Dr Oliver Mathematics
AQA Further Maths Level 2
June 2025 Paper 1
1 hour 45 minutes

The total number of marks available is 80.
You must write down all the stages in your working.
You are **not** permitted to use a scientific or graphical calculator in this paper.

1. (a) The function f is given by (1)

$$f(x) = 5x + 3.$$

The range is

$$f(x) < 23.$$

Work out the domain of the function, giving your answer as an inequality.

Solution
Well,

$$f(x) < 23 \Rightarrow 5x < 20$$
$$\Rightarrow \underline{\underline{x < 4}}.$$

(b) The function g is given by (2)

$$g(x) = x^3 - 4.$$

The domain is

$$-3 < x < 2.$$

Work out the range of the function, giving your answer as an inequality.

Solution
Now,

$$g(-3) = (-3)^3 - 4 = -31$$

and

$$g(2) = 2^3 - 4 = 4.$$

As it is a one-to-one function,

$$\underline{\underline{-31 < g(x) < 4}}.$$

(c) The function h is given by (2)

$$\mathrm{h}(x) = \sqrt{2x+1}.$$

Work out $\mathrm{h}^{-1}(x)$.

Solution

Finally,

$$\begin{aligned} y = \sqrt{2x+1} &\Rightarrow y^2 = 2x+1 \\ &\Rightarrow y^2 - 1 = 2x \\ &\Rightarrow \tfrac{1}{2}(y^2-1) = x, \end{aligned}$$

and so

$$\underline{\underline{\mathrm{h}^{-1}(x) = \tfrac{1}{2}(x^2-1).}}$$

2. (2)

$$\begin{pmatrix} 5 & 0 \\ 1 & -6 \end{pmatrix} \begin{pmatrix} 12 \\ 7 \end{pmatrix} = 15 \begin{pmatrix} a \\ b \end{pmatrix}.$$

Work out the values of a and b.

Solution

Well,

$$\begin{aligned} & \begin{pmatrix} 5 & 0 \\ 1 & -6 \end{pmatrix} \begin{pmatrix} 12 \\ 7 \end{pmatrix} = 15 \begin{pmatrix} a \\ b \end{pmatrix} \\ \Rightarrow \quad & \begin{pmatrix} 60 \\ -30 \end{pmatrix} = 15 \begin{pmatrix} a \\ b \end{pmatrix} \\ \Rightarrow \quad & \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}; \end{aligned}$$

hence,

$$\underline{\underline{a = 4 \text{ and } b = -2.}}$$

3. A linear sequence starts (3)

$$2 \qquad 5 \qquad 8 \qquad 11.$$

Work out the **position** of the term that is the first to have a value greater than 500.

Solution

Let the

$$n\text{th term} = an + b.$$

$$\begin{array}{ccccccc} 2 & & 5 & & 8 & & 11 \\ & 3 & & 3 & & 3 & \\ a+b & & 2a+b & & 3a+b & & 4a+b \\ & a & & a & & a & \end{array}$$

We compare terms:

$$a = 3$$

and

$$\begin{aligned} a + b = 2 &\Rightarrow 3 + b = 2 \\ &\Rightarrow b = -1. \end{aligned}$$

Hence,

$$n\text{th term} = 3n - 1.$$

Now,

$$\begin{aligned} n\text{th term} > 500 &\Rightarrow 3n - 1 > 500 \\ &\Rightarrow 3n > 501 \\ &\Rightarrow n > 167. \end{aligned}$$

Finally, the position of the term that is the first to have a value greater than 500 is 168.

4. AC is a straight line.

- A is the point $(-3, 2)$.
- B is the midpoint of the line AC.
- B is the point $(12, 10)$.

(a) Work out the coordinates of C. (2)

Solution

Well,

$$\begin{aligned}
\overrightarrow{OC} &= \overrightarrow{OA} + \overrightarrow{AC} \\
&= \overrightarrow{OA} + 2\overrightarrow{AB} \\
&= \overrightarrow{OA} + 2(\overrightarrow{AO} + \overrightarrow{OB}) \\
&= \overrightarrow{OA} + 2(-\overrightarrow{OA} + \overrightarrow{OB}) \\
&= \begin{pmatrix} -3 \\ 2 \end{pmatrix} + 2\left[-\begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 12 \\ 10 \end{pmatrix} \right] \\
&= \begin{pmatrix} -3 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 15 \\ 8 \end{pmatrix} \\
&= \begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 30 \\ 16 \end{pmatrix} \\
&= \begin{pmatrix} 27 \\ 18 \end{pmatrix};
\end{aligned}$$

hence, $\underline{\underline{C(27, 18)}}$.

