Dr Oliver Mathematics
GCSE Mathematics
2004 June Paper 5H: Non-Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. (a) Use the information that (2)

$$13 \times 17 = 221$$

to write down the value of

(i) 1.3×1.7,

Solution
Divide each term by 10 and so you divide the final answer by 100:

$$1.3 \times 1.7 = \underline{\underline{2.21}}.$$

(ii) $22.1 \div 1\,700$.

Solution
Start with

$$221 \div 17 = 13.$$

So, you divide the first term by 10 and you multiply the second term by 100: combined, you divide the answer by 1 000:

$$22.1 \div 1\,700 = \underline{\underline{0.013}}.$$

(b) Use the information that (2)

$$13 \times 17 = 221$$

to find the Lowest Common Multiple (LCM) of 39 and 17.

Solution
It is

$$39 \times 17 = 3 \times (13 \times 17) = 3 \times 221 = \underline{\underline{663}}.$$

2. The table shows some expressions. (3)
The letters a, b, c, and d represent lengths.
π and 2 are numbers that have no dimensions.
Three of the expressions could represent areas.
Tick (✓) the boxes underneath the three expressions which could represent areas.

$\frac{\pi abc}{2d}$	πa^3	$2a^2$	$\pi a^2 + b$	$\pi(a+b)$	$2(c^2+d^2)$	$2ad^2$

Solution

$\frac{\pi abc}{2d}$	πa^3	$2a^2$	$\pi a^2 + b$	$\pi(a+b)$	$2(c^2+d^2)$	$2ad^2$
✓		✓			✓	

3. The probability that a biased dice will land on a four is 0.2.
Pam is going to roll the dice 200 times.

(a) Work out an estimate for the number of times the dice will land on a four. (2)

Solution

$$0.2 \times 200 = \underline{\underline{40 \text{ times}}}.$$

The probability that the biased dice will land on a six is 0.4.
Ted rolls the biased dice once.

(b) Work out the probability that the dice will land on either a four or a six. (2)

Solution

$$0.2 + 0.4 = \underline{\underline{0.6}}.$$

4. (a) Express 108 as the product of powers of its prime factors. (3)

Solution

$$\begin{array}{r|l} & 108 \\ \hline 2 & 54 \\ \hline 2 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline 3 & 1 \\ \hline \end{array}$$

So

$$108 = 2 \times 2 \times 3 \times 3 \times 3 = \underline{\underline{2^2 \times 3^3}}.$$

(b) Find the Highest Common Factor (HCF) of 108 and 24. (1)

Solution

$$\begin{array}{r|l} & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 2 & 3 \\ \hline 3 & 1 \\ \hline \end{array}$$

So

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3.$$

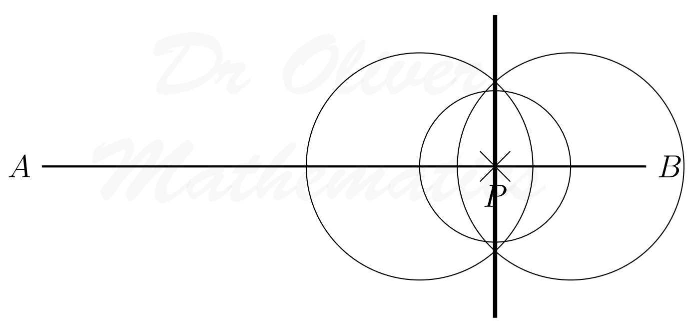
So,

$$\text{HCF}(24, 108) = 2^2 \times 3 = \underline{\underline{12}}.$$

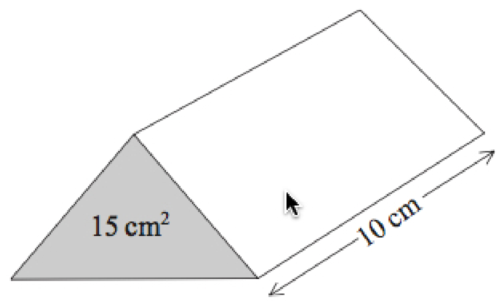
5. Use ruler and compasses to **construct** the perpendicular to the line segment AB that passes through the point P. (2)
You must show all construction lines.

Solution

First, draw a circle, centred at P.
Next, with your compasses slightly larger, draw the circle from the left and the right.
Finally, draw a straight line through the points.



6. The diagram shows a wedge in the shape of a triangular prism. (3)



The cross section of the prism is shown as a shaded triangle.
The area of the triangle is 15 cm^2.
The length of the prism is 10 cm.
Work out the volume of the prism.

Solution

$$15 \times 10 = \underline{\underline{150 \text{ cm}^3}}.$$

7. (a) Simplify $k^5 \div k^2$. (1)

Solution

$$\frac{k^5}{k^2} = \underline{\underline{k^3}}.$$

(b) Expand and simplify (4)

(i) $4(x + 5) + 3(x - 7)$,

Solution

$$\begin{aligned} 4(x + 5) + 3(x - 7) &= 4x + 20 + 3x - 21 \\ &= \underline{\underline{7x - 1}}. \end{aligned}$$

(ii) $(x + 3y)(x + 2y)$.

