}}.$$

(b) Solve (2)

$$7x + 6 > 1 + 2x.$$

Solution

$$7x + 6 > 1 + 2x \Rightarrow 5x > -5 \Rightarrow \underline{\underline{x > -1}}.$$

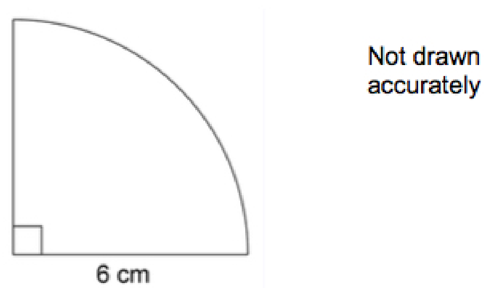
6. Work out the value of (2)

$$(\sqrt{3})^2 \times (\sqrt{2})^2.$$

Solution

$$\begin{aligned}(\sqrt{3})^2 \times (\sqrt{2})^2 &= 3 \times 2 \\ &= \underline{\underline{6}}.\end{aligned}$$

7. Here is a quarter circle of radius 6 cm. (2)



Work out the area of the quarter circle.
Give your answer in terms of π.

Solution

$$\begin{aligned}\text{Area} &= \tfrac{1}{4} \times \pi \times 6^2 \\ &= \tfrac{1}{4} \times \pi \times 36 \\ &= \underline{\underline{9\pi \text{ cm}^2}}.\end{aligned}$$

8. Three **whole** numbers are each rounded to the nearest 10. (2)

The sum of the rounded numbers is 70.

Work out the **maximum** possible sum for the original three numbers.

Solution

E.g., 14, 24, and 44 and

$$14 + 24 + 44 = \underline{\underline{82}}.$$

Rounding: 10, 20, and 40.
Sum: $10 + 20 + 40 = 70$.

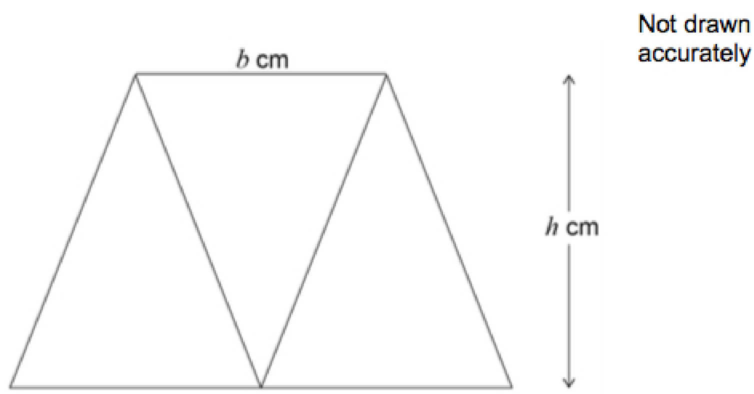
9. Circle the expression for the range of n consecutive integers. (1)

$$\frac{n+1}{2} \quad n-1 \quad n \quad n+1.$$

Solution

$$\frac{n+1}{2} \quad \underline{\underline{n-1}} \quad n \quad n+1.$$

10. Three identical isosceles triangles are joined to make this trapezium.



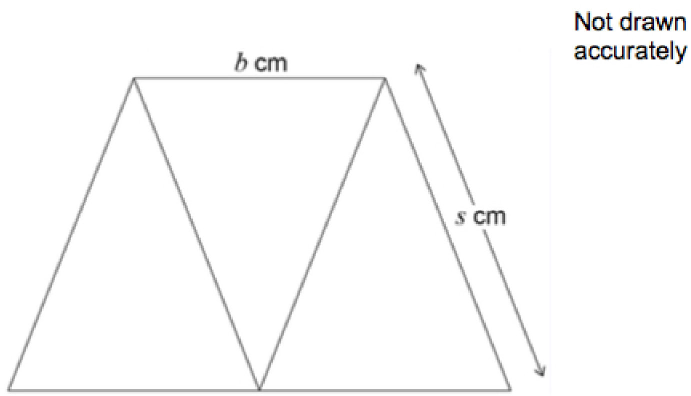
Each triangle has base b cm and perpendicular height h cm.