(b) Work out the distance AB. (2)

Solution

Finally,

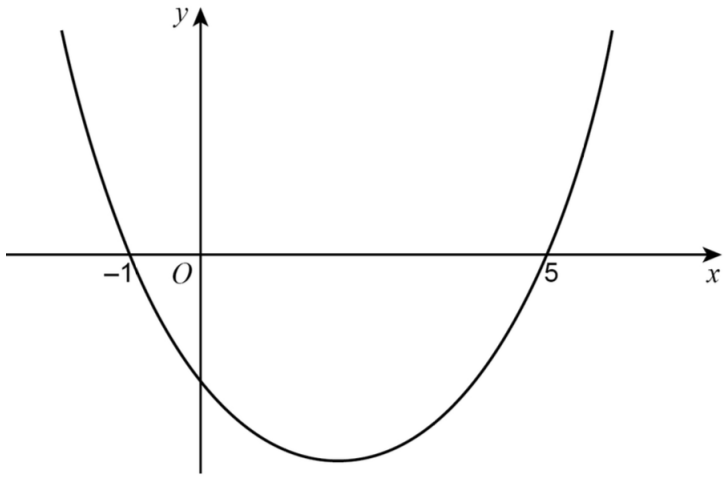
$$\begin{aligned}
\text{distance} &= \sqrt{[12 - (-3)]^2 + (10 - 2)^2} \\
&= \sqrt{15^2 + 8^2} \\
&= \sqrt{225 + 64} \\
&= \sqrt{289} \\
&= \underline{\underline{17}}.
\end{aligned}$$

5. Here is a sketch of (3)

$$y = x^2 + bx + c.$$

The graph passes through $(-1, 0)$ and $(5, 0)$.

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Work out the values of b and c.

Solution

Well,

$\times$	x	-5
x	x^2	$-5x$
$+1$	$+x$	-5

and

$$\begin{aligned} y &= (x+1)(x-5) \\ &= x^2 - 4x - 5; \end{aligned}$$

hence,

$$\underline{\underline{b = -4 \text{ and } c = -5}}.$$

6. Simplify fully (3)

$$\frac{\sqrt{75} - \sqrt{27}}{\sqrt{48}}.$$

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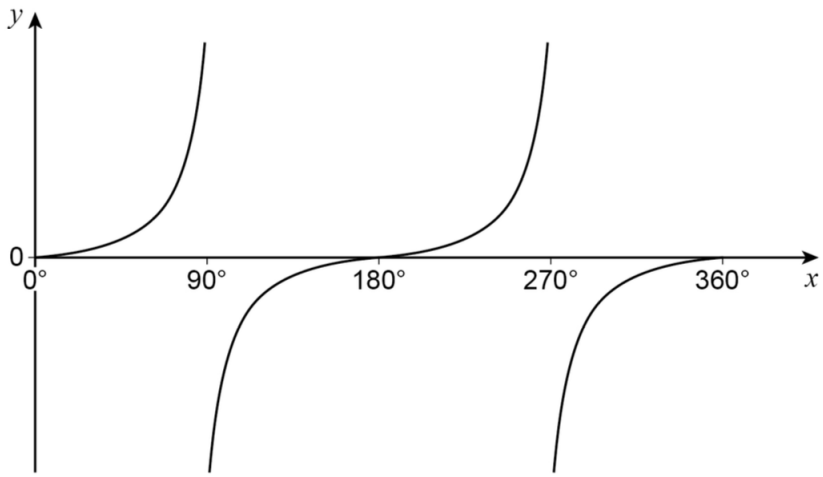
Solution

Well,

$$\begin{aligned}\frac{\sqrt{75}-\sqrt{27}}{\sqrt{48}} &= \frac{\sqrt{25\times 3}-\sqrt{9\times 3}}{\sqrt{16\times 3}}\\ &= \frac{\sqrt{25}\times\sqrt{3}-\sqrt{9}\times\sqrt{3}}{\sqrt{16}\times\sqrt{3}}\\ &= \frac{5\sqrt{3}-3\sqrt{3}}{4\sqrt{3}}\\ &= \frac{2\sqrt{3}}{4\sqrt{3}}\\ &= \underline{\underline{\tfrac{1}{2}}}.\end{aligned}$$