Solution

×	x	$+3y$
x	x^2	$+3xy$
$+2y$	$+2xy$	$+6y^2$

Hence,

$$(x+3y)(x+2y) = \underline{\underline{x^2 + 5xy + 6y^2}}.$$

(c) Factorise $(p+q)^2 + 5(p+q)$. (1)

Solution

$$\begin{aligned}(p+q)^2 + 5(p+q) &= (p+q)[(p+q)+5] \\ &= \underline{\underline{(p+q)(p+q+5)}}.\end{aligned}$$

(d) Simplify $(m^{-4})^{-2}$. (1)

Solution

$$(m^{-4})^{-2} = \underline{\underline{m^8}}.$$

(e) Simplify $2t^2 \times 3r^3t^4$. (2)

Solution

$$2t^2 \times 3r^3t^4 = \underline{\underline{6r^3t^6}}.$$

8. Each side of a regular pentagon has a length of 101 mm, correct to the nearest millimetre. (2)

(a) Write down the **least** possible length of each side.

Solution

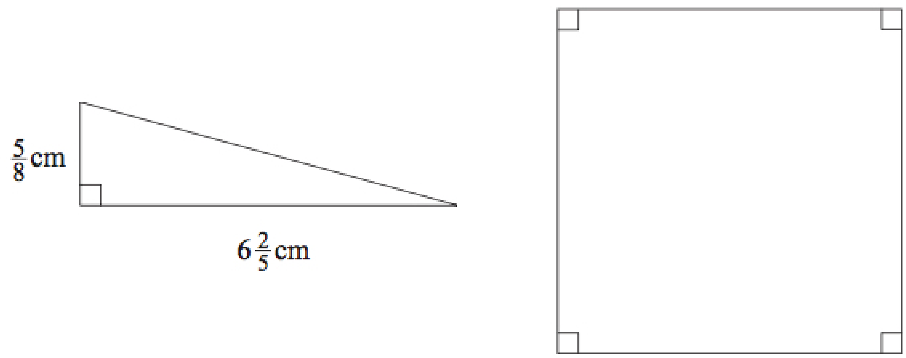
The least possible length of each side is $\underline{\underline{100.5\text{ mm}}}$.

(b) Write down the **greatest** possible length of each side.

Solution

The greatest possible length of each side is 101.5 mm.

9. The area of the square is 18 times the area of the triangle. (5)



Work out the **perimeter** of the square.

Solution

$$\begin{aligned}\text{Area of the triangle} &= \tfrac{1}{2} \times \tfrac{5}{8} \times \tfrac{32}{5} \\ &= 2 \text{ cm}^2\end{aligned}$$

and so

$$\text{area of the square} = 18 \times 2 = 36 \text{ cm}^2.$$

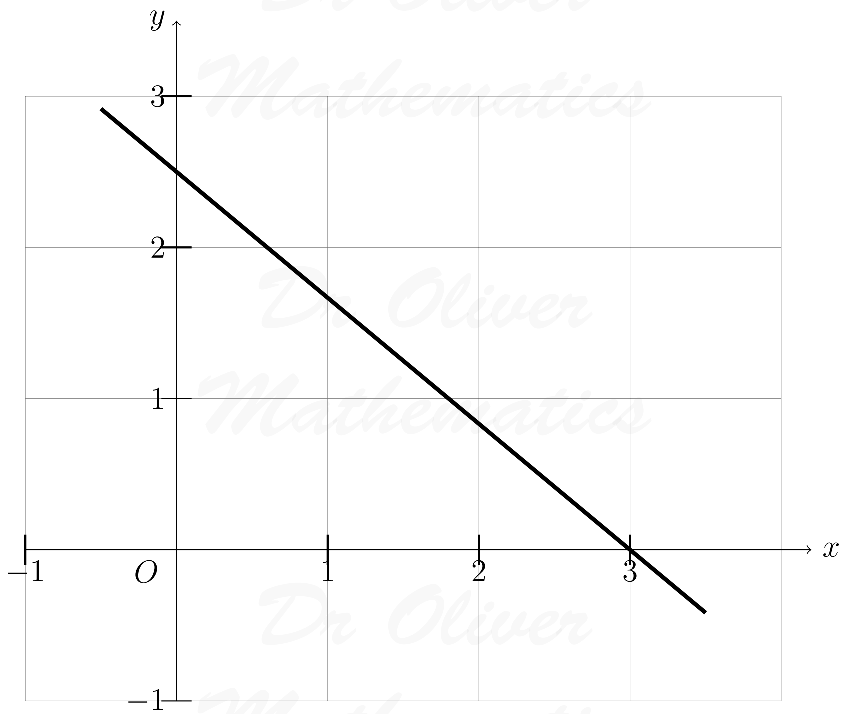
Next, one side of the square is

$$\sqrt{36} = 6 \text{ cm}$$

and so the perimeter of the square is

$$6 \times 4 = \underline{\underline{24 \text{ cm}}}.$$

10. The line with equation $6y + 5x = 15$ is drawn on the grid below.



(a) Rearrange the equation $6y + 5x = 15$ to make y the subject. (2)

Solution

$$6y + 5x = 15 \Rightarrow 6y = -5x + 15$$
$$\Rightarrow \underline{\underline{y = -\tfrac{5}{6}x + \tfrac{5}{2}}}.$$

(b) The point $(-21, k)$ lies on the line. (2)
Find the value of k.

