

## Dr Oliver Mathematics

### Highest Common Factor

Although you can simply check to see if a collection of numbers is divisible by 1, 2, 3, ... and so on until there are no possibilities left to check the easiest way to find the highest common factor of some numbers is to use their prime factorisations. Once you have these, proceed as follows:

- if the prime factorisations have no primes in common then the highest common factor is 1;
  - for each prime that they have in common, choose the largest power of that prime that is common to each number, e.g., if  $a$  contains  $3^5$ ,  $b$  contains  $3^2$ , and  $c$  contains  $3^{17}$ , then  $3^2$  is the relevant power to take;
  - the highest common factor is then the product of these powers of primes.
- Find the highest common factor of 1 890 and 2 772.

#### Solution

First, express each number as a product of its prime factors:

$$1\,890 = 2 \times 3^3 \times 5 \times 7 \text{ and } 2\,772 = 2^2 \times 3^2 \times 7 \times 11.$$

Each number contains the primes 2, 3, and 7:

- 1 890 contains  $2 = 2^1$  and 2 772 contains  $2^2$  and so for the HCF we take 2.
- 1 890 contains  $3^3$  and 2 772 contains  $3^2$  and so for the HCF we take  $3^2$ .
- both 1 890 and 2 772 contain  $7 = 7^1$  and so for the HCF we take 7.

Hence

$$\text{HCF}(1\,890, 2\,772) = 2 \times 3^2 \times 7 = \underline{\underline{126}}.$$