7. Here is a sketch of (2)

$$y = \tan x, \text{ for } 0^\circ \leqslant x \leqslant 360^\circ.$$



You are given that

$$\tan 72^\circ = k.$$

Work out the two values of x between 0° and 360° for which

$$\tan x = -k.$$

Solution

Well,

$$90 + (90 - 72) = \underline{\underline{108^\circ}}$$

and

$$180 + 108 = \underline{\underline{288^\circ}}.$$

8. Work out (1)

$$\left(\frac{x^{\frac{9}{2}}}{x^{\frac{1}{2}}}\right)^3$$

as a power of x.

Circle your answer.

$$x^7 \qquad x^{12} \qquad x^{15} \qquad x^{27}$$

Solution

Now,

$$\begin{aligned}\left(\frac{x^{\frac{9}{2}}}{x^{\frac{1}{2}}}\right)^3 &= \left(x^{\frac{9}{2}-\frac{1}{2}}\right)^3 \\ &= \left(x^4\right)^3 \\ &= x^{12},\end{aligned}$$

and so

$$x^7 \qquad \underline{\underline{x^{12}}} \qquad x^{15} \qquad x^{27}$$

9. 3-digit integers are made using three **different** digits. (3)

The digits are chosen from

$$1, 2, 3, 4, 5, 6, 7, 8, 9.$$

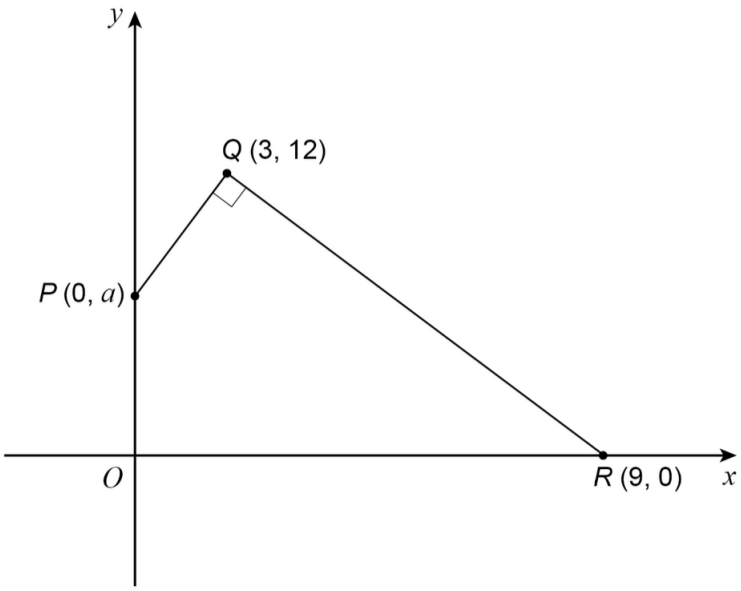
How many **more** odd integers than even integers can be made?

Solution

Now,

$$\begin{aligned}\text{many more} &= \text{end with an odd digit} - \text{end with an even digit}\\ &= (5 \times 8 \times 7) - (4 \times 8 \times 7)\\ &= 280 - 224\\ &= \underline{\underline{56}}.\end{aligned}$$

10. PQ is perpendicular to QR. (4)



Work out the value of a.

Solution

Well,

$$\begin{aligned}m_{QR} &= \frac{0-12}{9-3}\\ &= \frac{-12}{6}\\ &= -2,\end{aligned}$$

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and so

$$m_{PQ} = -\frac{1}{-2} = \tfrac{1}{2}.$$

Now, the equation of PQ is

$$y - 12 = \tfrac{1}{2}(x - 3) \Rightarrow y - 12 = \tfrac{1}{2}x - \frac{3}{2}$$
$$\Rightarrow y = \tfrac{1}{2}x + \tfrac{21}{2}.$$

Finally,

$$x = 0 \Rightarrow \underline{\underline{a = 10\tfrac{1}{2}}}.$$

11. Solve (4)

$$x^3 + 10x^2 + 8x = 0.$$

Where appropriate give your answers in surd form.
Simplify your answers where possible.