Solution

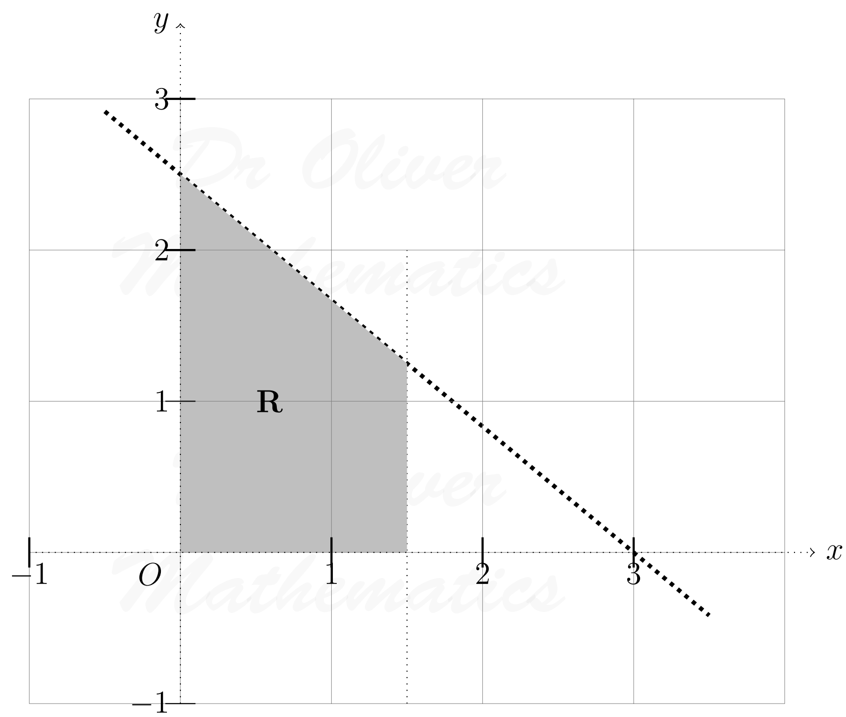
$$6y + 5 \times (-21) = 15 \Rightarrow 6y - 105 = 15$$
$$\Rightarrow 6y = 120$$
$$\Rightarrow \underline{\underline{y = 20}}.$$

(c) (i) On the grid, shade the region of points whose coordinates satisfy the four inequalities (3)

$$y > 0, \quad x > 0, \quad 2x < 3, \quad 6y + 5x < 15.$$

Label this region **R**.

Solution

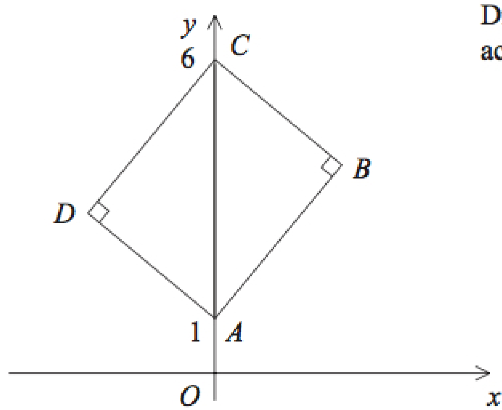


P is a point in the region **R**.
The coordinates of P are both integers.

(ii) Write down the coordinates of P.

Solution
$\underline{\underline{P(1, 1)}}$.

11. $ABCD$ is a rectangle.



A is the point $(0, 1)$.
C is the point $(0, 6)$.
The equation of the straight line through A and B is $y = 2x + 1$.

(a) Find the equation of the straight line through D and C. (2)

Solution
$\underline{\underline{y = 2x + 6}}$.

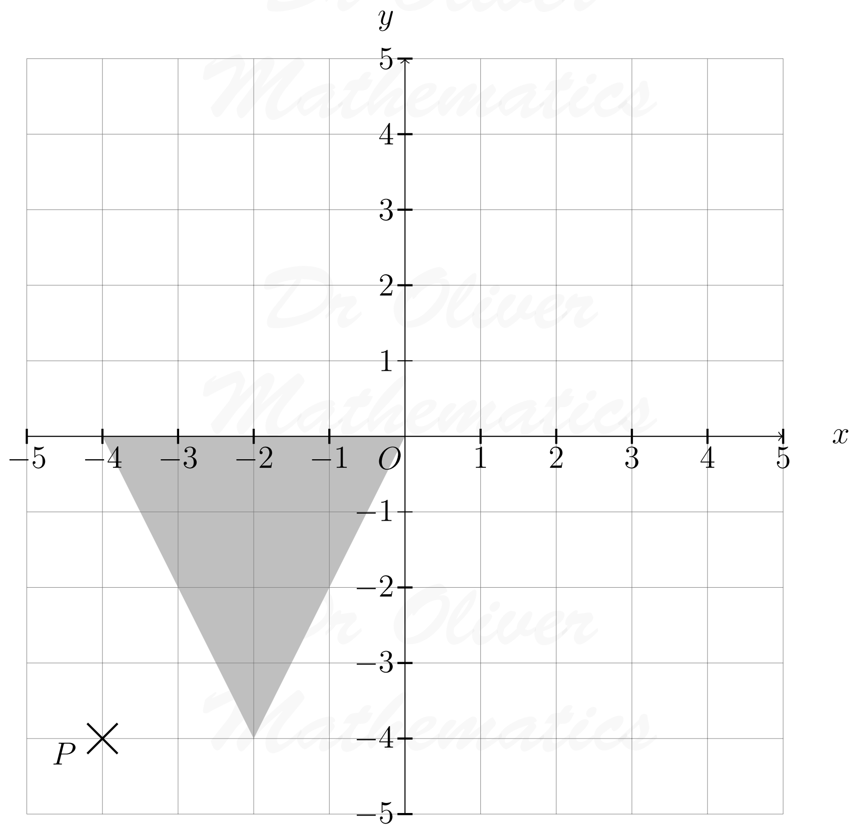
(b) Find the equation of the straight line through B and C. (2)