(a) Work out an expression, in terms of b and h, for the area of the trapezium. Give your answer in its simplest form. (2)

Solution

$$\begin{aligned} \text{Area} &= 3 \times \tfrac{1}{2}bh \\ &= \underline{\underline{\tfrac{3}{2}bh \text{ cm}^2}}. \end{aligned}$$

This diagram shows the same trapezium.



$b : s = 2 : 3$.

(b) Work out an expression, in terms of b, for the perimeter of the trapezium. (2)

Solution

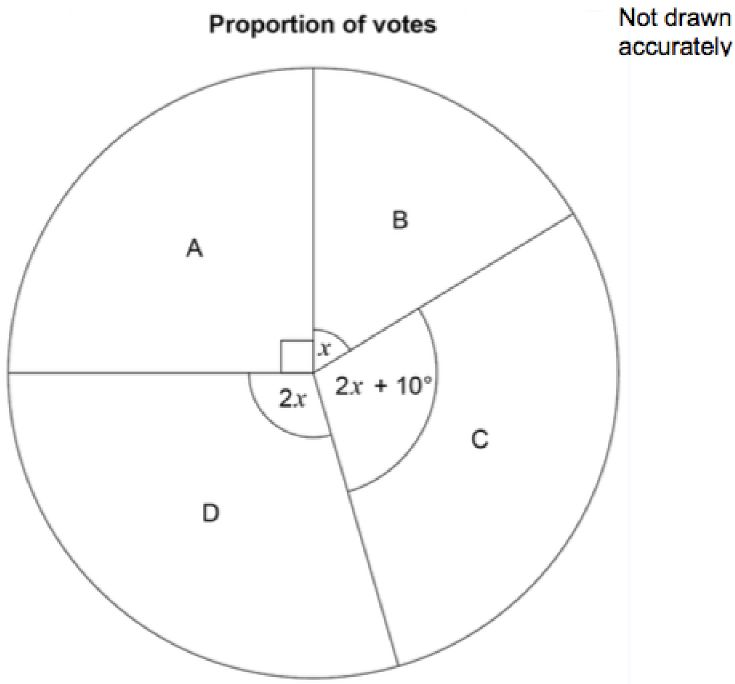
Well,

$$\frac{s}{b} = \frac{3}{2} \Rightarrow s = \tfrac{3}{2}b$$

and

$$\begin{aligned} \text{perimeter} &= 3b + 2s \\ &= 3b + 2(\tfrac{3}{2}b) \\ &= 3b + 3b \\ &= \underline{\underline{6b \text{ cm}}}. \end{aligned}$$

11. The four candidates in an election were A, B, C, and D. (4)
The pie chart shows the proportion of votes for each candidate.



Work out the probability that a person who voted, chosen at random, voted for C.

Solution

Well,

$$\begin{aligned}90 + x + (2x + 10) + 2x = 360 &\Rightarrow 5x + 100 = 360\\ &\Rightarrow 5x = 260\\ &\Rightarrow x = 52\\ &\Rightarrow 2x = 104\\ &\Rightarrow 2x + 10 = 114\end{aligned}$$

and the

$$\begin{aligned}\text{probability} &= \tfrac{114}{360}\\ &= \tfrac{19\times 6}{60\times 6}\\ &= \underline{\underline{\tfrac{19}{60}}}.\end{aligned}$$

12. Use approximations to 1 significant figure to estimate the value of (3)

$$\frac{0.526 \times 39.6^2}{\sqrt{97.65}}.$$

You **must** show your working.

Solution

1 sf:

$$\begin{aligned}\frac{0.526 \times 39.6^2}{\sqrt{97.65}} &\approx \frac{0.5 \times 40^2}{\sqrt{100}} \\ &= \frac{0.5 \times 1\,600}{10} \\ &= \frac{800}{10} \\ &= \underline{\underline{80}}.\end{aligned}$$

13. (3)

$$\begin{aligned}x : y &= 7 : 4 \\ x + y &= 88.\end{aligned}$$

Work out the value of

$$x - y.$$

Solution

Well,

$$\begin{aligned}x : y = 7 : 4 &\Rightarrow \frac{y}{x} = \frac{4}{7} \\ &\Rightarrow y = \tfrac{4}{7}x\end{aligned}$$