Solution

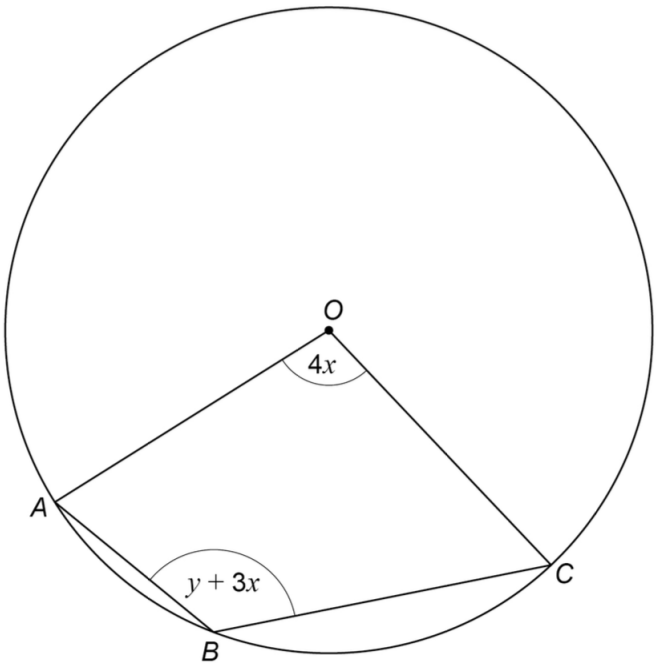
Well,

$$x^3 + 10x^2 + 8x = 0 \Rightarrow x(x^2 + 10x + 8) = 0$$

coefficient of x : $+10$
half it: $+5$
square it: $(+5)^2 = +25$

$$\Rightarrow x[(x^2 + 10x + 25) - 25 + 8] = 0$$
$$\Rightarrow x[(x + 5)^2 - 17] = 0$$
$$\Rightarrow x = 0 \text{ or } (x + 5)^2 - 17 = 0$$
$$\Rightarrow x = 0 \text{ or } (x + 5)^2 = 17$$
$$\Rightarrow x = 0 \text{ or } x + 5 = \pm\sqrt{17}$$
$$\Rightarrow \underline{\underline{x = 0 \text{ or } x = -5 \pm \sqrt{17}}}.$$

12. A, B, and C are points on a circle, centre O. (4)



Prove that

$$y = 180 - 5x.$$

Solution

Let D be a point directly above O on the circle.

Then $\angle ADC = 2x$ (the angles which an arc of a circle subtends at the centre is equal to twice the angle the arc subtends at any point on the remaining circumference)

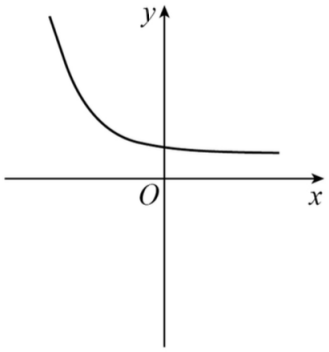
Now, the opposite angles of a cyclic quadrilateral are supplementary and so

$$2x + (y + 3x) = 180 \Rightarrow \underline{\underline{y = 180 - 5x}},$$

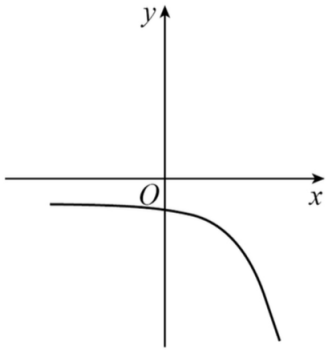
as required.

13. Here are four sketch graphs. (1)

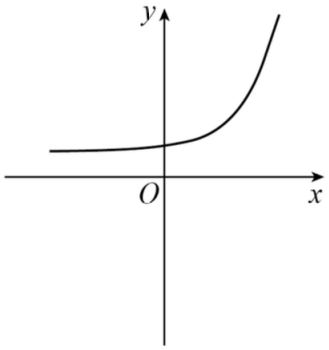
A



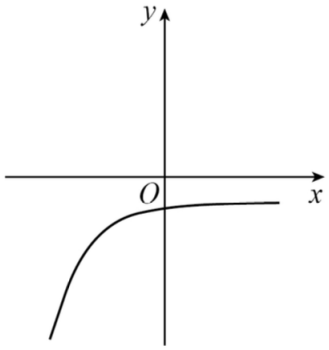
B



C



D



Circle the letter of the sketch graph that represents

$$y = 5^x.$$

Solution

$\underline{\underline{C}}$.