Solution
The gradient is
$$\frac{-1}{2} = -\tfrac{1}{2}$$
and the equation is
$$\underline{\underline{y = -\tfrac{1}{2}x + 6}}.$$

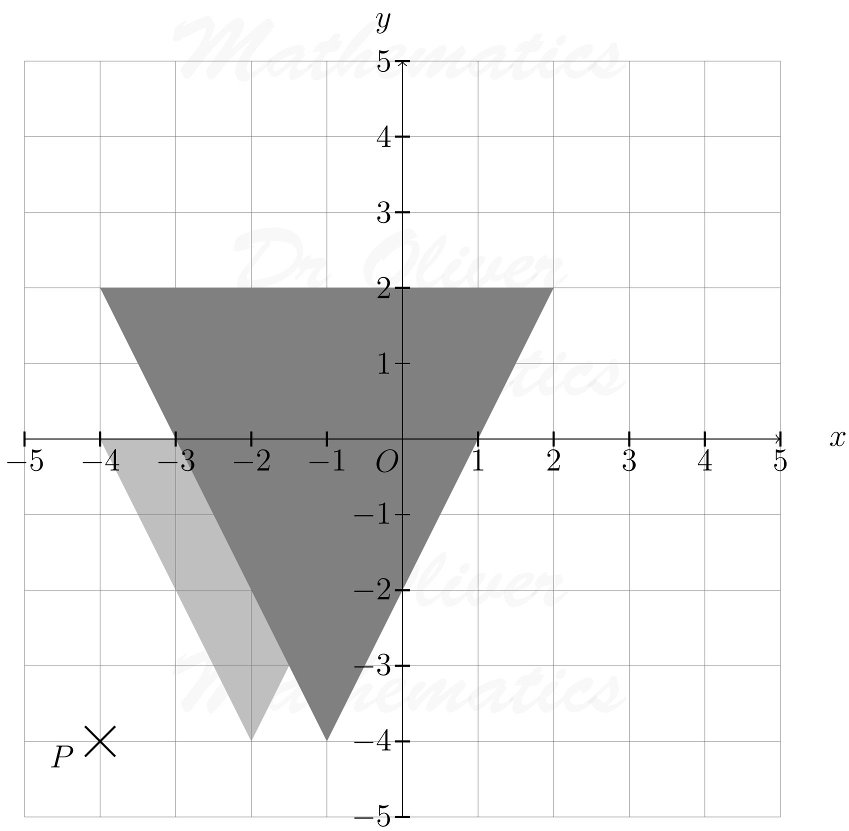
(c) It is always possible to draw a circle which passes through all four vertices of a rectangle. (1)
Explain why.

Solution
E.g., the lines from the point of intersection of diagonals of rectangle to all four vertices are equal, rectangle is a cyclic quadrilateral, etc.

12. Enlarge the shaded triangle by a scale factor $1\frac{1}{2}$, centre P. (3)



Solution

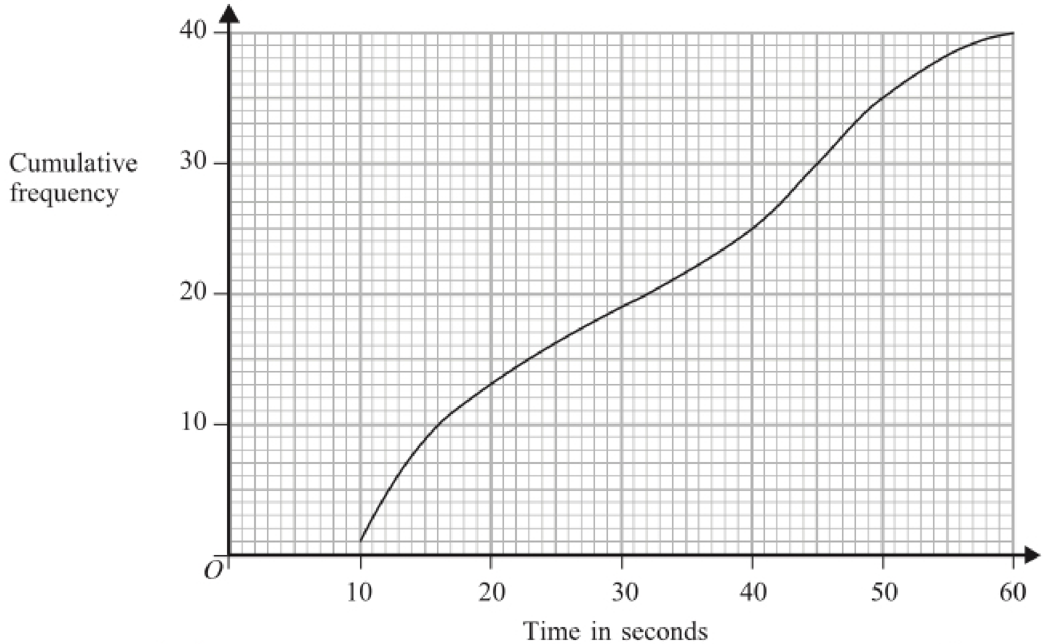


13. 40 boys each completed a puzzle.
The cumulative frequency graph opposite gives information about the times it took them to complete the puzzle.