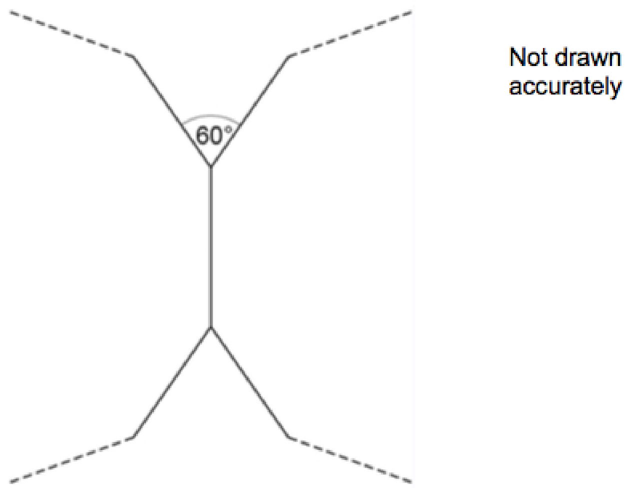
and

$$\begin{aligned}x + y = 88 &\Rightarrow x + \tfrac{4}{7}x = 88 \\ &\Rightarrow \tfrac{11}{7}x = 88 \\ &\Rightarrow \tfrac{1}{7}x = 8 \\ &\Rightarrow x = 56 \\ &\Rightarrow y = 32.\end{aligned}$$

Finally,

$$x - y = 56 - 32 = \underline{\underline{24}}.$$

14. Two congruent regular polygons are joined together. (3)



Work out the number of sides on each polygon.

Solution

Half the 60° and we get 30°. Finally,

$$\begin{aligned}\text{the number of sides on each polygon} &= \frac{360}{30} \\ &= \underline{\underline{12}}.\end{aligned}$$

15. There are

- 7 different sandwiches
- 5 different drinks, and
- 3 different snacks.

Meal Deal
Choose one sandwich, one drink and one snack

(a) How many different Meal Deal combinations are there? (2)

Solution

$$7 \times 5 \times 3 = \underline{\underline{105}}.$$

Two of the sandwiches have cheese in them.

Three of the drinks are fizzy.

Eva picks a Meal Deal at random.

(b) Work out the probability that the sandwich has cheese in it **and** the drink is fizzy. Give your answer as a fraction. (2)

Solution

Now, the sandwich has cheese in it **and** the drink is fizzy:

$$2 \times 3 = 6.$$

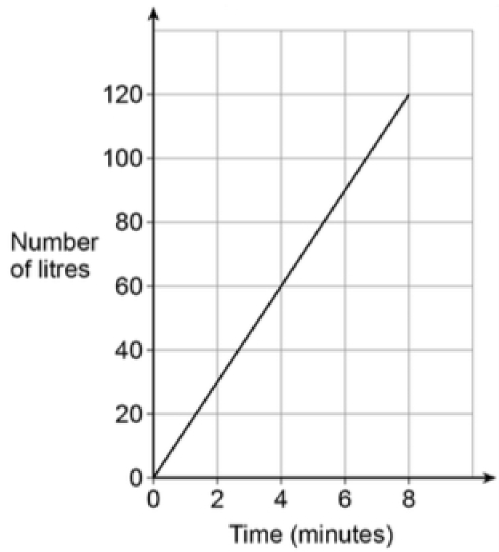
Multiply that by the number of snacks:

$$6 \times 3 = 18.$$

So,

$$\begin{aligned} \text{probability} &= \frac{18}{105} \\ &= \underline{\underline{\tfrac{6}{35}}}. \end{aligned}$$

16. Water is poured into a tank. (1)
The graph shows the number of litres of water in the tank.



How much water is poured into the tank each minute?
Circle your answer.

1.5 litres   15 litres   30 litres   120 litres.

Solution

The line goes through $(0, 0)$ and $(4, 60)$:

$$\text{gradient} = \frac{60 - 0}{15 - 0} = 15$$

so

1.5 litres   <u>15 litres</u>   30 litres   120 litres.

