Dr Oliver Mathematics
OCR FMSQ Additional Mathematics
2018 Paper
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.
Final answers should be given correct to three significant figures where appropriate.

Section A

1. Solve the inequality (3)

$$2 - x < 1 + 3(x - 2).$$

2. The gradient function of a curve is given by (4)

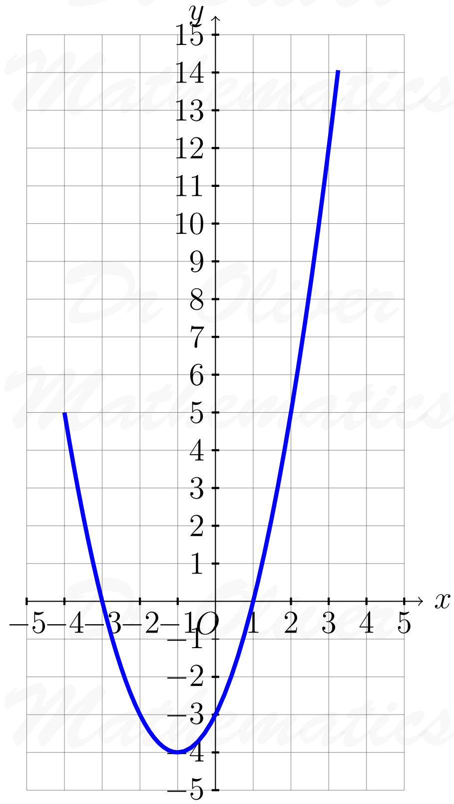
$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2 + 2x - 3x^2.$$

Find the equation of the curve given that it passes through the point $(2, 3)$.

3. The graph of

$$y = x^2 + 2x - 3$$

is given below.



(a) Write down the solution to the equation (1)

$$x^2 + 2x - 3 = 0.$$

(b) By plotting the appropriate straight line on the grid, find the solution to the equation (3)

$$x^2 - x - 6 = 0.$$

4. You are given that the acute angle θ is such that

$$\sin\theta = \tfrac{1}{5}.$$

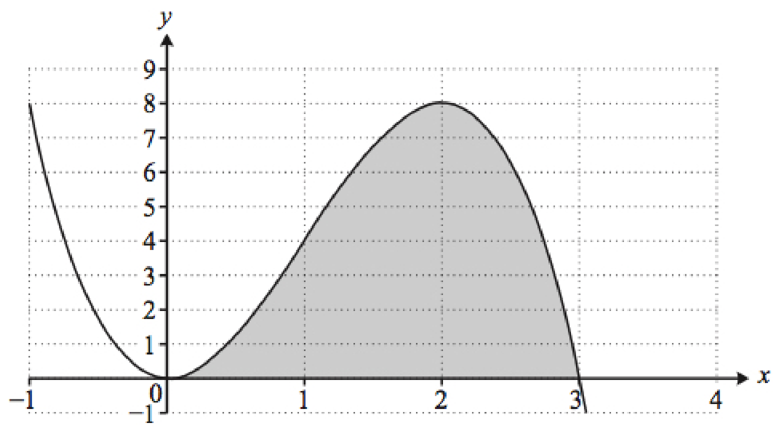
Find the exact value of each of the following.

(a) $\cos\theta$, (2)

(b) $\tan\theta$. (2)

5. The figure below shows part of the graph of the curve with equation (4)

$$y = 6x^2 - 2x^3.$$



Find the area of the shaded region enclosed by the curve and the x-axis.

6. (a) Solve these simultaneous equations. (4)

$$3x + 4y = 18$$
$$7x - 3y = 5.$$

(b) Draw a rough sketch of the lines to demonstrate graphically the solution to part (a). (2)

7. (a) Find the coordinates of the points where the line (4)

$$y = 7x - 9$$

cuts the curve

$$y = x^2 + 2x - 5.$$

(b) Determine whether the line is a normal to the curve at either of the points of intersection. (3)

8. (a) Simplify the equation (3)

$$\frac{x + a}{x} + \frac{x - 2}{4} = 0,$$

leaving your answer in the form $(x + p)^2 = q$, where p is an integer and q is given in terms of the constant a.

(b) Hence write down the range of values of a for which the equation has real roots. (2)

(c) Using your answer to part (a), solve the equation when $a = -1$, giving your answers **exactly**. (2)

9. The proportion of people who are left-handed is 20%.

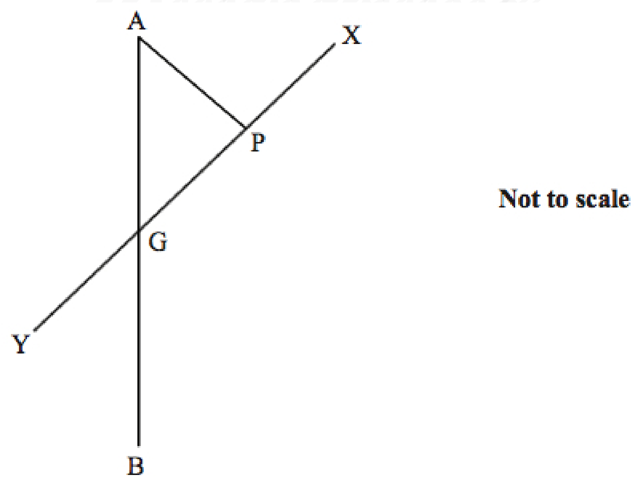
(a) For a group of 10 students chosen at random, use the binomial distribution to find the probability that

  (i) no student is left-handed, (2)

  (ii) exactly 4 students are left-handed. (3)

(b) State the conditions necessary for the binomial distribution to be valid. (2)

10. The diagram shows an "up and over" garage door, XY, that is 200 cm long. There is a small wheel at the point G on the door. The wheel runs freely up a groove in a fixed vertical door frame, AB. A metal rod AP is fixed to the top of the door frame, A, and is also fixed to the point P on the door. The rod is hinged at both ends.