(a) Use the graph to find an estimate for the median time. (1)

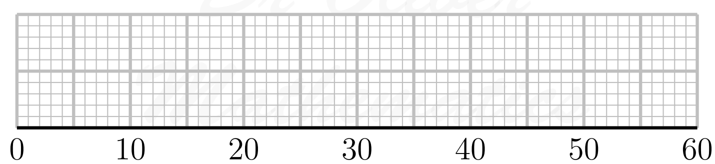
Solution

Draw a straight line from 20 until it crosses the curve: approximately 32 seconds.

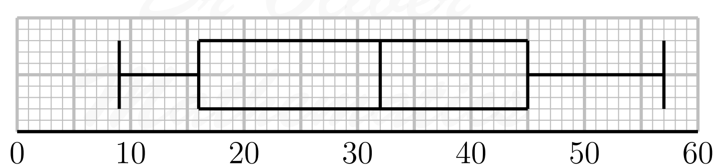


For the boys, the minimum time to complete the puzzle was 9 seconds and the maximum time to complete the puzzle was 57 seconds.

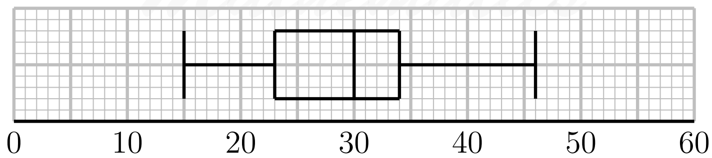
(b) Use this information and the cumulative frequency graph to draw a box plot showing information about the boys' times. (3)



Solution



The box plot below shows information about the times taken by 40 girls to complete the same puzzle.



(c) Make two comparisons between the boys' times and the girls' times. (2)

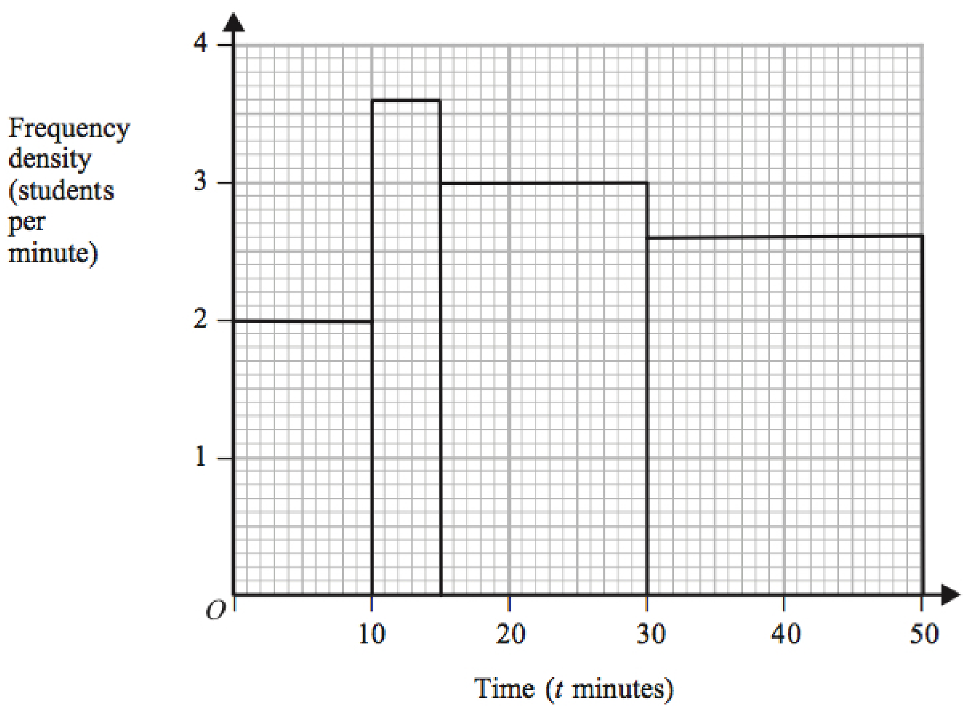
Solution

E.g., the median time for the boys' (32 s) is slower than the median time for the girls' (30 s)

EITHER the IQR is faster for the girls' ($34 - 23 = 11$ s) than the boys' ($45 - 16 = 29$ s)

OR the range is faster for the girls' ($46-15 = 31$ s) than the boys' ($57-9 = 48$ s)

14. The histogram gives information about the times, in minutes, 135 students spent on the Internet last night. (2)



Use the histogram to complete the table.

Time (t minutes)	Frequency
$0 < t \leqslant 10$	
$10 < t \leqslant 15$	
$15 < t \leqslant 30$	
$30 < t \leqslant 50$	
Total	135

Solution

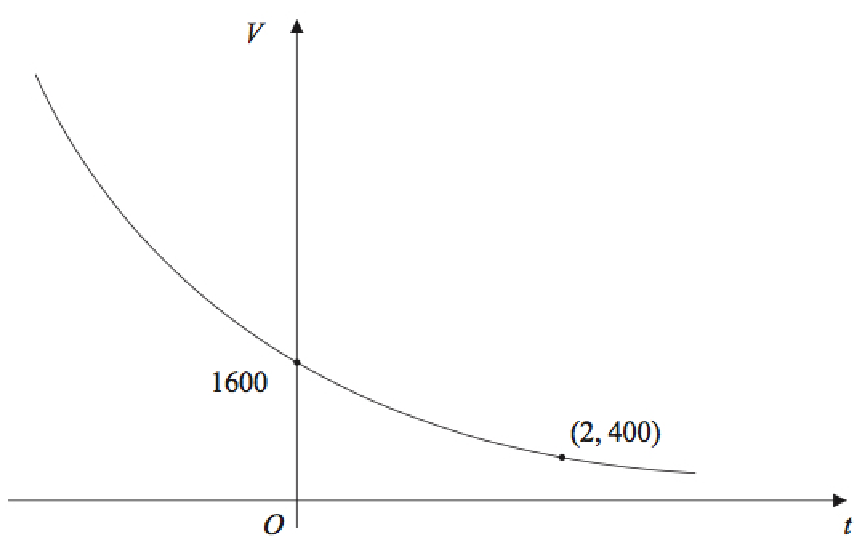
The area of the histogram is

$$\begin{aligned}(10 \times 2) + (5 \times 3.6) + (15 \times 3) + (20 \times 2.6) &= 20 + 18 + 45 + 52 \\ &= 135\end{aligned}$$