17. A and B are similar solids. (1)

Solid	Length (cm)
A	l
B	$2l$

Alex says, "The volume of B is double the volume of A because the length of B is double the length of A."'

Is he correct?
Tick a box.

Yes ☐ No ☐

Give a reason for your answer.

Solution

No: the volume of B is

$$2^3 = \underline{\underline{8}}$$

times the volume of A

18. Circle the two roots of (1)

$$(2x+3)(5x-2) = 0:$$

$$-\tfrac{3}{2} \quad -\tfrac{2}{5} \quad \tfrac{2}{5} \quad \tfrac{3}{2}.$$

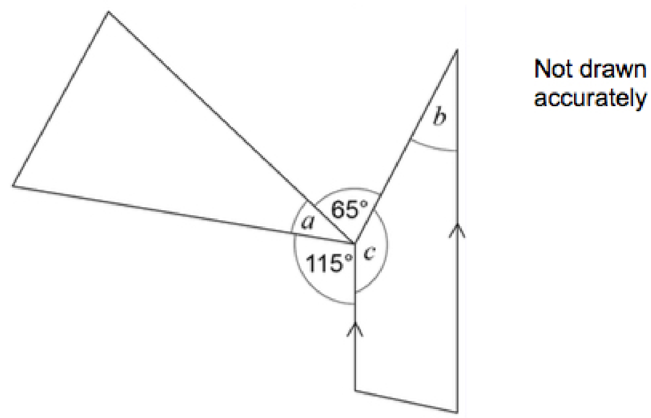
Solution

$$\begin{aligned}(2x+3)(5x-2) = 0 &\Rightarrow 2x+3 = 0 \text{ or } 5x-2 = 0\\ &\Rightarrow 2x = -3 \text{ or } 5x = 2\\ &\Rightarrow x = -\tfrac{3}{2} \text{ or } x = \tfrac{2}{5}\end{aligned}$$

so

$$\underline{\underline{-\tfrac{3}{2}}} \quad -\tfrac{2}{5} \quad \underline{\underline{\tfrac{2}{5}}} \quad \tfrac{3}{2}.$$

19. The diagram shows a triangle and a trapezium. (3)



Prove that

$$a = b.$$

Solution

Well,

$$a + 65 + c + 115 = 360 \Rightarrow a + c + 180 = 360$$
$$\Rightarrow c = 180 - a.$$

Now, the interior angles are equal so

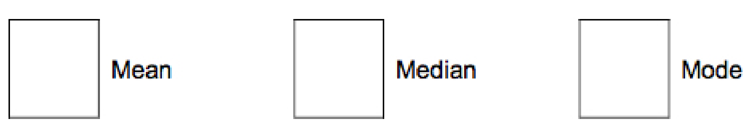
$$180 + (180 - a) + b = 360 \Rightarrow 360 - a + b = 360$$
$$\Rightarrow \underline{\underline{a = b}},$$

as required.

20. In one month, the number of hours of exercise taken by 10 people are (2)

$$4 \quad 7 \quad 2 \quad 8 \quad 6 \quad 5 \quad 1 \quad 82 \quad 3 \quad 9.$$

Which is the appropriate average to use in this situation?
Tick a box.

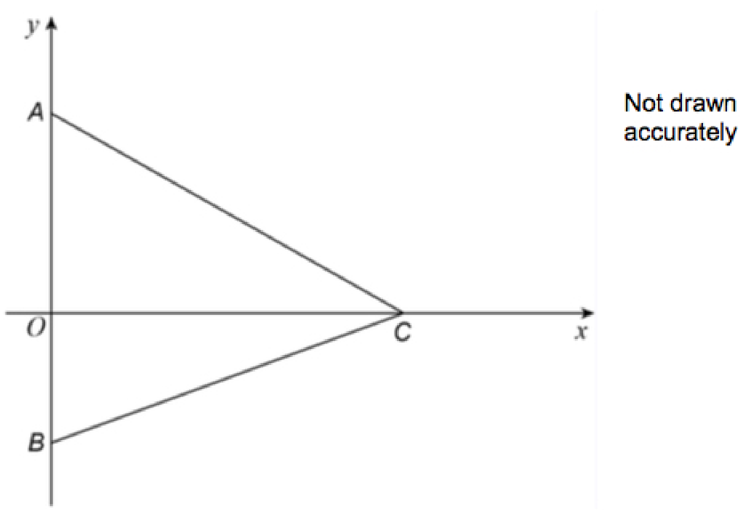


Give one reason for each of the other two averages as to why they are **not** appropriate.