14. Simplify fully (4)

$$\frac{2x^2 - 4x - 6}{x^2 - 3x} \div \frac{4x + 4}{7x^2}.$$

Solution

Well,

$$2x^2 - 4x - 6 = 2(x^2 - 2x - 3)$$

$$\left.\begin{array}{ll} \text{add to:} & -2 \\ \text{multiply to:} & -3 \end{array}\right\} +1, -3$$

and so

$$2x^2 - 4x - 6 = 2(x+1)(x-3).$$

Now,

$$\begin{aligned} \frac{2x^2 - 4x - 6}{x^2 - 3x} \div \frac{4x+4}{7x^2} &= \frac{2(x+1)(x-3)}{x(x-3)} \times \frac{7x^2}{4(x+1)} \\ &= \frac{2}{1} \times \frac{7x}{4} \\ &= \underline{\underline{\tfrac{7}{2}x}}. \end{aligned}$$

15. (3)

$$(x-3)(x+4)(x+p) \equiv x^3 + qx^2 + rx + 24.$$

Work out the values of p, q, and r.

Solution

Well,

$\times$	x	$+4$
x	x^2	$+4x$
-3	$-3x$	-12

and

$$(x-3)(x+4) = x^2 + x - 12.$$

Now,

$\times$	x^2	$+x$	-12
x	x^3	$+x^2$	$-12x$
$+p$	$+px^2$	$+px$	$-12p$

and so

$$(x-3)(x+4)(x+p) = x^3 + (p+1)x^2 + (p-12)x - 12p.$$

Constant term:

$$-12p = 24 \Rightarrow \underline{\underline{p = -2}}.$$

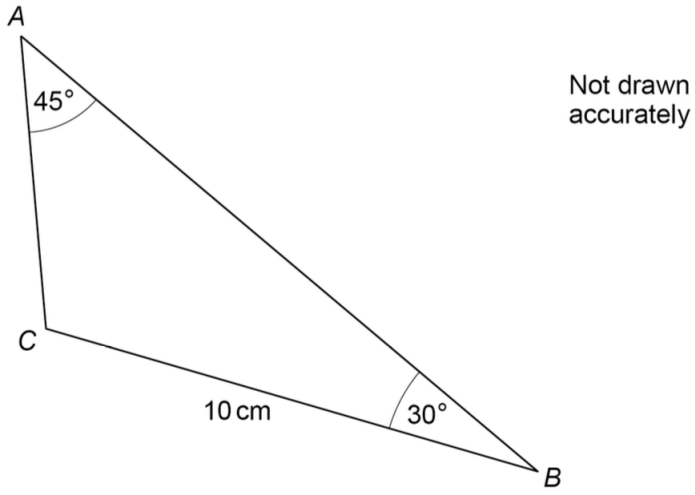
x^2 term:

$$\underline{\underline{q = -1}}.$$

x term:

$$\underline{\underline{r = -14}}.$$

16. Work out the length AC. (3)



Give your answer in the form $a\sqrt{2}$, where a is an integer.

Solution

Sine rule:

$$\frac{AC}{\sin ABC} = \frac{BC}{\sin BAC} \Rightarrow \frac{AC}{\sin 30^\circ} = \frac{10}{\sin 45^\circ}$$
$$\Rightarrow AC = \frac{10 \sin 30^\circ}{\sin 45^\circ}$$
$$\Rightarrow AC = 5\sqrt{2};$$

hence, $\underline{\underline{a = 5}}$.

17. The expansions of (3)

$$(4a + x)^4$$

and

$$(5 - 2x)^3$$

have the same coefficient of x^3.

Work out the value of a.

Solution

In $(4a + x)^4$, the coefficient of x^3 is

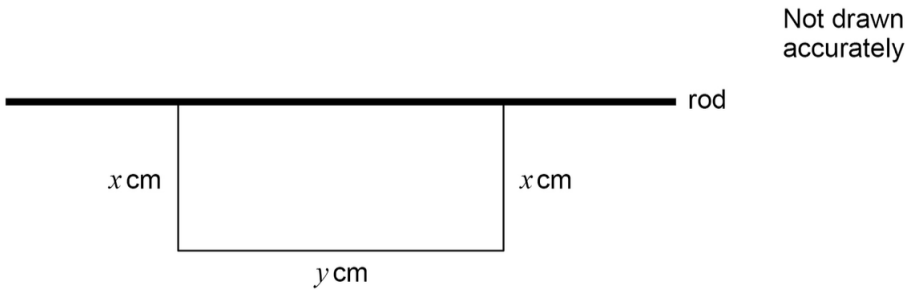
$$4(1)(4a)^1 = 16a$$

and, in $(5 - 2x)^3$, the coefficient of x^3 is -8.