and so the complete table is

Time (t minutes)	Frequency
$0 < t \leqslant 10$	$\underline{\underline{20}}$
$10 < t \leqslant 15$	$\underline{\underline{18}}$
$15 < t \leqslant 30$	$\underline{\underline{45}}$
$30 < t \leqslant 50$	$\underline{\underline{52}}$
Total	135

15. Mr Patel has a car.



The value of the car on January 1st 2000 was £1600.
The value of the car on January 1st 2002 was £400.
The sketch graph shows how the value, £V, of the car changes with time.
The equation of the sketch graph is

$$V = pq^t,$$

where t is the number of years after January 1st 2000.
p and q are positive constants.

(a) Use the information on the graph to find the value of p and the value of q. (2)

Solution

$$t = 0 \Rightarrow \underline{\underline{p = 1\,600}}$$

and

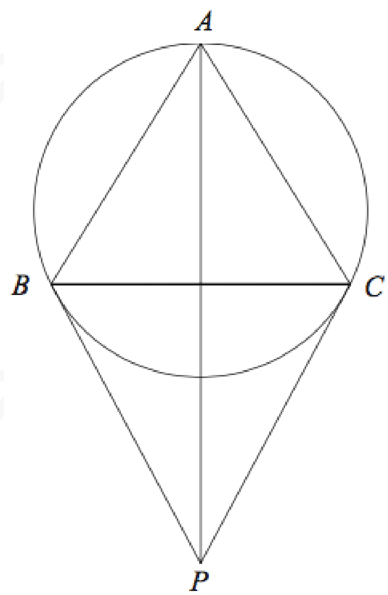
$$\begin{aligned} 400 = 1\,600q^2 &\Rightarrow q^2 = \tfrac{1}{4} \\ &\Rightarrow \underline{\underline{q = \tfrac{1}{2}}}. \end{aligned}$$

(b) Using your values of p and q in the formula $V = pq^t$ find the value of the car on January 1st 1998. (3)

Solution

$$\begin{aligned} V &= 1\,600 \times (\tfrac{1}{2})^{-2} \\ &= 1\,600 \times 4 \\ &= \underline{\underline{£6\,400}}. \end{aligned}$$

16. A, B, and C are three points on the circumference of a circle.



Angle ABC = Angle ACB.
PB and PC are tangents to the circle from the point P.

(a) Prove that triangle APB and triangle APC are congruent. (3)

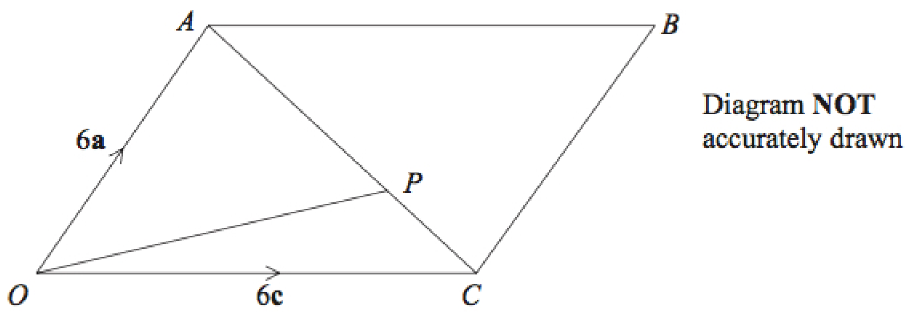
> **Solution**
> AP is shared.
> $PB = PC$ (tangents to the circle from the point P).
> $AB = AC$ (triangle ABC is isosceles).
> $\triangle APB$ and $\triangle APC$ are congruent (SSS).

Angle $BPA = 10°$.

(b) Find the size of angle ABC. (4)

> **Solution**
> $BPC = 20°$ (double the angle)
> $PBC = 90 - (\frac{1}{2} \times 20) = 80°$ (angles in a triangle)
> $BAC = 80°$ (alternating segment theorem)
> $ABC = 90 - (\frac{1}{2} \times 80) = 50°$ (angles in a triangle).

17. $OABC$ is a parallelogram.



P is the point on AC such that $AP = \frac{2}{3}AC$.
$\overrightarrow{OA} = 6\mathbf{a}$.
$\overrightarrow{OC} = 6\mathbf{c}$.

(a) Find the vector $\overrightarrow{OP}$. (3)
Give your answer in terms of **a** and **c**.

Solution

$$\begin{aligned}\overrightarrow{OP} &= \overrightarrow{OA} + \overrightarrow{AP} \\ &= \overrightarrow{OA} + \tfrac{2}{3}\overrightarrow{AC} \\ &= 6\mathbf{a} + \tfrac{2}{3}(6\mathbf{c} - 6\mathbf{a}) \\ &= 6\mathbf{a} + (4\mathbf{c} - 4\mathbf{a}) \\ &= \underline{\underline{2\mathbf{a} + 4\mathbf{c}}}.\end{aligned}$$

The midpoint of CB is M.