Solution

Median.
It is not the mean: it is skewed by outlying number (82).
It is not the mode: the ten numbers are all different and so it does not have a mode.

21. A, B, and C are points on the axes as shown. (2)



The area of triangle ABC is 28 square units.

Work out possible coordinates for A, B, and C.

Solution

Well,

$$\tfrac{1}{2}bh = 26 \Rightarrow bh = 56,$$

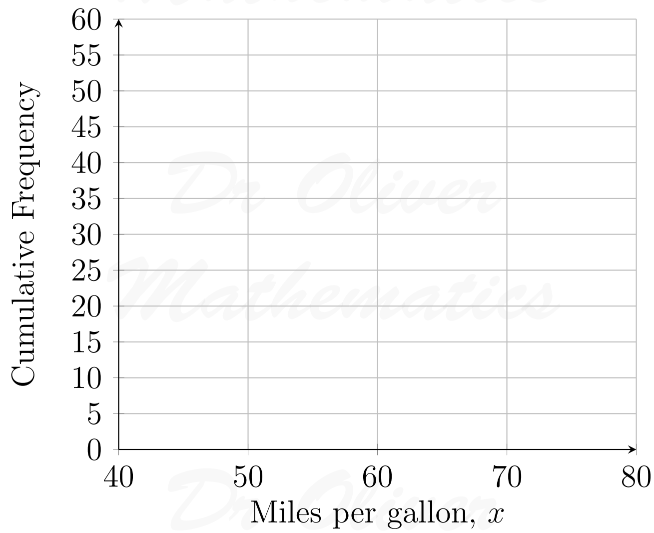
e.g.,

A	B	C
$(0, 1)$	$(0, -1)$	$(28, 0)$
$(0, 2.5)$	$(0, -1.5)$	$(14, 0)$
$(0, 2)$	$(0, -8)$	$(7, 0)$

22. Here is some information about the miles per gallon of 60 cars.

Miles per gallon, x	Frequency
$40 < x \leqslant 50$	6
$50 < x \leqslant 60$	16
$60 < x \leqslant 70$	28
$70 < x \leqslant 80$	10

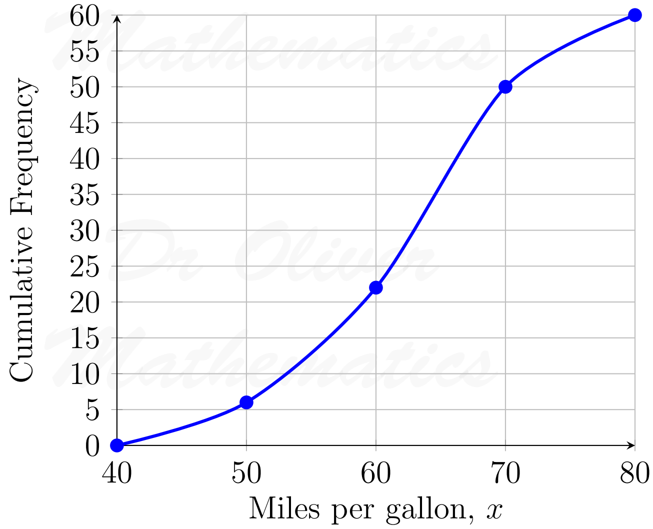
(a) Draw a cumulative frequency graph. (3)



Solution

Miles per gallon, x	Frequency
$40 < x \leqslant 50$	6
$40 < x \leqslant 60$	$6 + 16 = 22$
$40 < x \leqslant 70$	$22 + 28 = 50$
$40 < x \leqslant 80$	$50 + 10 = 60$

and



(b) Use the graph to work out the interquartile range. (2)