Hence,

$$16a = -8 \Rightarrow \underline{\underline{a = -\tfrac{1}{2}}}.$$

18. A 60 cm piece of wire is bent to form three sides of a rectangle.
It is then attached to a fixed rod as shown in the diagram.



(a) The area of the rectangle is A cm^2. (2)

Show that

$$A = x(60 - 2x).$$

Solution
Well,

$$2x + y = 60 \Rightarrow y = 60 - 2x$$

and

$$\begin{aligned} A &= xy \\ &= \underline{\underline{x(60 - 2x)}}, \end{aligned}$$

as required.

(b) Work out the maximum value of A as x varies. (4)

Solution
Now,

$$\begin{aligned} A = x(60 - 2x) &\Rightarrow A = 60x - 2x^2 \\ &\Rightarrow A = -2[x^2 - 30x] \end{aligned}$$

add to: -30
half it: -15
square it: $(-15)^2 = +225$

$$\begin{aligned} &\Rightarrow A = -2[(x^2 - 30x + 225) - 225] \\ &\Rightarrow A = -2[(x - 15)^2 - 225] \\ &\Rightarrow A = -2(x - 15)^2 + 450; \end{aligned}$$

hence,

$$x = 15 \Rightarrow \underline{\underline{A = 450 \text{ cm}^2}}.$$

19. (a) $(x - 1)$ is a factor of (3)

$$3x^3 + 2x^2 - 3x - 2.$$

Factorise fully

$$3x^3 + 2x^2 - 3x - 2.$$

Solution
We use synthetic division:

1	3	2	−3	−2
	↓	3	5	2
	3	5	2	0

and so

$$3x^3 + 2x^2 - 3x - 2 = (x-1)(3x^2 + 5x + 2)$$

add to: $+5$
multiply to: $(+3) \times (+2) = +6$ $\Big\}$ $+2, +3$

e.g.,

$$\begin{aligned} &= (x-1)[3x^2 + 2x + 3x + 2] \\ &= (x-1)[x(3x+2) + 1(3x+2)] \\ &= \underline{\underline{(x-1)(x+1)(3x+2)}}. \end{aligned}$$

(b) (2)

$$\mathrm{f}(x) = (x-2)(5x+4)(3x+2).$$

c is a positive integer.

The value of $\mathrm{f}(c)$ can be written as the product of two prime factors.

Deduce the value of $\mathrm{f}(c)$.

Solution

The only way it could make sense is $x = 3$ (why?):

$$\begin{aligned} \mathrm{f}(3) &= (3-2)(5 \times 3 + 4)(3 \times 3 + 2) \\ &= (1)(19)(11) \\ &= \underline{\underline{209}}. \end{aligned}$$

20. (4)

$$y = \tfrac{1}{3}x^3 - 5x^2 + 28x + 11.$$

Show that y is an increasing function for all values of x.

Solution

Well,

$$y = \tfrac{1}{3}x^3 - 5x^2 + 28x + 11 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = x^2 - 10x + 28$$

we complete the square:

add to: -10
half it: -5
square it: $(-5)^2 = +25$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = (x^2 - 10x + 25) + 3$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = (x-5)^2 + 3$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} > 0.$$

Hence, y is an increasing function for all values of x.

21. Show that (2)

$$\frac{20\sin^2 x}{3 + \cos^2 x - 3\sin^2 x}$$

can be simplified to

$$k\tan^2 x,$$

where k is an integer.

Solution

Well,

$$\begin{aligned}\frac{20\sin^2 x}{3 + \cos^2 x - 3\sin^2 x} &= \frac{20\sin^2 x}{3 + \cos^2 x - 3(1 - \cos^2 x)} \\ &= \frac{20\sin^2 x}{3 + \cos^2 x - 3 + 3\cos^2 x)} \\ &= \frac{20\sin^2 x}{4\cos^2 x} \\ &= \underline{5\tan^2 x};\end{aligned}$$

so, $\underline{k = 5}$.

22. A circle has centre $(2, 3)$ and radius 5. (6)

A straight line passes through $(3, 7)$ and has a gradient of 2.