(b) Prove that OPM is a straight line. (2)

Solution

$$\begin{aligned}\overrightarrow{PM} &= \overrightarrow{PO} + \overrightarrow{OM} \\ &= -(2\mathbf{a} + 4\mathbf{c}) + (3\mathbf{a} + 6\mathbf{c}) \\ &= \mathbf{a} + 2\mathbf{c} \\ &= \tfrac{1}{2}(2\mathbf{a} + 4\mathbf{c}) \\ &= \tfrac{1}{2}\overrightarrow{OP};\end{aligned}$$

hence, $\underline{\underline{OPM \text{ is a straight line}}}$.

18. (a) Find the value of $16^{\frac{1}{2}}$. (1)

Solution

$$16^{\frac{1}{2}} = \underline{\underline{4}}.$$

(b) Given that $\sqrt{40} = k\sqrt{10}$, find the value of k. (1)

Solution

$$\sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10};$$

hence, $\underline{\underline{k = 2}}$.

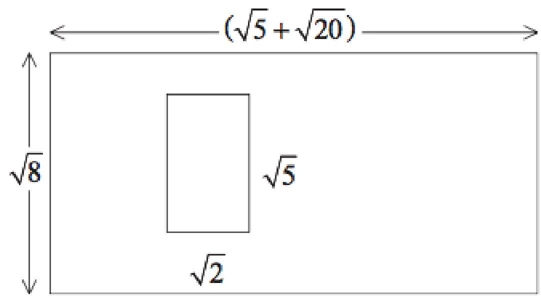


Diagram **NOT** accurately drawn

A large rectangular piece of card is $(\sqrt{5} + \sqrt{20})$ cm long and $\sqrt{8}$ cm wide.
A small rectangle $\sqrt{2}$ cm long and $\sqrt{5}$ cm wide is cut out of the piece of card.

(c) Express the area of the card that is left as a percentage of the area of the large rectangle. (4)

Solution

The large rectangular piece of card is

$$\begin{aligned}\sqrt{5} + \sqrt{20} &= \sqrt{5} + 2\sqrt{5} \\ &= 3\sqrt{5}\end{aligned}$$

and its width is

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2};$$

hence, its area is

$$3\sqrt{5} \times 2\sqrt{2} = 6\sqrt{10}.$$

The small rectangle is

$$\sqrt{5} \times \sqrt{2} = \sqrt{10}.$$

Finally, the area of the card that is left as a percentage of the area of the large rectangle is

$$\begin{aligned}\frac{6\sqrt{10}-\sqrt{10}}{6\sqrt{10}} \times 100\% &= \frac{5\sqrt{10}}{6\sqrt{10}} \times 100\% \\ &= \tfrac{5}{6} \times 100\% \\ &= \underline{\underline{83\tfrac{1}{3}\%}}.\end{aligned}$$

19. (a) (i) Factorise $2x^2 - 35x + 98$. (3)

Solution

$$\left.\begin{array}{ll}\text{add to:} & -35 \\ \text{multiply to:} & (+2)\times(+98) = 196\end{array}\right\} -7,\ -28$$

Finally,

$$\begin{aligned}2x^2 - 35x + 98 &= 2x^2 - 7x - 28x + 98 \\ &= x(2x-7) - 14(2x-7) \\ &= \underline{\underline{(x-14)(2x-7)}}.\end{aligned}$$

(ii) Solve the equation $2x^2 - 35x + 98 = 0$.

Solution

$$\begin{aligned}2x^2 - 35x + 98 = 0 &\Rightarrow (x-14)(2x-7) = 0 \\ &\Rightarrow x - 14 = 0 \text{ or } 2x - 7 = 0 \\ &\Rightarrow \underline{\underline{x = 14 \text{ or } x = 3\tfrac{1}{2}}}.\end{aligned}$$

A bag contains $(n + 7)$ tennis balls.
n of the balls are yellow.
The other 7 balls are white.
John will take at random a ball from the bag.
He will look at its colour and then put it back in the bag.

(b) (i) Write down an expression, in terms of n, for the probability that John will take a white ball. (3)

Solution

The probability that John will take a white ball is $\underline{\underline{\frac{7}{n+7}}}$.

Bill states that the probability that John will take a white ball is $\frac{2}{5}$.

(ii) Prove that Bill's statement cannot be correct.

Solution

$$\begin{aligned}\frac{7}{n+7} = \frac{2}{5} &\Rightarrow 35 = 2(n+7)\\ &\Rightarrow 17\tfrac{1}{2} = n + 7\\ &\Rightarrow n = 10\tfrac{1}{2};\end{aligned}$$

hence, it is not an integer and <u>Bill's statement cannot be correct</u>.

After John has put the ball back into the bag, Mary will then take at random a ball from the bag.
She will note its colour.

(c) Given that the probability that John and Mary will take balls with different colours is $\frac{4}{9}$, prove that (5)

$$2n^2 - 35n + 98 = 0.$$

Solution

$$\begin{aligned}2 \times \frac{7}{n+7} \times \frac{n}{n+7} = \frac{4}{9} &\Rightarrow 18 \times 7 \times n = 4(n+7)^2\\ &\Rightarrow 126n = 4(n^2 + 14n + 49)\\ &\Rightarrow 126n = 4n^2 + 56n + 196\\ &\Rightarrow 4n^2 - 70n + 196 = 0\\ &\Rightarrow \underline{\underline{2n^2 - 35n + 98 = 0}},\end{aligned}$$

as required.