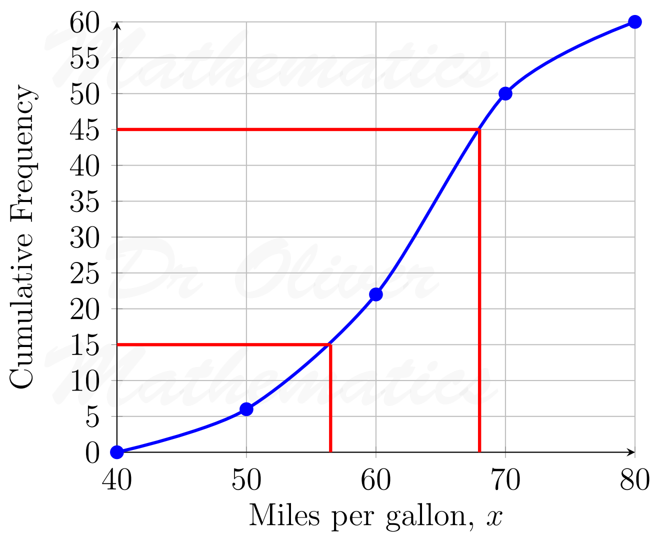
Solution

Now, the LQ is at

$$\tfrac{1}{4}(60) = 15\text{th place}$$

and UQ is at

$$\tfrac{3}{4}(60) = 45\text{th place.}$$



Finally,

$$\begin{aligned}\text{IQR} &= \text{UQ} - \text{LQ} \\ &= 68 - 56.5 \\ &= \underline{\underline{11.5 \text{ miles per gallon}}}.\end{aligned}$$

23. The equation of a curve is (1)

$$y = (x+3)^2 + 5.$$

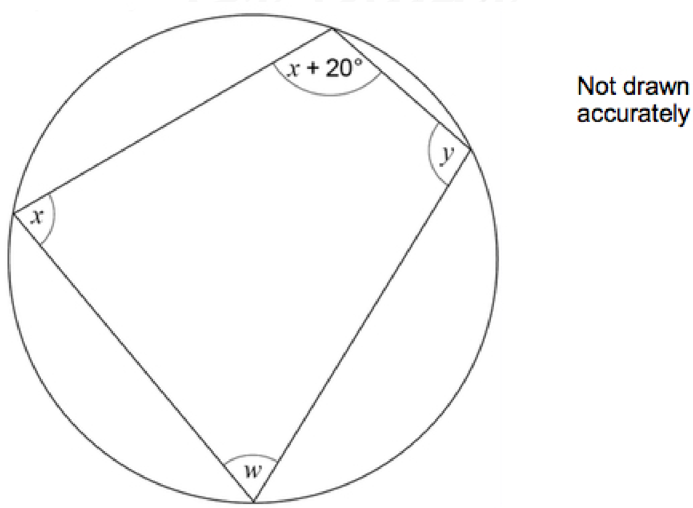
Circle the coordinates of the turning point.

$$(5,3) \quad (5,-3) \quad (3,5) \quad (-3,5).$$

Solution

$$(5,3) \quad (5,-3) \quad (3,5) \quad \underline{\underline{(-3,5)}}.$$

24. Here is a cyclic quadrilateral. (4)



$x : y = 5 : 7.$

Work out the size of angle w.

Solution

Now,

$$\begin{aligned} x : y = 5 : 7 &\Rightarrow \frac{y}{x} = \frac{7}{5} \\ &\Rightarrow y = \tfrac{7}{5}x. \end{aligned}$$

Next, as the opposite angles add up to 180°,

$$\begin{aligned} x + y = 180 &\Rightarrow x + \tfrac{7}{5}x = 180 \\ &\Rightarrow \tfrac{12}{5}x = 180 \\ &\Rightarrow \tfrac{1}{5}x = 15 \\ &\Rightarrow x = 75 \end{aligned}$$

and

$$\begin{aligned} (x + 20) + w = 180 &\Rightarrow (75 + 20) + w = 180 \\ &\Rightarrow 95 + w = 180 \\ &\Rightarrow \underline{\underline{w = 85^\circ}}. \end{aligned}$$

25. 15 machines work at the same rate. (3)

Together, the 15 machines can complete an order in 8 hours.

3 of the machines break down after working for 6 hours.