$GP = PX = AP = 60$ cm and $YG = 80$ cm.
When the door is closed, Y is at B and X is at A. When the door is fully open, G is at A and the door is horizontal, 200 cm above the horizontal ground.

(a) Explain why P is the centre of the circle through A, G, and X. (1)

(b) Hence show that AX is horizontal whatever the position of the garage door. (1)

(c) Find the height of Y above the ground when angle $AGP = 40°$. (4)

Section B

11. A circle has centre $(0, 3)$ and radius 3.

(a) Show that the equation of the circle is (2)

$$x^2 + y^2 - ky = 0,$$

where k is to be determined.

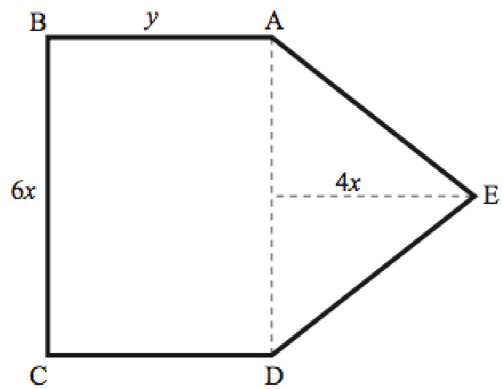
The line $y = mx - 2$ passes through the point $P(0, -2)$ and is a tangent to the circle.

(b) Find the two possible values of m. (6)

The two tangents from P meet the circle at the points A and B respectively.

(c) Find the lengths PA and PB. (4)

12. The shape shown in the figure is made of metal rods.



Not to scale

$ABCD$ is a rectangle. $AB = CD = y$ cm and $BC = DA = 6x$ cm.
AED is an isosceles triangle with height $4x$ cm and $AE = ED$.

(a) Show that the perimeter, p cm, can be written as (3)

$$p = 16x + 2y.$$

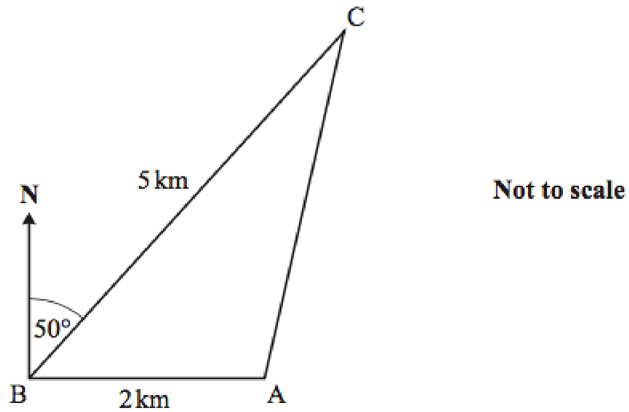
You are given that $p = 96$.

(b) Show that the area of the shape, A cm^2, can be written as (3)

$$A = 288x - 36x^2.$$

(c) Find the maximum area of the shape as x and y vary and find the values of x and y for this area. (6)

13. Jessie walks at 3 km per hour in a straight line from a point B to a point C, a distance of 5 km. C is on a bearing 050° from B, as shown in the figure below.

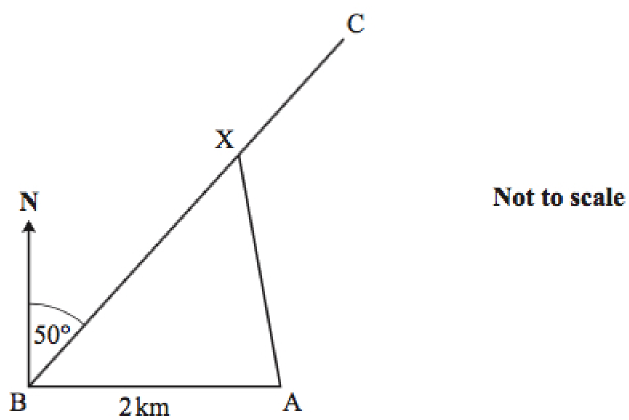


Brandon sets out at the same time as Jessie. He starts from a point A which is 2 km due East of B. He walks at 2 km per hour directly to C.

(a) Calculate the distance AC, correct to 3 significant figures. (4)

(b) Show that Brandon arrives at C approximately 11 minutes after Jessie arrives. (3)

Charlie also sets out at the same time as Jessie. He walks in a straight line from A at 2 km per hour to meet Jessie at a point X on BC, as shown in the figure below.



He arrives at the point X at the same time as Jessie.

(c) Show that there are two possible positions for X and find the bearing on which Charlie must walk in each case. (5)

14. Two cars, P and Q, accelerate from rest from a point O at the same time.

(a) P accelerates uniformly at 2 ms^{-2}.

(i) Write down the formula for the displacement, s metres, of P at time t seconds after leaving O. (1)

(ii) Using appropriate units, find the time taken for P to reach a speed of $90\ \text{km}\,\text{h}^{-1}$. (3)

(b) Q accelerates from rest with variable acceleration $a\ \text{ms}^{-2}$ where, at time t seconds, $a = 1 + kt$, where k is a positive constant. Q passes P when $t = 10$.

(i) Find the value of k. (5)

(ii) Show that at the time when P reaches $90\ \text{km}\,\text{h}^{-1}$, Q is travelling at a speed just less than $130\ \text{km}\,\text{h}^{-1}$. (3)