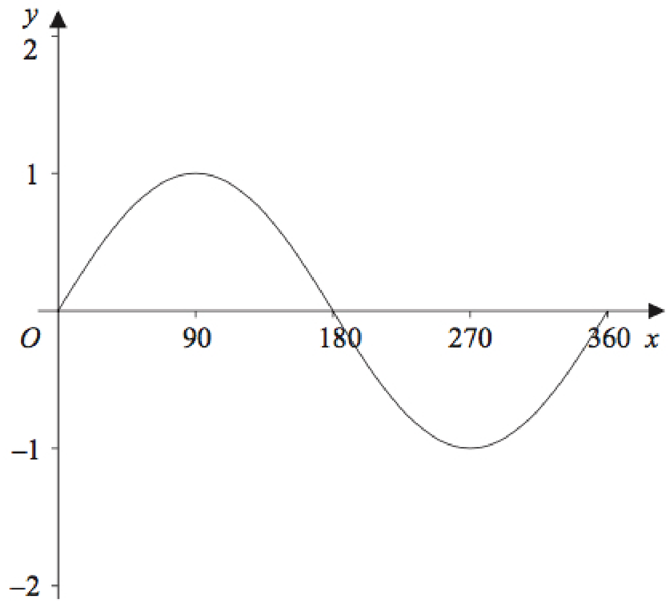
(d) Using your answer to part (a) (ii) or otherwise, calculate the probability that John and Mary will both take white balls. (2)

Solution

There are $7 + 14 = 21$ tennis balls and

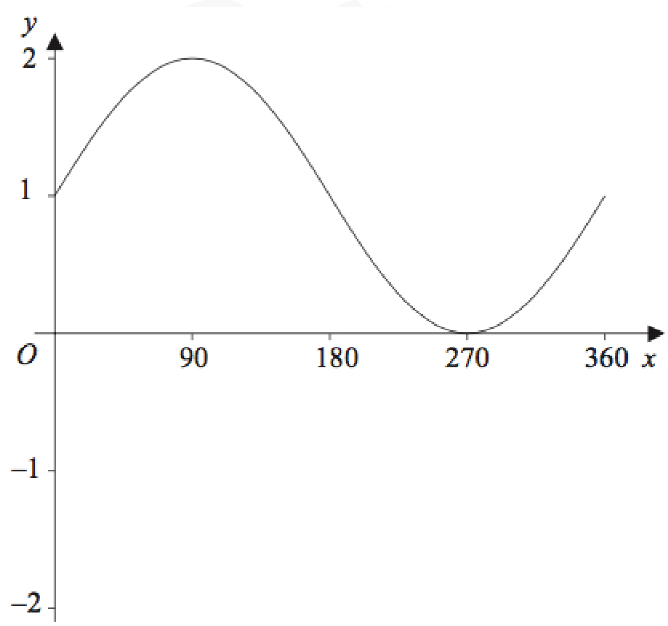
$$\frac{7}{21} \times \frac{7}{21} = \underline{\underline{\frac{1}{9}}}.$$

20. A sketch of the curve $y = \sin x^\circ$ for $0 \leqslant x \leqslant 360$ is shown below.



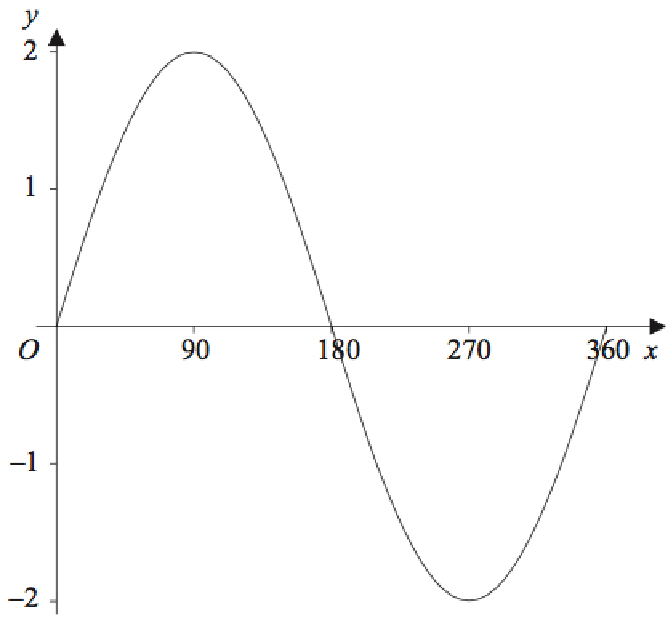
(a) Using the sketch above, or otherwise, find the equation of each of the following two curves. (2)

(i) Here is the first.



Solution
$\underline{\underline{y = 1 + \sin x^\circ}}$.

(ii) Here is the second.



Solution
$\underline{\underline{y = 2\sin x^\circ}}$.

(b) Describe fully the sequence of two transformations that maps the graph of $y = \sin x^\circ$ onto the graph of $y = 3\sin 2x^\circ$. (3)

Solution
(1): $\underline{y = \sin x^\circ \to y = \sin 2x^\circ}$: horizontally stretch by a factor of $\frac{1}{2}$, and
(2): $\underline{y = \sin 2x^\circ \to y = 3\sin 2x^\circ}$: vertically stretch by a factor of 3.