The other machines carry on working until the order is complete.

In total, how many hours does each of the other machines work?

Solution

Together, they run

$$15 \times 8 = 120 \text{ machine} \cdot \text{hour},$$

if 3 break down,

$$120 - (3 \times 6) = 120 - 18 = 102 \text{ machine} \cdot \text{hour}.$$

That leaves 12 machines to do

$$\frac{102}{12} = \underline{\underline{8\tfrac{1}{2}}} \text{ hours}.$$

26.

$$0.\dot{7} = \tfrac{7}{9}.$$

(a) Use this fact to show that (1)

$$0.0\dot{7} = \tfrac{7}{90}.$$

Solution
Divide each expression by 10:

$$0.\dot{7} = \tfrac{7}{9} \Rightarrow \underline{\underline{0.0\dot{7} = \tfrac{7}{90}}}.$$

(b) Using part (a) or otherwise, convert (3)

$$0.2\dot{7}$$

to a fraction.
Give your answer in its simplest form.

Solution
Let $x = 0.2\dot{7}$. Then

$$\begin{aligned} 10x &= 2.\dot{7} \quad (1) \\ 100x &= 27.\dot{7} \quad (2). \end{aligned}$$

Do (2) − (1):

$$\begin{aligned} 90x = 25 &\Rightarrow x = \tfrac{25}{90} \\ &\Rightarrow x = \tfrac{5\times5}{18\times5} \\ &\Rightarrow \underline{\underline{x = \tfrac{5}{18}}}. \end{aligned}$$

27. There are 11 pens in a box. (4)

8 are black and 3 are red.

Two pens are taken out at random **without** replacement.

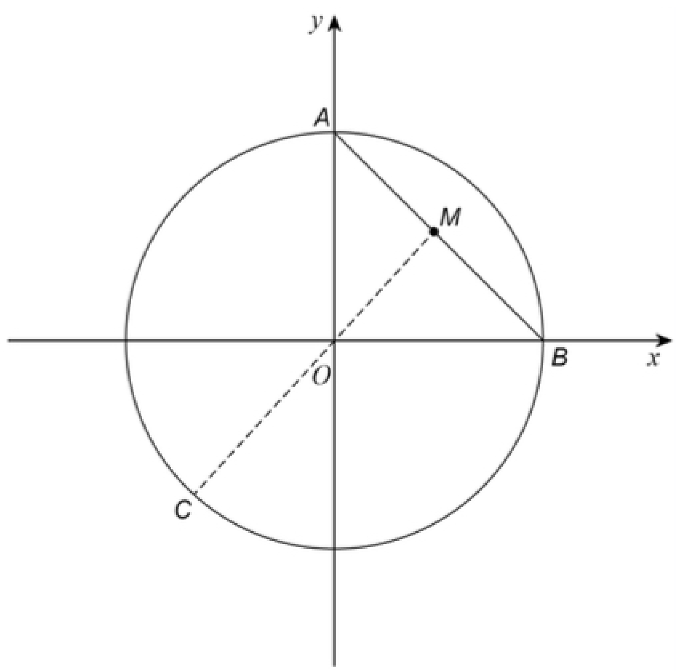
Work out the probability that the two pens are the **same** colour.

Solution

$$\begin{aligned}\text{P(same colour)} &= \text{P}(BB) + \text{P}(RR)\\ &= \left(\tfrac{8}{11} \times \tfrac{7}{10}\right) + \left(\tfrac{3}{11} \times \tfrac{2}{10}\right)\\ &= \tfrac{56}{110} + \tfrac{6}{110}\\ &= \tfrac{62}{110}\\ &= \underline{\underline{\tfrac{31}{55}}}.\end{aligned}$$

28. A, B, and C are points on the circle

$$x^2 + y^2 = 36.$$



- A is on the y-axis.
- B is on the x-axis.
- M is the midpoint of AB.
- COM is a straight line.

(a) Show that the coordinates of A are $(0, 6)$. (1)

Solution

$$\begin{aligned} x = 0 &\Rightarrow y^2 = 36 \\ &\Rightarrow y = \pm 6. \end{aligned}$$

We reject the negative solution (why?) and $\underline{\underline{A(0, 6)}}$.