Work out the coordinates of the **two** points where the circle and the straight line intersect.

Solution

The circle is

$$(x-2)^2 + (y-3)^2 = 25 \quad (1)$$

and the straight line is

$$\begin{aligned} y - 7 = 2(x-3) &\Rightarrow y - 7 = 2x - 6 \\ &\Rightarrow y = 2x + 1 \quad (2). \end{aligned}$$

Insert (2) into (1):

$$\begin{aligned} (x-2)^2 + (y-3)^2 = 25 &\Rightarrow (x-2)^2 + (2x+1-3)^2 = 25 \\ &\Rightarrow (x-2)^2 + (2x-2)^2 = 25 \end{aligned}$$

$\times$	x	-2
x	x^2	$-2x$
-2	$-2x$	$+4$

$\times$	$2x$	-2
$2x$	$4x^2$	$-4x$
-2	$-4x$	$+4$

$$\begin{aligned} &\Rightarrow (x^2 - 4x + 4) + (4x^2 - 8x + 4) = 25 \\ &\Rightarrow 5x^2 - 12x - 17 = 0 \end{aligned}$$

add to: -12
multiply to: $(+5) \times (-17) = -85$ $\Big\}$ $+5,\ -17$

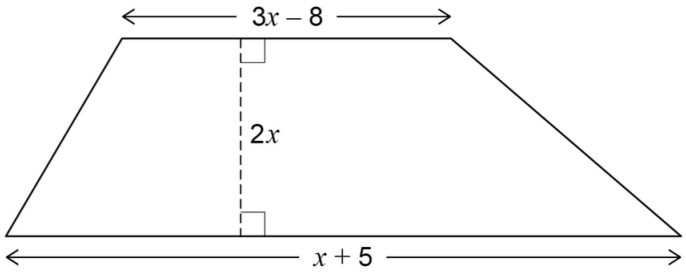
e.g.,

$$\begin{aligned}
&\Rightarrow 5x^2 + 5x - 17x - 17 = 0 \\
&\Rightarrow 5x(x+1) - 17(x+1) = 0 \\
&\Rightarrow (5x - 17)(x+1) = 0 \\
&\Rightarrow 5x - 17 = 0 \text{ or } x + 1 = 0 \\
&\Rightarrow x = 3\tfrac{2}{5} \text{ or } x = -1 \\
&\Rightarrow y = 7\tfrac{4}{5} \text{ or } y = -1;
\end{aligned}$$

hence, the coordinates are

$$\underline{\underline{(3\tfrac{2}{5}, 7\tfrac{4}{5}) \text{ and } (-1, -1)}}.$$

23. The diagram shows a trapezium. (5)
All lengths are in centimetres.



The area of the trapezium is less than 22 cm^2.

Work out the greatest possible range of values of x. Give your answer in the form

$$m < x < n,$$

where m and n are both positive numbers.

Solution

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Well,

$$\begin{aligned}\text{area} < 22 &\Rightarrow \tfrac{1}{2}(2x)[(3x-8)+(x+5)] < 22\\ &\Rightarrow x(4x-3) < 22\\ &\Rightarrow 4x^2 - 3x < 22\\ &\Rightarrow 4x^2 - 3x - 22 < 0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & -3\\ \text{multiply to:} & (+4)\times(-22) = -88\end{array}\right\} +8,\ -11$$

e.g.,

$$\begin{aligned}&\Rightarrow 4x^2 + 8x - 11x - 22 < 0\\ &\Rightarrow 4x(x+2) - 11(x+2) < 0\\ &\Rightarrow (4x-11)(x+2) < 0.\end{aligned}$$

We need a 'table of signs':

	$x < -2$	$x = -2$	$-2 < x < \frac{11}{4}$	$x = \frac{11}{4}$	$x > \frac{11}{4}$
$(x+2)$	$-$	0	$+$	$+$	$+$
$(4x-11)$	$-$	$-$	$-$	0	$+$
$(x+2)(4x-11)$	$+$	0	$-$	0	$+$

And

$$-2 < x < \tfrac{11}{4}$$

should the final answer?

Well, no. Because

$$\begin{aligned}3x - 8 > 0 &\Rightarrow 3x > 8\\ &\Rightarrow x > \tfrac{8}{3}.\end{aligned}$$

Hence,

$$\underline{\underline{\tfrac{8}{3} < x < \tfrac{11}{4}}}.$$

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