(b) Work out the coordinates of B. (1)

Solution

$$\begin{aligned} y = 0 &\Rightarrow x^2 = 36 \\ &\Rightarrow x = \pm 6. \end{aligned}$$

We reject the negative solution (why?) and $\underline{\underline{B(6, 0)}}$.

(c) Show that the equation of the straight line passing through C, O, and M is $y = x$. (2)

Solution
Well, C is

$$\left(\frac{0+6}{2}, \frac{6+0}{2} \right) = (3, 3)$$

and

$$\begin{aligned} \text{gradient} &= \frac{3-0}{3-0} \\ &= 1. \end{aligned}$$

Hence, the equation is

$$y - 0 = 1(x - 0) \Rightarrow \underline{\underline{y = x}},$$

as required.

(d) Work out the coordinates of C. (3)
Give your answers in surd form.

Solution

Well,

$$
\begin{aligned}
x^2 + y^2 = 36 &\Rightarrow 2x^2 = 36 \\
&\Rightarrow x^2 = 18 \\
&\Rightarrow x = \pm 18 \\
&\Rightarrow x = \pm 9 \times 2 \\
&\Rightarrow x = \pm\sqrt{9} \times \sqrt{2} \\
&\Rightarrow x = \pm 3\sqrt{2}.
\end{aligned}
$$

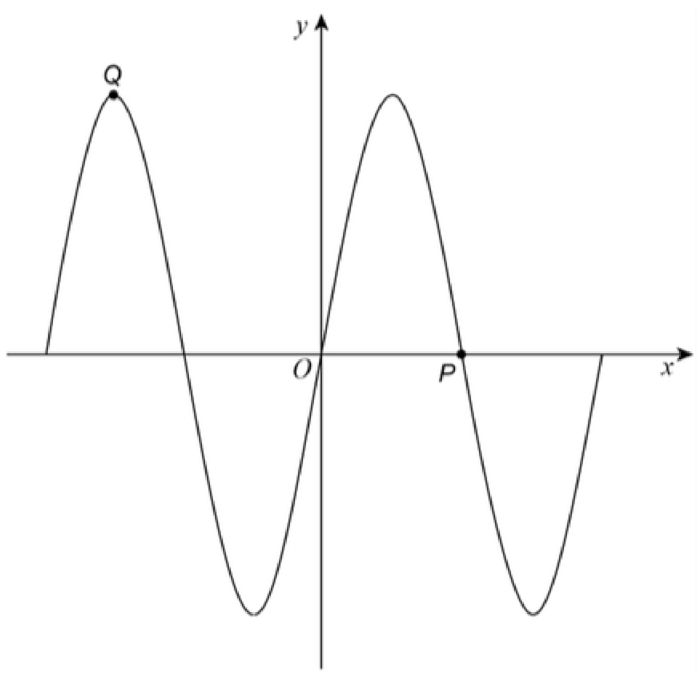
Hence,

$$\underline{\underline{C(-3\sqrt{2}, -3\sqrt{2})}}.$$

29. Here is a sketch of

$$y = \sin x^\circ,$$

for $-360 \leqslant x \leqslant 360$.



(a) Write down the coordinates of P. (1)

Solution

$\underline{\underline{P(180, 0)}}$.

(b) Write down the coordinates of Q. (1)

Solution

$\underline{\underline{Q(-270, 1)}}$.

30. (a) Work out the value of (2)

$$81^{-\frac{1}{4}}.$$

Solution

$$\begin{aligned} 81^{-\frac{1}{4}} &= \frac{1}{81^{\frac{1}{4}}} \\ &= \frac{1}{(3^4)^{\frac{1}{4}}} \\ &= \underline{\underline{\tfrac{1}{3}}}. \end{aligned}$$

(b) Write (3)

$$16 \times 8^{2x}$$

as a power of 2 in terms of x.

Solution

$$\begin{aligned} 16 \times 8^{2x} &= 2^4 \times (2^3)^{2x} \\ &= 2^4 \times 2^{3(2x)} \\ &= 2^4 \times 2^{6x} \\ &= \underline{\underline{2^{4+6x}}}. \end{aligned}$$